1. **Kronecker delta.** Evaluate the following.

   (a) $\delta_{02}$
   (b) $\sum_{i=1}^{\infty} \delta_{ik} a_i$
   (c) $\sum_{i=1}^{\infty} \sum_{j=-\infty}^{\infty} \delta_{ik} \delta_{jl} (-1)^j b_{ij}$
   (d) $\sum_{i=2}^{4} \sum_{j=-1}^{1} \delta_{ij}$
   (e) $\sum_{i=2}^{4} \sum_{j=2}^{3} \delta_{ij}$
   (f) $\sum_{i=2}^{4} \sum_{j=2}^{3} \delta_{ij} (i + j)^2$

2. Let $V$ be the vector space of all polynomials in $x$ of degree 2 with real coefficients, with an inner product defined by

   $$\langle p(x) | q(x) \rangle = \int_0^1 p(x) q(x) \, dx.$$ 

   (a) What is the dimension of $V$?
   (b) Is there a real value of $c$ such that the polynomials $x^2 - cx$ and $x$ are orthogonal?

3. Consider the following three vectors in $\mathbb{R}^3$:

   $$v_1 = (1, 0, 1), \quad v_2 = (1, \sqrt{6}, -1), \quad v_3 = (\sqrt{3}, -\sqrt{2}, -\sqrt{3}).$$

   (a) Show that $\{v_1, v_2, v_3\}$ is an orthogonal set with respect to the usual dot product.
   (b) Because the number of vectors in the set above is equal to the dimension of the vector space, they form a basis. Expand the vector $v = (1, 2, 1)$ in terms of this basis: find real coefficients $a_1, a_2, a_3$ such that

   $$v = a_1 v_1 + a_2 v_2 + a_3 v_3.$$ 

4. Define the following operation on $\mathbb{R}^2$:

   $$\langle (x_1, x_2) | (y_1, y_2) \rangle = 2x_1 y_1 + x_1 y_2 + x_2 y_1 + 2x_2 y_2$$

   (a) Show that this operation is symmetric, bilinear, and positive definite. Therefore it defines an inner product on $\mathbb{R}^2$.
   (b) With this definition, compute the inner products of the following pairs of vectors:
      • $(1, 0)$ and $(0, 1)$
      • $(1, 0)$ and $(-2, 1)$
      • $(0, 1)$ and $(-2, 1)$
   (c) With respect to this same inner product, write an orthonormal set with 2 elements.