School of Mathematics

MA3429 — Differential Geometry 2011-12
(SS Theoretical Physics
JS & SS Mathematics )

Lecturer: Prof. P. Taylor

Requirements/prerequisites:

Duration: Michaelmas Term, 11 weeks

Number of lectures per week: 3 including tutorials

Assessment:

ECTS credits: 5

End-of-year Examination: This module will be examined jointly with MA4448 in a 3-
hour examination in Trinity term, except that those taking just one of the two modules
will have a 2 hour examination. However there will be separate results for MA3429 and
MA4448.

Description:

Textbooks:

Learning Outcomes: On successful completion of this module, students will be able to:

- Obtain a coordinate-induced basis for the tangent space and cotangent space at points of
  a differentiable manifold, construct a coordinate induced basis for arbitrary tensors and
  obtain the components of tensors in this basis.

- Determine whether a particular map is a tensor by either checking multi-linearity or by
  showing that the components transform according to the tensor transformation law.

- Construct manifestly chart-free definitions of the Lie derivative of a function and a vector,
  to compute these derivatives in a particular chart and hence compute the Lie derivative
  of an arbitrary tensor.

- Compute, explicitly, the covariant derivative of an arbitrary tensor.

- Define parallel transport, derive the geodesic equation and solve problems involving par-
  allel transport of tensors.

- Obtain an expression for the Riemann curvature tensor in an arbitrary basis for a manifold
  with vanishing torsion, provide a geometric interpretation of what this tensor measures,
  derive various symmetries and results involving the curvature tensor.

- Define the metric, the Levi-Civita connection and the metric curvature tensor and com-
  pute the components of each of these tensors given a particular line-element.
• Re-derive the geodesic equation from an action principle and compute null, timelike, or spacelike geodesics on a particular space-time.

• Derive the Einstein equations or equations for similar metric theories from an action principle.