School of Mathematics

Course 442 - Differential Geometry and General Relativity 2008-09
(SS Theoretical Physics, JS & SS Mathematics )

Lecturer: Dr. Calin Lazariou

Requirements/prerequisites: Analytical mechanics (241) and Classical Electrodynamics (432); Differential Geometry (manifolds, vector and tensor fields, differential forms and vector bundles)

Duration: 19 weeks.

Number of lectures per week: 3

Assessment:

End-of-year Examination: One 3-hour examination

Description: The course is an introduction to general relativity. See also http://www.maths.tcd.ie/~calin/teaching/442.html

Course content:

Differential Geometry

• basic theory of abstract manifolds (chart, atlas, differentiable structure, tangent space, smooth map, differential of a smooth map at a point)

• algebra of smooth real-valued functions on a manifold; vector fields on a manifold; Lie bracket of vector fields

• basic theory of vector bundles; sections of vector bundles; the tangent bundle; the differential of a map; natural operations on vector bundles (direct sum, tensor product, dual, symmetric and antisymmetric product)

• reminder on modules over commutative rings and their basic operations

• the module of sections of a vector bundle; its behavior under the natural operations on bundles

• natural vector bundles on a manifold; tensor fields, polyvector fields and differential forms on manifolds; symmetric and antisymmetric tensor fields

• connections in a vector bundle; curvature of a connection; induced connections

• pull-back of a vector bundle; pull-back connection

• parallel transport of a connection

• affine connections and their torsion tensor; covariant derivation of tensor fields; autoparallel curves with respect to an affine connection
• pseudo-Riemannian manifolds; signature; Riemannian, Minkowskian and Minkowsky manifolds.

• the Levi-Civita connection, geodesics, variational principle for geodesics.

• Riemann curvature tensor, Ricci tensor and Einstein tensor.

• orientability, volume form determined by a metric

• natural differential operators on a pseudo-Riemannian manifold

**General Relativity**

• Minkowsky manifolds, light cones and time orientability; space-times; timelike, lightlike and spacelike geodesics;

• observers, proper time

• The axioms of general relativity; action principle for gravity coupled to mater.

• Matter stress-energy tensor, Einstein equations, positivity and causality conditions

• matter models: free falling particle, dust, real scalar field, electromagnetic field

• the weak field limit, recovering Newtonian gravity, gravitational waves

• gravitational red shift

• basic special solutions of Einstein’s equations (Schwarzschild, Robertson-Walker)

• motion in a central field; applications to gravitational lensing and precession of perihelia

Advanced topics (chosen at the interests of the class and as time allows): black hole physics; Kerr, Reisner-Nordstrom solutions; basic cosmology.

**Core textbook:**

March 13, 2009