School of Mathematics

Course 111 — Algebra
(JF Mathematics, Theoretical Physics & Two-subject Moderatorship )

Lecturer: Dr. C. Ó Dúnlaing
Requirements/prerequisites: None
Duration: 24 weeks

Number of lectures per week: 2 and 1 tutorial

Assessment: Continuous assessment through twenty homeworks and three short examinations spread over the academic year.

End-of-year Examination: No annual examination. Those who fail through continuous assessment must take a supplemental examination (in September)

Description:

Number theory: The natural number system and Peano’s axioms
The integers, divisibility, and congruence modulo n
Remainder modulo n and integer division

Groups: Semigroups, monoids, and groups
Groups
Additive subgroups of \( \mathbb{Z} \)
The symmetric group \( S_n \)
Generators for \( S_n \)
Parity and the alternating group
Binary relations, equivalence relations, and partitions
Cosets, Lagrange’s Theorem, and Fermat’s Theorem
Normal subgroups and quotient groups
Greatest common divisor
Multiplicative group \( \mathbb{Z}_n^* \)
First isomorphism theorem for groups
Prime factorisation theorem
A Sylow theorem

Rings and fields: Rings
Zero divisors, integral domains, and fields.
Ring homomorphisms
Characteristic of a ring
Polynomials
Division algorithm for polynomials over a field
Factorising polynomials
Gauss’s Lemma and Eisenstein’s Criterion
Ring homomorphisms and ideals
Principal ideal domains

Solving equations: Dimension of extension fields
Ruler-and-compass constructions
Cubic equations
The Galois group of an extension field
Normal extensions, stable intermediate fields, and splitting fields.
Certain standardised radical extensions have solvable group; radical splitting fields have solvable group.
A polynomial equation not solvable by radicals

A last result: Finite multiplicative subgroups of a field

Textbooks: John R. Durbin, Modern algebra – an introduction, contains some but not all of the material.