

Heavy hadrons on an anisotropic lattice

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Results from simulations of quarkonia and heavy-light mesons on an anisotropic lattice are presented. The improved quark action and action-parameter tuning used in this study are discussed.

1. DESIGNING A 3+1 ACTION

A fermion action, which is useful for simulations at extreme anisotropies and has discretisation errors of $\mathcal{O}(\alpha_s a, a_s^3, a_t^2)$, is constructed. The details of this action are in Refs [3,4]. Our starting point is the usual isotropic rotation, used in the Sheikholeslami-Wohlert (SW) and D234 actions

$$\begin{aligned}\psi &= [1 - \frac{ra}{4}(\not{D} - m)]\psi', \\ \bar{\psi} &= \bar{\psi}'[1 - \frac{ra}{4}(\not{D} - m)].\end{aligned}\quad (1)$$

For a fixed Wilson parameter, r (usually equal to one) doublers may reappear as a_t is made small, which hinders simulations with $a_t \ll a_s$. The solution proposed here is to treat the temporal and spatial directions differently. The temporal rotations are retained since such one-hop improvement terms ensure the transfer matrix is positive definite and no “ghost” states can propagate. Thus Eqs. 1 are modified to read

$$\begin{aligned}\psi &= [1 - \frac{ra_t}{4}(\gamma_0 D_0 - m)]\psi', \\ \bar{\psi} &= \bar{\psi}'[1 - \frac{ra_t}{4}(\gamma_0 D_0 - m)]\end{aligned}\quad (2)$$

and once the covariant temporal derivative is discretised the temporal doublers are removed. The spatial doublers remain and are removed through the addition of an (irrelevant) higher-dimensional operator, a D_i^4 term, to the Dirac operator. The resulting improved action can then be written

$$S' = \bar{\psi}' M_r \psi' - \frac{ra_t}{2} \bar{\psi}' \left(D_t^2 - \frac{g}{2} \epsilon_i E_i \right) \psi'$$

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$$+ sa_s^3 \bar{\psi}' \sum_i D_i^4 \psi', \quad (3)$$

where $M_r = \mu_r \gamma_i D_i + \gamma_0 D_t + \mu_r m$ and $\mu_r = (1 + \frac{1}{2} r a_t m)$. The Wilson-like parameter, s is chosen such that the doublers receive a sufficiently large mass. A lattice discretisation scheme is now quite simple. An improved discretisation is only needed for the $\gamma_i D_i$ term in Eq. 3 since the simplest discretisation would lead to $\mathcal{O}(a_s^2)$ errors. Including the gauge fields and the mean-link improvement coefficients, u_s and u_t the lattice fermion matrix reads

$$\begin{aligned}M_L \psi_x &= \frac{1}{a_t} \left\{ \left(\mu_r m a_t + \frac{18s}{\xi} + r + \frac{r a_t^2 g}{4} \epsilon_i E_i \right) \psi_x \right. \\ &- \frac{1}{2u_t} \left[(r - \gamma_0) U_t(x) \psi_{x+\hat{t}} + (r + \gamma_0) U_t^\dagger(x - \hat{t}) \psi_{x-\hat{t}} \right] \\ &- \frac{1}{\xi} \sum_i \left[\frac{1}{u_s} \left(4s - \frac{2}{3} \mu_r \gamma_i \right) U_i(x) \psi_{x+\hat{i}} \right. \\ &+ \frac{1}{u_s} \left(4s + \frac{2}{3} \mu_r \gamma_i \right) U_i^\dagger(x - \hat{i}) \psi_{x-\hat{i}} \\ &- \frac{1}{u_s^2} \left(s - \frac{1}{12} \mu_r \gamma_i \right) U_i(x) U_i(x + \hat{i}) \psi_{x+2\hat{i}} \\ &\left. \left. - \frac{1}{u_s^2} \left(s + \frac{1}{12} \mu_r \gamma_i \right) U_i^\dagger(x - \hat{i}) U_i^\dagger(x - 2\hat{i}) \psi_{x-2\hat{i}} \right] \right\}.\end{aligned}\quad (4)$$

Choosing $r = 1$ and $s = 1/8$ yields the sD34 action proposed in Ref [1]. The authors investigated the radiative corrections to this action and at one-loop in perturbation theory find no contribution from $\mathcal{O}(\alpha_s a_s m_q)$ terms, which would spoil simulations with heavy quarks. This is a key advantage of this action over anisotropic SW- or D234-type actions where such terms may appear, depending on the choice of the parameter, r [2].

2. RESULTS

Our quenched simulations use an improved gluon action designed for precision glueball simulations on anisotropic lattices [5]. We omit the temporal rotations in this exploratory study, leaving a leading discretisation error of $\mathcal{O}(a_t)$. Further details of the simulation and parameter values are in Table 1. A range of masses is investi-

# gauge configs.	100
Volume	$10^3 \times 120$
a_s	0.21fm
a_s/r_0	0.432
$\xi = a_s/a_t$	6 [†]

Table 1

Lattice details. [†]The gauge anisotropy was tuned nonperturbatively, from the static quark potential.

gated from $a_t m_q = -0.04$,² close to the strange quark mass, to $a_t m_q = 1.5$. Data were accumulated at spatial momenta $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$ and $(1, 1, 1)$ in units of $2\pi/a_s L$, averaging over equivalent momenta. Degenerate and nondegenerate combinations of quark propagators are considered. The nondegenerate combination is made with the lightest quark, and each of the heavier quarks. These mesons are denoted heavy-light while the degenerate combination is heavy-heavy. Effective masses for both pseudoscalar (PS) and vector (V) particles were determined from single cosh fits. For all masses and momenta good fits to long plateaux are observed. Fits to the lightest and heaviest PS masses are shown in Fig. 1.

2.1. Mass dependence of ξ

Quantum fluctuations renormalise the anisotropy so the input parameter, ξ in the action is not the same as the anisotropy determined from a physical observable in the simulation. The parameter in the action must be “tuned” such that the measured anisotropy takes its required value. The aim of this study was to investigate the mass dependence of the renormalised anisotropy, examining the extent to which the tuning must

²This corresponds to a positive quark mass since Wilson-type actions have an additive mass renormalisation.

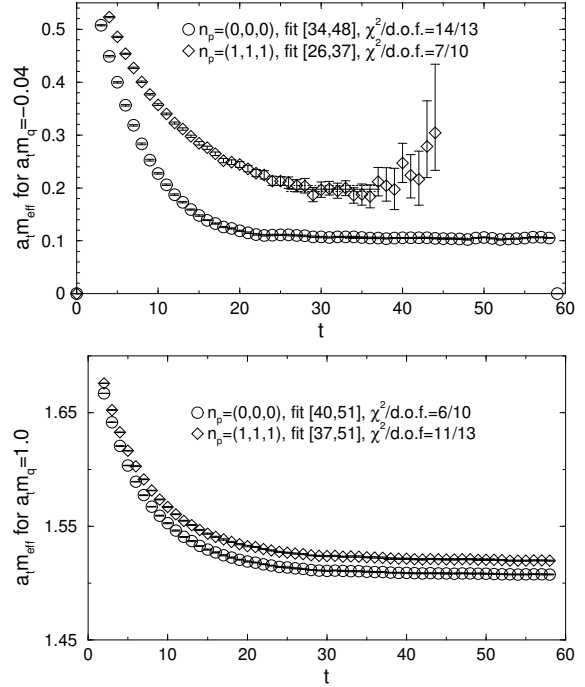


Figure 1. Effective mass plots $a_t m_q = -0.04, 1.0$ respectively. For $a_t m_q = -0.04$, $n_p = (1, 1, 1)$ the signal becomes very noisy at $t \geq 50$.

be repeated as $a_t m_q$ is varied. Both the precision of the determination and its deviation from the required anisotropy are of interest. We also examine the difference between the anisotropy determined from PS and V mesons.

ξ was tuned at the lightest mass by measuring the slope of the energy-momentum dispersion relation. A value of six from the dispersion relation requires $\xi = 6.17$ in the action. The anisotropy was then computed, for a range of masses from their dispersion relations. Fig. 2 is a representative sample of the dispersion relations for PS mesons. For clarity, $E^2(p) - E^2(0)$ is plotted as a function of $(a_s p)^2$. In general, very good linear (relativistic) behaviour is observed. The mass dependence of the measured anisotropy is shown in Fig. 3 for PS mesons. Similar mass-dependence, although with larger statistical errors, is seen for the vectors. The plots show that up to $a_t m_q \sim 0.3$ the heavy-heavy and heavy-light determinations are in agreement. In addition the values are con-

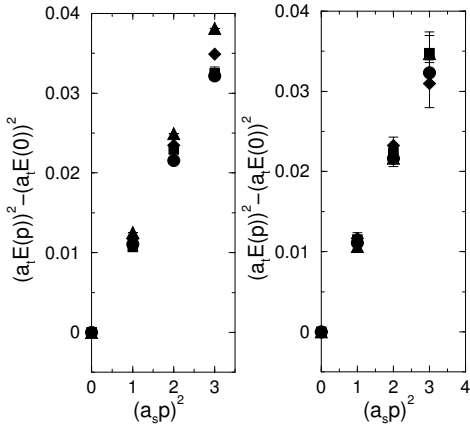


Figure 2. Dispersion relations for heavy-heavy (left) and heavy-light (right) PS mesons. The symbols \circ , \square , \diamond , \triangle are $a_t m_q = 0.1, 0.2, 0.5, 1.0$ respectively.

sistent, within errors, with the target anisotropy and are determined to $\approx 3\%$ or better accuracy. The charm quark mass on this lattice is close to $a_t m_q = 0.2$, indicating that charm physics is feasible computationally and requires little parameter tuning.

For $a_t m_q > 0.5$, the heavy-heavy and heavy-light dispersion relations do not give a consistent value of the anisotropy. This dependence of the measured anisotropy on the input parameter ξ in the action was studied at fixed quark mass. The results, for both PS and V particles are shown in Fig 4. The value of ξ , determined from the heavy-heavy dispersion relation, moves closer to its target value of six while ξ determined from the heavy-light physics moves away from this value. It is also interesting to note the agreement between determinations of ξ from pseudoscalar and vector particles. The tuning, described above was carried out for pions and it is reassuring that although the vector particles have larger statistical errors they nevertheless yield a consistent pattern for the mass dependence of ξ .

3. FUTURE WORK

A paper with more details and additional quark masses is in preparation [3]. We plan to investigate the feasibility of precision b -physics using the anisotropic lattice described here.

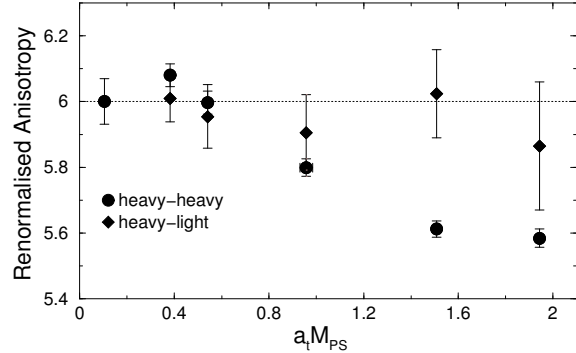


Figure 3. The mass dependence of the anisotropy for PS mesons as a function of the heavy-heavy meson mass in lattice units.

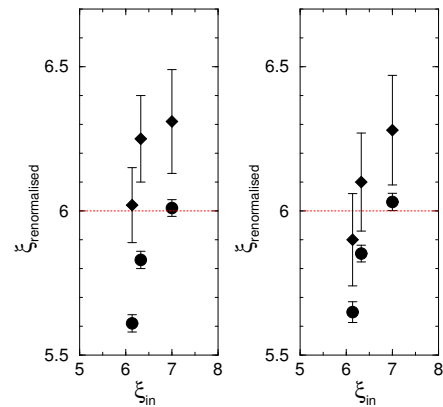


Figure 4. The dependence of the renormalised anisotropy on the input parameter ξ , for PS (left) and V mesons (right). \circ and \diamond are heavy-heavy and heavy-light respectively.

REFERENCES

1. S. Hashimoto and M. Okamoto, Phys. Rev. D67 (2003) 114503.
2. J. Harada *et al*, Phys. Rev. D64 (2001) 074501.
3. J. Foley, A. Ó Cais, M. Peardon and S.M. Ryan, in preparation.
4. M. Peardon, Nucl. Phys. Proc. Suppl. 109A (2002) 212-217.
5. C. Morningstar and M. Peardon, Nucl. Phys. Proc. Suppl. 83 (2000) 887-889.