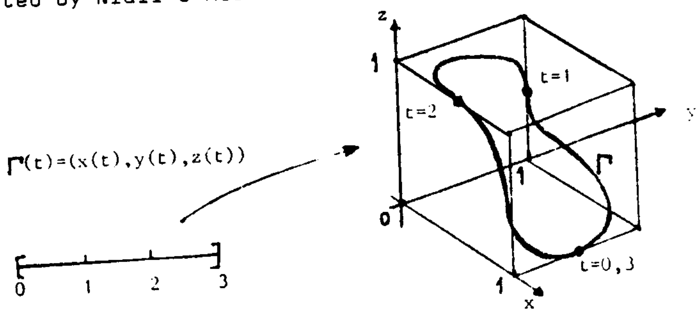
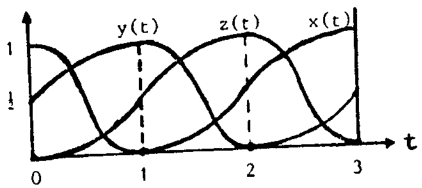
PROBLEMS

First, the solutions to the March problems.

1. Does there exist a smooth closed curve Γ in $\mathbb{R}^3$ such that no pair of tangent vectors to Γ are parallel but of opposite sense?

The answer is 'yes' as shown by the following curve, which was suggested by Niall O'Murchu:





If t_1, $t_2 \in [0,3]$ then at least one of the co-ordinate functions (say x) has the property that $x'(t_1)$, $x'(t_2)$ do not both vanish or have opposite signs. The tangent vectors at $\Gamma(t_1)$, $\Gamma(t_2)$ cannot then be 'parallel but of opposite sense'.

2. With the aid of a battery and a bell, how should an electrician identify n (>2) indistinguishable wires which run from the bottom to the top of a building? Only one visit to the top is allowed.

Here is one solution - there are others, based on the use of triangular numbers.

If $n = 2k + 1$, $k > 0$, connect the bottom ends in k pairs and label the singleton a. At the top the wires can then be labelled

$$\{a\},\ \{b_1,c_1\},\dots\dots,\ \{b_k,c_k\},$$

and reconnected in the form

$$\{a,b_1\},\ \{c_1,b_2\},\dots\dots,\ \{c_{k-1},b_k\},\ \{c_k\}.$$

At the bottom b_1 can now be labelled (it is connected to a at the top) and also, therefore, c_1. Knowing c_1 leads to b_2 etc.

If $n = 2k + 2$, $k > 0$, connect the bottom ends in k pairs leaving two unlabelled singletons. At the top the wires can be labelled

$$\{a_1\},\ \{a_2\},\ \{b_1.c_1\},\ \dots\ \{b_k,c_k\}.$$

and reconnected in the form

$$\{a_1\},\ \{a_2,b_1\},\ \{c_1,b_2\},\dots\dots,\ \{c_{k-1},b_k\},\ \{c_k\}.$$

At the bottom a_2 can now be labelled (it is connected to b_1 at the top) and also, therefore, a_1, b_1 and c_1. Knowing c_1 leads to b_2 etc.

It is intriguing that the problem can be solved in all cases except n=2, though even this can be done if the electrician has another piece of wire. If he also has a supply of resistances and an ammeter then he can solve the problem without coming back down again!

Now, some more problems.

1. The following problem was suggested by Pat Fitzpatrick who heard it from Peter Cameron. It appeared a few years ago on a problem page of another journal, but is so pretty that it's well worth repeating.

A car park has spaces numbered 1,2,...,n. One morning n drivers arrive in turn, each wishing to park in a certain space (which they each choose at random). A driver who wishes to park in space k does so unless it is occupied, in which case he chooses the first vacant space from k+1, k+2,...,n. If these are all occupied he leaves in disgust.

The probability that all the drivers can park is $(n+1)^{n-1}/n^n$. Prove this.

2. Ship A is moving due east at constant speed and, at a certain moment, ship B is moving due north at the same speed towards A. If B maintains this speed but continuously alters course towards A how closely can B approach A ?

3. Des McHale asks the following:

Say that a group G is <u>n-abelian</u>, $n \in \mathbb{Z}$, is

$$(xy)^n = x^n y^n, \quad x,y \in G).$$

For which n is it true that:

G n-abelian implies G(n+1)-abelian ?

Also, what happens if n+1 is replaced by n-1 ?

As usual I'd welcome any historical references to the problems discussed here.

Phil. Rippon,
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CONFERENCE REPORTS

<u>GROUPS IN GALWAY CONFERENCE</u> (13-14 May, 1983)

The main speakers at the Conference were Richard Watson (Maynooth), John Hannah (U.C.G.), Peter Cameron (Oxford), Martin Liebeck (Cambridge) and Colin Walter (U.C.D.). Gerard Enright (M.I.C., Limerick) was unable to give his talk due to illness and Tom Laffey (U.C.D.) and Sean Tobin (U.C.G.) very kindly stepped in at short notice to fill the gap. Other short communications were given by Roderick Gow (U.C.D.), James Ward (U.C.G.), Brian Abrahamson (Flinders, Adelaide), Michael Barry (Carysfort) and Martin Newell (U.C.G.).

The Conference certainly brought home the range and variety of Group Theory and its applications, a fact not fully appreciated nor understood by those not in the area. In the last year alone, three advanced (i.e. postgraduate at least) textbooks have come on the market, namely Curtis and Reiner "Methods of Representation Theory" Vol. I containing over 819 pages (with volume II to follow), W. Feit "The Representation Theory of Finite Groups" (502+ pages) and Huppert and Blackburn "Finite Groups", Vols. II and III containing over 1000 pages. These books are destined to become standard references in their own branches of Group Theory. In the 70's and early 80's also, numerous advanced textbooks on permutation groups, groups of graphs, cohomology of groups, Galois groups, varieties and classes of groups, topological groups, Lie groups, combinatorial group theory, crystallographic groups, group rings and matrix groups have appeared and the subject is expanding into most unlikely areas such as applications in Computer Science and Geology. (For all this and as a contribution to Mathematical Education note that it is, I believe, agreed among Group Theorists that Group Theory is certainly <u>not</u> a topic for Secondary Schools' curricula.)

The first main speaker at the conference was Richard Watson who spoke on "Some aspects of profinite groups". A