PROBLEMS

First, the solutions to the December problems.

1. $$I = \int_0^{\frac{1}{2}\pi} \frac{dt}{1+\tan^{\sqrt{2}} t} = \frac{\pi}{4}.$$

The trick with this is to substitute $x = \frac{1}{2}\pi - t$. Then

$$I = \int_0^{\frac{1}{2}\pi} \frac{dx}{1+\cot^{\sqrt{2}} t} = \int_0^{\frac{1}{2}\pi} \frac{\tan^{\sqrt{2}} x}{\tan^{\sqrt{2}} x+1}\, dx ,$$

so that

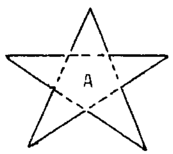
$$2I = \int_0^{\pi} \frac{1+\tan^{\sqrt{2}} t}{1+\tan^{\sqrt{2}} t}\, dt = \frac{\pi}{2}.$$

2. For a set A in $\mathbb{R}^n$, aSb means that the line segment joining a to b lies in A. Does the condition

(*) "If $a,b,c \in A$ then at least one of the following holds
aSb, bSc, cSa ".

imply that A is the union of two convex sets?

The answer is <u>no</u> as the following set shows.



To see that A cannot be written as the union of two convex sets, consider where the five outer vertices would end up.

Now, some more problems.

1. Let Γ be a C^1 closed curve in $\mathbb{R}^3$. Must there exist a pair of points in Γ at which the tangent vectors to Γ are parallel but of opposite sense?

2. An electrician is faced with a bundle of n (>2) unmarked identical wires running from the ground floor to the top floor of a block of flats.

With the aid of a bell and a battery, and making only one visit to the top floor, how should he proceed to match up and label the top and bottom ends of the wires?

3. Prove, or disprove, that

$$\inf\{|n \sin n| : n=1,2,3,\dots\} = 0$$

(submitted by *T. Laffey*)

4. The following problem arises in Control Theory (Simultaneous Stabilization):

Given rational functions d_1, d_2, n_1 and n_2 which have no poles in the closed unit disk $\bar{D}$ and whose zero sets are pairwise disjoint in $\bar{D}$, when is it possible to find a rational function m without poles in $\bar{D}$ such that neither of the functions d_1+mn_1, d_2+mn_2 has any zeros in $\bar{D}$?

If each of the functions d_1, d_2, n_1 and n_2 is self-conjugate (a function f is self-conjugate if $f(\bar{z}) = \overline{f(z)}$), when is it possible to find a self conjugate function m as above?

This problem is equivalent to the following (at least when the zeros of d_1, d_2, n_1 and n_2 are simple): When can one find a rational function r without poles or zeros in $\bar{D}$ such that

$$r(z) = \frac{n_1(z)}{n_2(z)}$$

if and only if $(n_1d_2 - n_2d_1)(z) = 0$.

Remark: If we only need $n_i + md_i$ to be non-vanishing, there is no restriction on solving the problem without the self conjugation condition: with this condition, the problem can be solved if and only if the values of d_i at all <u>real</u> zeros of n_i in $\overline{D}$ have the same sign.

(submitted by *J.J. Murray*)

P. Rippon,
Mathematics Faculty,
Open University,
Milton Keynes.

WORD CONSERVATION

To counter the recent British changeover from the English billion, which is worth one million million, to the American billion, of one thousand million, we advocate the introduction of the Irish thousand, worth one hundred hundred, as a substitute for the English and American thousand, which is ten hundred. This would give an Irish billion of ten to the power of sixteen, which the Americans would have to call ten quadrillion. By the time we in Ireland have used up one new word therefore the Americans will have used up three, and be well on the way to their fourth.

AUTONOMOUS NOTATION

$e^{\text{xponential}}$; $\log_a\text{rithm}$; $\int e^x = f(u_n)$; $\text{CAT}_{\text{A}_{\text{STROPHE}}}$

ALGEBRA CONFERENCE

MARY IMMACULATE COLLEGE OF EDUCATION, LIMERICK

12-13 NOVEMBER, 1982

The principal invited speaker was Professor John Thompson (Cambridge) who is well known to algebraists for his many fundamental contributions to the theory of finite groups and in particular to the classification of finite simple groups. His first lecture took the form of an anecdotal tour through the background to the Odd Order Paper highlighting the decisive influence of Frobenius, Burnside, Brauer and Suzuki. Without the individual achievements of these great mathematicians the project could not have been undertaken.

In his second, more technical, lecture Professor Thompson sought to demonstrate the fundamental and intimate connexions that exist between the theory of modular functions and finite group theory. The precise nature of the relationship is still not clearly understood and new facts are constantly emerging. In particular Professor Thompson announced his recently proved result that the group known as Monster fits remarkably well into this scheme as a Galois group over the rationals. He discussed the possibility that there may lie the foundation of an overall theory which would put the twenty six sporadic simple groups into a unified context, thereby satisfying those who, like himself, dislike regarding these exceptions as mere 'bumps in the universe'.

There were three other invited lectures given, appropriately, by speakers from Cork, Dublin and Galway.

Des McHale (University College Cork) described the lives of Boole and Hamilton seeking to explain why there was so little contact between them, despite the fact that they lived in Ireland at the same time for a period of fourteen years. In a detailed analysis drawn from his own work on Boole and a