PROBLEM SECTION

(4.1) (D. McHale) Let R be a finite commutative ring with identity and let $a,b \in R$ with $aR = bR$. Show that $a = bu$ for some unit u of R. Generalise!

(4.2) (M. Hall and M. Newman) Let $Q(x_1,\ldots,x_n)$ be a positive definite real quadratic form and suppose that Q can be expressed as $L_1^2 + \ldots + L_r^2$ where $L_1,\ldots,L_r$ are linear forms with non-negative coefficients. Show that Q has such a representation with $r \leqslant f(n)$ for some function $f(n)$ of n^2 and find the best such function f.

(4.3) Calculate explicitly $\sum_{n=0}^{\infty} e^{-\pi n^2}$, $\sum_{n=0}^{\infty} e^{-2\pi n^2}$.

(4.4) (P. Halmos) Let A,B be positive semi-definite matrices. Let $d_1(A,B) = \|A - B\|$ and $d_2(A,B) = \sup_{\|x\|=1} \left| \|Ax\| - \|Bx\| \right|$. Prove that there exists a function $c(n)$ such that $d_2 \leqslant d_1 \leqslant c(n)d_2$ and find the best function c.

(4.5) For which natural numbers n does there exist an $n \times n$ magic square whose entries are $1,2,\ldots,n$. [According to the Evening Press, one has been constructed recently with $n = 243$.]

(4.6) (G. Myerson) Given two co-prime integer polynomials $f(x)$, $g(x)$, find a formula for the least positive integer d for which there exist integral polynomials $h(x)$, $k(x)$ with $d = f(x)h(x)+g(x)k(x)$. [Myerson points out that the standard Sylvester resultant formula does not necessarily give the smallest d.]