RECENT ADVANCES IN MATHEMATICS

It seems a good idea to have a section of the Newsletter devoted to informing readers of recent major break-throughs in Mathematics, which should be of interest to mathematicians in general. In this issue, we list two results of this type.

(1) The Classification of the Finite Simple Groups

A paper by G. Mason, now in the course of completion, will bring to an end the classification of the finite simple groups. This is arguably the greatest mathematical achievement of all time, the proof that the list of known simple groups is complete adding up to many thousands of pages. The classification has involved the work of many of the greatest mathematicians of our time and has involved major contributions by so many people that it is difficult to single out any aspects for special mention. However, the development of techniques by Chevalley to actually construct the simple groups of Lie type in the 1950's, the proof of the solvability of groups of odd order by Feit and Thompson (1963), the determination of the minimal simple groups by Thompson (1968) and the work of Aschbacher and Thompson on the B-conjecture in the 1970's are universally recognised as milestones along the way.

Of course, quite a lot of work has still to be done to try and shorten and simplify the work to make it comprehensible to ordinary mortals. Referring to the view among some mathematicians that finite group theory was finished now that the simple groups are known, Gorenstein (in Pittsburgh in August) reminded his audience that though the real numbers have been classified for a long time, there are still many real analysts.

(2) <u>The van der Waerden conjecture</u>

Let $A = (a_{ij})$ be an nxn matrix. The <u>permanent</u>, per A, of A is defined by

$$\text{per } A = \sum_{\sigma \in S_n} a_{\sigma(1)1}\, a_{\sigma(2)2} \cdots a_{\sigma(n)n}$$

(where S_n denotes the symmetric group of degree n)

[per A is thus the same as det A except that sign (σ) is replaced by +1 throughout.]

A is called <u>doubly stochastic</u> if its entries are (real) <u>non-negative</u> and the sum of the entries in each row is 1 and the sum of the entries in each column is 1. Thus, in symbols

$$\sum_{i=1}^{n} a_{ij} = 1 = \sum_{i=1}^{n} a_{ji} \qquad (j = 1,\ldots,n).$$

Examples of doubly stochastic matrices are I_n and J_n where J_n is the nxn matrix all of whose entries are 1/n.

The famous van der Waerden conjecture (1925) states that the minimum of per A as A ranges through the set of all doubly stochastic nxn matrices is achieved precisely for $A = J_n$.

Many authors worked on this problem and many special cases were resolved. Of particular elegance is the solution by Marcus and Minc for the case of symmetric A. See Minc's book on Permanents or the article by Marcus and Minc in the American Mathematical Monthly Vol.75 (1965) for the history and results on the problem.

Recently the conjecture has been proved in full generality by Egoryshev in Krasnoyarsk in the U.S.S.R. A very nice account of the solution (incorporating some simplifications due to himself and Seidel) can be found in the article by van Lint in Linear Algebra and its Applications, Vol. 1981.

T.J. Laffey