EDITORIAL

This issue includes an article by Peter Lynch in which he makes a strong case for applying the name *O'Brien's Equation* to a classical equation in vector analysis which is of central importance in meteorology and which appears to lack a name, to date. A rather long article by Shannon Guerrero has interesting insights and suggestions about secondary mathematics education in Ireland. Bernd Kreussler's report on the IMO is encouraging.

We have agreed a new reciprocity agreement with the Moscow Mathematical Society, and this includes the exchange of our Bulletin for their periodical *Uspekhi Matematicheskii Nauk* (the Russian version, not the AMS translation). Please send expressions of interest in housing the exchange copies to me at the address below. The IMS Committee will then decide on the allocation.

John Miller was in contact last year, urging some action to record and preserve historical information about the IMS. There have been a few developments:

Richard Timoney of TCD and Michael Mackey of UCD have completed the task of assembling and scanning all issues of the IMS Newsletter and Bulletin, back to the very beginning, and these are now available online at `http://www.maths.tcd.ie/pub/ims/bulletin/`. Michael, the incoming President, has also compiled a list of the former Presidents and other officers. This is almost complete, and is currently at `http://banach.ucd.ie/~mackey/ims/imsofficers.html`. Members who can supply the missing data are asked to contact him.

Readers will be interested in a recent article on the history of Linear Algebra research in Ireland by Tom Laffey in IMAGE 52 (the Bulletin of the International Linear Algebra Society). See `http://www.ilasic.org/IMAGE/IMAGES/image52.pdf` This was drawn to our attention by Niall Madden of NUI Galway.

Just before Christmas Colm Mulcahy of Spelman College launched an effort to document Irish research mathematicians. His website `http://www.cardcolm.org/AIMM.html` has a table that is intended to provide some basic data about people from Ireland or who worked or studied here and who made mathematics. The stated goal is: "To develop and maintain a dynamic and comprehensive Archive of Irish

Mathematics and Mathematicians, at the third level. We also wish to track the influence and contribution of Irish mathematicians to the profession worldwide via their progeny in the discipline, as measured by the supervision of doctoral theses, and, on another webpage, the publication of books." Already there are over 600 entries. Readers are urged to check the entries about people of whom they they might have knowledge, and to send word to Colm about corrections and additions.

The San Francisco Declaration on Research Assessment, which has now been endorsed by the Society (as well as by many individual members) may be viewed at `http://am.ascb.org/dora/`.

In Ireland, the madness continues. We are informed that the administrative powers-that-be in one of our leading universities have adopted a definition of "research active staff" which will classify as "research inactive" a professor who does nothing but teach, discover new results and publish them in peer-reviewed journals. The recent visit of J-P Bourguignon, President of the European Research Council, and his trenchant public lecture at the Royal Irish Academy may have given some of those who hold the purse-strings for publicly-funded research cause to doubt the wisdom of their policies. Apart from urging support for basic research, it was also helpful that he stated bluntly that citation-metrics are quite useless in the evaluation of mathematical work.

The next main scientific meeting of the Society will take place in UCC, and will form a part of the Boole Centenary celebrations there.

AOF. Department of Mathematics and Statistics, NUI, Maynooth, Co. Kildare

E-mail address: `ims.bulletin@gmail.com`

LINKS FOR POSTGRADUATE STUDY

The following are the links provided by Irish Schools for prospective research students in Mathematics:

DCU: (Olaf Menkens)
`http://www.dcu.ie/info/staff_member.php?id_no=2659`
DIT: (Chris Hills)
`mailto://chris.hills@dit.ie`
NUIG:
`mailto://james.cruickshank@nuigalway.ie`
NUIM:
`http://www.maths.nuim.ie/pghowtoapply`
QUB:
`http://www.qub.ac.uk/puremaths/Funded_PG_2012.html`
TCD:
`http://www.maths.tcd.ie/postgraduate/`
UCD:
`mailto://nuria.garcia@ucd.ie`
UU:
`http://www.compeng.ulster.ac.uk/rgs/`

The remaining schools with Ph.D. programmes in Mathematics are invited to send their preferred link to the editor, a url that works. All links are live, and hence may be accessed by a click, in the electronic edition of this Bulletin[1].

E-mail address: `ims.bulletin@gmail.com`

[1] `http://www.maths.tcd.ie/pub/ims/bulletin/`

LETTER TO THE EDITOR

Dear Editor,

My book "The Joy of Understanding and Solving problems: A Guide to School Mathematics" was recently reviewed in the IMS Bulletin. Unfortunately the reviewer, Dr Brennan, did not get round to giving details of either the contents or of special features of the book, such as, for instance, its inclusion of historical remarks. These details are usually expected in a review. I am writing to very briefly remedy this lacuna.

I should make it clear that the joy of understanding stated in the title of my book will not come from simply reading a sentence or two of my book but involves a process of work and concentration. Once it happens it is a transformative experience.

Let me illustrate by a few examples what I have done in the book. There are altogether 15 chapters that cover what can be called core mathematics. They include numbers, equations, geometry, coordinate geometry, permutation/combinations, complex numbers, exponentials and logs, trigonometry, calculus, group theory, probability and statistics. There are additional topics such as quaternions, boolean algebra, diophantine equations and so on. Each topic is introduced by some motivational remarks, and there are short biographies of great contributors.

A broad and very brief picture of what is done at the level of concepts in four sections: numbers, equations, geometry and calculus is as follows: Mathematics is first presented as the science of significant forms and then the operations with numbers which underpin the subject are explained in geometrical terms. This converts, for example,the algebraic results such as $(a+b)^2 = a^2 + b^2 + 2ab$ into a geometrical property of a square of side $(a+b)$. Understanding concepts is crucial for making progress in mathematics. Thus we explain the basis for the rule $(-)(-) = +$ and carefully present fractions is parts of a whole i.e as the ratio of positive integers, while rational numbers have an additional rule which determines the sign of the ratio of signed integers. If this is not clear we have the paradox that $\frac{+1}{-1}$ has to be greater than one as $-$ is less than $+1$.

Moving to geometry we point out that from the parallel line axiom a measurable prediction, namely, that the three angles of a triangle add to 180° follows. This is a stunning result which can be appreciated only when explained using words. William Rowan Hamilton, when young, wrote a lovely discussion between Papus and Euclid in which the method for choosing axioms was discussed. Such discussions give students a sense of appreciation of the human element of the subject responsible for creating the structures of mathematics introduces in school. The formula that relates the sum of the three angles of a great circle triangle on a sphere of given radius to its area is mentioned in the book to show students that other geometries with elegant properties are possible. Understanding of concepts comes from highlighting hidden structures. They need words.

Turning to calculus we stress that calculus introduces a new operation into mathematics: the operation of taking a "limit" This new operation acts on functions just as addition subtraction etc are operations that act on numbers. The example of the way an infinite series can represent the number 2 is used to gently make the notion of this "limit" operation clear. New ideas need words to be properly understood.

There are more many examples of this nature in the book, and many more results to excite interest such as, for instance a way to find transcendental numbers, how to use group theory to get Kepler's law and how trigonometry on its way to measure areas using angles and ends up with the beautiful idea of a periodic function.

I believe this style of presentation is not present in current Irish school text books. I also believe if a generation, used to exploring and finding things out for themselves from the internet, have their imagination restricted by a narrow view regarding what is mathematics a great disservice will be done to the young.

My book covers material required for school leavers in India and I am using it in my enthusiastic interaction with trainee teachers and students when I am in India.

Siddhartha Sen
CRANN, Trinity College Dublin

NOTICES FROM THE SOCIETY

Officers and Committee Members 2014

President	Dr M. Mathieu	Queen's University Belfast
Vice-President	Dr M. Mackey	University College Dublin
Secretary	Dr R. Quinlan	NUI Galway
Treasurer	Prof G. Pfeiffer	NUI Galway

Dr P. Barry, Prof J. Gleeson, Dr B. Kreussler, Dr M. Mac an Airchinnigh, Dr A. Mustata, Dr S. O'Rourke, Dr J. O'Shea, Dr R. Levene Dr C. Stack, .

Local Representatives

Belfast	QUB	Dr M. Mathieu
Carlow	IT	Dr D. Ó Sé
Cork	IT	Dr D. Flannery
	UCC	Dr S. Wills
Dublin	DIAS	Prof Tony Dorlas
	DIT	Dr C. Hills
	DCU	Dr M. Clancy
	SPD	Dr S. Breen
	TCD	Prof R. Timoney
	UCD	Dr R. Higgs
Dundalk	IT	Mr Seamus Bellew
Galway	UCG	Dr J. Cruickshank
Limerick	MIC	Dr B. Kreussler
	UL	Mr G. Lessells
Maynooth	NUI	Prof S. Buckley
Tallaght	IT	Dr C. Stack
Tralee	IT	Dr B. Guilfoyle
Waterford	IT	Dr P. Kirwan

Applying for I.M.S. Membership

(1) The Irish Mathematical Society has reciprocity agreements with the American Mathematical Society, the Deutsche Mathematiker Vereinigung, the Irish Mathematics Teachers Association, the Moscow Mathematical Society, the New Zealand Mathematical Society and the Real Sociedad Matemática Española.

(2) The current subscription fees are given below:

Institutional member	€160
Ordinary member	€25
Student member	€12.50
DMV, I.M.T.A., NZMS or RSME reciprocity member	€12.50
AMS reciprocity member	$15

The subscription fees listed above should be paid in euro by means of a cheque drawn on a bank in the Irish Republic, a Eurocheque, or an international money-order.

(3) The subscription fee for ordinary membership can also be paid in a currency other than euro using a cheque drawn on a foreign bank according to the following schedule:

If paid in United States currency then the subscription fee is US$ 30.00.
If paid in sterling then the subscription is £20.00.
If paid in any other currency then the subscription fee is the amount in that currency equivalent to US$ 30.00.

The amounts given in the table above have been set for the current year to allow for bank charges and possible changes in exchange rates.

(4) Any member with a bank account in the Irish Republic may pay his or her subscription by a bank standing order using the form supplied by the Society.

(5) Any ordinary member who has reached the age of 65 years and has been a fully paid up member for the previous five years may pay at the student membership rate of subscription.

(6) Subscriptions normally fall due on 1 February each year.

(7) Cheques should be made payable to the Irish Mathematical Society. If a Eurocheque is used then the card number should be written on the back of the cheque.

(8) Any application for membership must be presented to the Committee of the I.M.S. before it can be accepted. This Committee meets twice each year.

(9) Please send the completed application form with one year's subscription to:

The Treasurer, IMS
School of Mathematics, Statistics and Applied Mathematics
National University of Ireland
Galway
Ireland

E-mail address: `subscriptions.ims@gmail.com`

IMS President's Report 2014

My second year as President of the Society started in January 2014. It was generally quieter than expected, however the organisation of the Society's Annual Meeting in Belfast (5 and 6 September 2014) took up a large part of my time. In the days before our meeting, an international workshop on operator theory was held (organised by me as well) which attracted more international participants to our meeting than usual. While the international workshop received a large grant from the LMS and some support from the School of Mathematics and Physics at Queen's University, the IMS meeting was solely funded by the Society.

Once again I represented the Society at the *Annual Meeting of the Presidents of the National Member Societies of the EMS* which was held in Istanbul on 12 April 2014. Details of the programme can be found at

`http://bricksite.com/emspresidents2014/velkommen`

together with a report by the EMS secretary Stephen Huggett as well as slides of the presentations.

It has not been decided where the next meeting of the presidents will be held in April 2015. In 2016, the IMS will celebrate its 40th anniversary and one might consider whether we should use this opportunity to invite the Presidents' Meeting over to Ireland. This would have to be proposed to the EMS firmly at the meeting in 2015.

The EMS Council Meeting was held in San Sebastian on 28–29 June 2014; however I could not attend this, partly for family reasons but also as the Society can probably only fund attendance at one such meeting per year.

The Fergus Gaines Cup 2014 was presented to Luke Gardiner at the Society's Annual Meeting.

Following an invitation by the EMS Newsletter's editor, I wrote a short piece on the history of the IMS and its current activities which is published in this year's December issue, pp. 50-51 under the title "*A brief history of the Irish Mathematical Society*".

Conference Support. The Society supported the following conferences in 2014:

- Women in Mathematics Day Ireland, NUIG, May 1, 2014
- Groups in Galway, NUIG, May 23–24, 2014

- Irish Geometry Conference, NUIG, May 9–10, 2014
- Irish Mathematical Society Workshop on Kähler Geometry and Geometric Analysis, UCC, May 12–13, 2014
- 2nd Irish Linear Algebra and Matrix Theory meeting, UCD, August 29, 2014.

Together with the IMS treasurer I worked on clearer guidelines for applicants to the Society's conference support fund.

Future Meetings: In 2015 we will meet in UCC on 27-28 August as part of the bicentenary celebrations of George Boole; in 2016 the annual meeting is likely to be held at TCD.

In concluding my term I would like to thank all members of the IMS for their support and their advice over the past two years and all the committee for several stimulating discussions.

Martin Mathieu, President
10 December 2014

Minutes of the Meeting of the Irish Mathematical Society
Annual General Meeting
Queen's University Belfast
September 6 2014

The meeting commenced at 13.00. There were 18 members present.

(1) **Minutes of 2013 AGM**
The minutes of the 2013 AGM were approved and signed.

(2) **Matters Arising**
The Society now has a functioning list of members' email addresses. An email message will shortly be sent to IMS members offering individuals the option not to receive a paper copy of the Bulletin.

(3) **Correspondence**
- Prof. John Miller has proposed that the society should maintain a list of its presidents since its foundation in 1976. This will be initiated in the near future.
- Prof. Graham Ellis has suggested that the Society consider signing up to the San Francisco Declaration on Research Assessment, which concerns appropriate use of quantitative measures of research productivity. There was general support for this move.
- M. Mathieu has received some correspondence from the European Mathematical Society about its regular actvities.

(4) **Membership Applications**
Applications from the following persons have been received and were approved.

Alan Compelli	M.Sc. student, DIT
Alexander Rahm	NUI Galway
Oliver Mason	NUI Maynooth

(5) **Chairman's Business**
M. Mathieu reported that he attended the annual meeting of presidents of member societies of the European Mathematical Society in Istanbul on April 12. The venue for the 2015 meeting is yet to be decided. The 40th anniversary of the establishment of the IMS will be in 2016, and it was suggested that one way of marking this milestone would be to host the

meeting of society presidents in that year, perhaps in conjuction with a wider mathematical event. This idea will be considered at the next committee meeting in December.

M. Mathieu has written an invited article on the history of the IMS for the EMS newsletter.

Five conferences received financial support from the IMS this year.

(6) **Dissolved Mathematics Committee of the RIA**
The Royal Irish Academy has now established a Physical, Chemical and Mathematical Sciences Committee. At present this committee has 15 members including five mathematicians; it is expected that the number of members will increase to 20 in the next few weeks. The structure and operation of the new multidisciplinary committees of the RIA will be reviewed after two years.

Since these committees have not yet commenced their work, how they will serve the needs of relevant disciplinary communities remains to be seen. However there are reasons for the IMS and wider mathematical community to be concerned about current developments. One specific concern is how affiliation to bodies such as the International Mathematical Union will be managed, in the absence of a national committee for Mathematics. More generally, a committee that could be in a position to advise on matters of policy relating to mathematics would need to be geographically diverse and sufficiently broadly based to include representation across a wide range of constituencies connected (for example) to education at all levels, to research, to academia and to industry.

A. O'Farrell suggested that the mathematical community in Ireland needs to have a national committee that can appropriately serve its interests, and that the IMS is in a position to initiate the establishment of such a group. There was general support for this idea, although it remains unclear what formal status such a group might have and how it would interact with the RIA Committee. C. Stack raised a question about how the members of such a committee would be appointed and whether this has always been done in a transparent manner. It was agreed to establish a working group at the meeting, with the immediate role of considering the evolving situation and advising the IMS on possible ways forward. This working group will intially consist of L. Creedon, M. Mackey, G. McGuire, A. O'Farrell, C. Stack and R. Timoney.

(7) **Treasurer's Report**
G. Pfeiffer circulated the Treasurer's Report for 2013. Members were reminded that some subscription fees for 2013 and 2014 remain unpaid.

(8) **Bulletin**
IMS members are encouraged to consider submitting articles of general interest to the Bulletin. Organizers of conferences that received financial support from the IMS are reminded to submit short reports to the Bulletin. Abstracts of completed PhD theses should also be sent to the editor for inclusion in the Bulletin.

(9) **Education sub-Committee**
It was proposed that the Society establish an Education sub-Committee with the following brief:

- To advise the IMS Committee on all matters concerning mathematical education;
- In particular, to suggest to the IMS Committee responses to policy documents in the field of the sub-Committee's interest;
- To act on behalf of the Society in such matters when authorized to do so;
- To liaise as appropriate with other bodies with interests in the area (for example, with sister societies and government agencies).

This proposal was approved. R. Quinlan will seek expressions of interest in serving on the Education sub-Committee, with the expectation that its proposed membership can be brought to the December committee meeting and that the sub-committee can commence its work in January 2015.

(10) **Elections to Committee**
C. Stack will conclude her current period of service on the IMS Committee on December 31 2014, and M. Mathieu will conclude his service as President on the same date.

M. Mackey was nominated by M. Mathieu for the position of President, and seconded by R. Timoney. S. Buckley was nominated for the position of vice-President by M. Mackey, and seconded by M. Mathieu. Both candidates were elected unopposed.

The numbers of years that each continuing member will have served on the IMS Committee as of January 2015 are tabulated below.

P. Barry	1
J. Gleeson	3
B. Kreussler	3
R. Levene	1
M. Mac an Airchinnigh	2
M. Mackey	5
M. Mathieu	4
A. Mustata	1
G. Pfeiffer	1
R. Quinlan	5
J. O'Shea	1

(11) **Future Meetings**
C. Koestler announced that the 2015 IMS meeting will be held at University College Cork in the last week of August, as part of a series of events celebrating the bicentenary of the birth of George Boole.

The meeting concluded at 14.25.

Rachel Quinlan
NUI Galway

27th Annual Meeting of the Irish Mathematical Society Queen's University, Belfast

SEPTEMBER 5–6, 2014

TIMETABLE

Friday 5 September

11:00–12:00	Registration
12:00	Opening remarks by Prof. James McElnay, Pro-VC of QUB
12:10–13:00	**Gilles Godefroy** The Lipschitz-free Banach Spaces Associated with a metric space
13:00–14:00	Lunch
14:00–14:50	**Raúl Curto** Truncated Moment Problems
15:00–15:30	**Tony Wickstead** Two-dimensional Unital Riesz Algebras
15:30–16:00	Coffee/Tea
16:00–16:30	**Claus Kostler** Thompson Group F from the Viewpoint of Noncommutative Probability
16:30–17:00	**Stephen Gardiner** Universal Taylor series
17:00–17:50	**Frank Lutz** On the Topology of Steel
19:30	Conference Dinner in the Great Hall

Saturday 6 September

08:30–10:00	**IMS Committee Meeting**,
10:00–10:30	**James O'Shea** Multiples of Pfister Forms
10:30–11:20	**Brigitte Lutz-Westphal** Bringing Authenticity to the Mathematics Classroom

11:30–12:00	Coffee/Tea
12:00–12:50	**Gary McGuire** Online Security in Bits: Recent Developments in Public-key Cryptography
13:00–14:30	IMS Annual General Meeting
14:30–15:20	**Götz Pfeiffer** Computing with groups and Algebras
15:30–16:00	**Steve Buckley** Groups that are Isoclinic to Rings
16:00–16:30	Coffee/Tea
16:30–17:20	**Alain Valette** The Kadison-Singer Problem
17:30	Close of the Meeting

Abstracts

Stephen Buckley (NUI Maynooth)

"Groups that are Isoclinic to Rings"

Let F be a finite algebraic system in which multiplication is denoted by juxtaposition. The *commuting probability of F* is

$$\Pr(F) = \frac{|\{(x,y) \in F \times F \ : \ xy = yx\}|}{|F|^2},$$

where $|\cdot|$ denotes cardinality. Much has been written on $\Pr(G)$ where G is a finite group. Together with MacHale and Ní Shé, we previously studied $\Pr(R)$ where R is a finite ring, making use of an associated notion of ring isoclinism.

In this talk, we compare and contrast the values attained by $\Pr(G)$ and $\Pr(R)$ as G and R range over certain classes of groups and rings, respectively. We show in particular that the set of values that arise for finite rings and for finite class-2 nilpotent groups are the same. Proving this involves the consideration of certain triples $T = (A, B, k)$ associated with both class-2 groups and rings. Isomorphism of such triples generalizes the previous notions of group isoclinism and ring isoclinism, and commuting probability of groups and rings is an isomorphic invariant of the associated triples.

This talk is based on joint work with Des MacHale (UCC).

Raúl Curto (University of Iowa)

"Truncated Moment Problems: The Interplay Between Functional Analysis, Algebraic Geometry and Optimization"

Inverse problems naturally occur in many branches of science and mathematics. An inverse problem entails finding the values of one or more parameters using the values obtained from observed data. A typical example of an inverse problem is the inversion of the Radon transform. Here a function (for example of two variables) is deduced from its integrals along all possible lines. This problem is intimately connected with image reconstruction for X-ray computerized tomography.

Moment problems are a special class of inverse problems. While the classical theory of moments dates back to the beginning of the 20th century, the systematic study of *truncated* moment problems began only a few years ago. In this talk we will first survey the elementary theory of truncated moment problems, and then focus on moment problems admitting cubic column relations.

For a degree $2n$ real d-dimensional multisequence $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{i \in Z_+^d, |i| \leq 2n}$ to have a representing measure μ, it is necessary for the associated *moment matrix* $M(n)$ to be positive semidefinite, and for the corresponding *algebraic variety*, V_β, to satisfy rank $M(n) \leq$ card V_β as well as the following *consistency condition*: if a polynomial $p(x) \equiv \sum_{|i| \leq 2n} a_i x^i$ vanishes on V_β, then $p(\beta) := \sum_{|i| \leq 2n} a_i \beta_i = 0$. In previous joint work with L. Fialkow and M. Möller, we proved that for the *extremal* case (rank $M(n) =$ card V_β), positivity and consistency are sufficient for the existence of a (unique, rank $M(n)$-atomic) representing measure.

In recent joint work with Seonguk Yoo we have considered cubic column relations in $M(3)$ of the form (in complex notation) $Z^3 = itZ + u\bar{Z}$, where u and t are real numbers. For (u, t) in the interior of a real cone, we prove that the algebraic variety V_β consists of exactly 7 points, and we then apply the above mentioned solution of the extremal moment problem to obtain a necessary and sufficient condition for the existence of a representing measure. To

check consistency, one needs a new representation theorem for sextic polynomials in Z and $\bar{Z}$ which vanish in the 7-point set V_β.
Our proof of this representation theorem relies on two successive applications of the Fundamental Theorem of Linear Algebra. For other extremal moment matrices admitting cubic column relations, one can appeal to the Division Algorithm from real algebraic geometry to obtain similar representations; the Cayley–Bacharach Theorem also plays a role.

Stephen Gardiner (UCD)

"Universal Taylor Series"

In various mathematical contexts it is possible to find a single object which, when subjected to a countable process, yields approximations to the whole universe under study. Such an object is termed "universal" and, contrary to expectations, such objects often turn out to be generic rather than exceptional. This talk will focus on this phenomenon in respect of the Taylor series of a holomorphic function, and how the partial sums behave outside the domain of the function.

Gilles Godefroy (Paris VI)

"The Lipschitz-free Banach Spaces Associated with a Metric Space"

Let M be a metric space equipped with a distinguished point 0. Given such a pointed metric space M, the Lipschitz-free space $\mathcal{F}(M)$ over M is the linear span of the Dirac measures in the dual of the space $Lip(M)$ of Lipschitz functions on M which vanish at 0. This space $\mathcal{F}(M)$, which is actually a predual of $Lip(M)$, allows linearization of the Lipschitz maps between metric spaces, and its structure somehow reflects the properties of the metric space M. But although their definition is pretty simple, the Banach spaces $\mathcal{F}(M)$, which are separable when M is separable, are far from being well understood.
We will present some recent results e.g. on approximation properties of the free spaces, and natural open problems on this "new" class of Banach spaces.

Claus Koestler (UCC)
"Thompson Group F from the Viewpoint of Noncommutative Probability"

Recently we have given a characterization of all extremal characters of the Thompson group F which is motivated from our new proof of Thom's theorem about extremal characters of the infinite symmetric group. My talk will introduce our approach and address some open questions. This is joint work with Rolf Gohm.

Frank Lutz (Technical University Berlin)
"On the Topology of Steel"

Polycrystalline materials, such as metals, are composed of crystal grains of varying size and shape. Typically, the occuring grain cells have the combinatorial types of 3-dimensional simple polytopes, and together they tile 3-dimensional space.
We will see that some of the occurring grain types are substantially more frequent than others—where the frequent types turn out to be "combinatorially round". Here, the classification of grain types gives us, as an application of combinatorial low-dimensional topology, a new starting point for a topological mircostructure analysis of steel.

Brigitte Lutz-Westphal
(Free University Berlin and MATHEON)
"'Bringing Authenticity to the Mathematics Classroom"

Activities in the mathematics classroom should give an authentic feeling of doing mathematics. How can we close the gap between classroom mathematics and the fascination of mathematical research? Teachers and students need to learn to look at their own living environment with a mathematical eye. They have to get involved directly and individually in mathematical problems. Within the framework of learning in mutual dialogue they can start to act as little researchers. As "Math Investigators", students discover mathematics and are trained to ask questions with a mathematical content. We will report from our experience in a 4-year project "Mathe.Forscher"

(Math Investigators) of the Stiftung Rechnen (German Numeracy Foundation).

Gary McGuire (UCD)

"Online Security in Bits: Recent Developments in Public Key Cryptography"

All schemes and protocols in Public Key Cryptography today are based on one of two hard problems, the Integer Factorisation Problem or the Discrete Logarithm Problem. We will give a full introduction to these matters, including the historical development and connections between the IFP and the DLP. We will explain some recent developments in the DLP and their consequences. In particular, we will explain how polynomials of a certain shape were important in our results. Based on joint work with Faruk Gologlu, Robert Granger, and Jens Zumbragel. Our paper won the best paper award at the CRYPTO 2013 conference.

James O'Shea (NUI Maynooth)

"Multiples of Pfister Forms"

Forms with diagonalization $\langle 1, a_1\rangle \otimes \ldots \otimes \langle 1, a_n\rangle$ for some scalars $a_1, \ldots, a_n$, known as Pfister forms, play a central role in the theory of quadratic forms. The sums and multiples of such forms in the Witt ring of a field have been subjects of much study, with Elman and Lam establishing long-standing results regarding their isotropy (non-trivial representations of zero).

We will consider the isotropy of multiples of Pfister forms over field extensions, establishing an improved lower bound on their Witt index (the dimension of a maximal totally-isotropic subspace). This bound enables us to add "maximal splitting" to the list of properties that are preserved under multiplication by a Pfister form. Conversely, in the case where the Pfister form is generated by variables, we will show that "going-down" results also hold with respect to multiplication.

Götz Pfeiffer (NUI Galway)
"Computing with Groups and Algebras"

Many algorithms in Computational Group Theory follow the simple pattern of a breadth first search. I will discuss classical examples like the Todd–Coxeter procedure for coset enumeration in this context. The results of such algorithms can be regarded as graphs which frequently reveal surprising symmetries. Examples arise from finite Coxeter groups (like symmetric or dihedral groups), or more generally, from complex reflection groups. Linear versions of the algorithms can be used to compute with modules for finitely presented algebras. A cyclotomic Hecke algebra is a deformation of the group algebra of a complex reflection group that plays an important role in the representation theory of finite groups of Lie Type. I will present an application of a linear variant of the Todd–Coxeter algorithm to the cyclotomic Hecke algebras of some complex reflection groups which represents the algebra as a free module over a parabolic subalgebra. This establishes previously unknown cases of the freeness conjecture by Brou, Malle and Rouquier, which claims that a cyclotomic Hecke algebra, like the Iwahori–Hecke algebra of a finite Coxeter group, has a basis in bijection to the elements of the group.

Alain Valette (Université de Neuchatel)
"The Kadison–Singer Problem"

In 1959, R.V. Kadison and I.M. Singer asked whether each pure state of the algebra of bounded diagonal operators on ℓ^2 admits a unique state extension to $B(\ell^2)$. The positive answer was given in June 2013 by A. Marcus, D. Spielman and N. Srivastava, who took advantage of a series of translations of the original question, due to C. Akemann, J. Anderson, P. Casazza, N. Weaver,... Ultimately, the problem boils down to an estimate of the largest zero of the expected characteristic polynomial of the sum of independent random variables taking values in rank-one positive matrices in the algebra of n-by-n matrices. In turn, this is proved by studying a special class of polynomials in d variables, the so-called real stable polynomials. The talk will highlight the main steps in the proof.

Anthony W. Wickstead (Queen's University Belfast)

"Two-dimensional unital Riesz Algebras"

A *Riesz algebra* is an associative algebra over the reals that is simultaneously a vector lattice with the two structures connected by the implication $x, y \geq 0 \implies xy \geq 0$. My own interest is in *Banach lattice algebras* where there is also a complete norm that is compatible with the algebra and order structures in a very strict way. Although there are several nice examples, there is no general theory. In order to get a better understanding of the general situation, I decided to look at simple (but not particularly special) examples. Following Polya's dictum *If you can't solve a problem, look for a simpler problem that you can't solve.* which I combined with transfinite induction, I sought the simplest possible class of Banach lattice algebras that I didn't understand. I think I understand one-dimensional Banach lattice algebras, so I started thinking about two-dimensional Banach lattice algebras with an identity. Even these are varied enough to illustrate the kind of problems that a general theory of Banach lattice algebras will encounter. In this talk, I will describe the surprisingly rich family of all two dimensional unital Riesz algebras and (if time permits) start to explain why they are interesting to me.

Report by Martin Mathieu, Queen's University, Belfast.
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Reports of Sponsored Meetings

Irish Mathematics Learning Support Network 8th Annual Workshop
28 January 2014, IT Tallaght Dublin

The Irish Mathematics Learning Support Network (IMLSN) was founded in 2009. Its aim is to act as an informal focus point for all those who are interested in the provision of mathematics and statistics support at higher education institutions in Ireland. This gathering, the 8th Annual Workshop of the IMSLN, had a theme of *Diversity: challenges and opportunities - enabling and supporting mathematics learning in a diverse student population*.

This workshop was motivated by the increasingly diverse population of students studying mathematics as part of their higher education. This diversity manifests itself in many ways but in particular:

- A higher proportion of students entering higher education are not coming straight from second level.
- The number of students with particular learning needs is increasing.

The workshop was co-hosted by the Institute of Technology Tallaght Dublin (ITTD) and the Dublin Institute of Technology (DIT), and was held in ITTD.

The day began with a Welcoming Address by the President of ITTD, Pat McLaughlin. This was then followed by the first of the keynote addresses, given by Clare Trott, a Mathematics Support Tutor at Loughborough University. Clare gave a fascinating and very informative presentation on supporting neurodiverse students with their mathematics and statistics studies. She outlined many of the different forms of neurodiversity that tutors in a Maths Learning Centre (MLC) may encounter and illustrated, with the aid of case studies, some strategies on how tutors may enable neurodiverse students to overcome educational barriers.

Following this were two further short presentations. Brien Nolan of Dublin City University (DCU) gave a very engaging outline of a group work tutorial that was run for a large service teaching module (work done in collaboration with Eabhnat Ní Fhloinn). Anthony Cronin of University College Dublin (UCD) discussed an interesting study he undertook involving the use of a novel software package, Realizeit, to provide Maths Support for adult learners.

The workshop resumed in the afternoon with an address by the second Keynote Speaker- Terry Maguire, who is the Chair of Adults Learning Mathematics - An International Research Forum and the Director of the National Forum for the Enhancement of Teaching and Learning in Higher Education. Terry gave a very interesting presentation on the topic of Adult Learners and Mathematics Support. She discussed some of the characteristics of adult learners and the implications for support offered to these students by MLCs. Finally the speaker introduced the Forum for the Enhancement of Teaching and Learning in Higher Education and outlined areas of possible collaboration between the Forum and the IMLSN community.

The final session in the conference consisted of two further short presentations. Ciarán O'Sullivan of ITTD gave an illuminating overview of some of the results from a large scale survey of Mathematics Learning Support in Irish higher education institutions that relate to adult learners (work done in collaboration with Olivia Fitzmaurice , Eabhnat Ní Fhloinn and Ciarán Mac an Bhaird). Timothy J. Crawford from Queens University Belfast (QUB) spoke on the MLS offered to mature students in QUB, which is primarily delivered through an appointment based model as well as online support that is tailored to specific student groups (work done in collaboration with Jonathan S. Cole). This informative presentation led to an open discussion on some of the issues surrounding MLS for mature students/adult learners at the various institutes represented at the workshop.

The day concluded with a discussion session, chaired by Ciarán Mac an Bhaird of NUI Maynooth (NUIM), where the possibility of sharing support materials between MLCs via Dropbox, among other topics, was discussed.

A total of 27 people from 18 different institutions (UCD, NUI Galway, DCU, Cork IT, IT Sligo, Sheffield Hallam University, Newcastle University, IT Carlow, Dun Laoghaire Institute of Art, Design and Technology, ITTD, QUB, NUIM, University of Limerick, Dundalk IT, Loughborough University, DIT, National Forum for the Enhancement of Teaching and Learning in Higher Education, Athlone IT) were in attendance. This workshop was funded by the Registrar's Office in ITTD, the School of Mathematical Sciences in DIT and the Irish Mathematical Society.

A full list of abstracts and slides from each presentation can be found here:
`http://supportcentre.maths.nuim.ie/mathsnetwork/workshop8`

Report by Cormac Breen, Dublin Institute of Technology.
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Groups in Galway 2014
23–24 May 2014, NUIG

Groups in Galway, an annual conference on group theory and related topics which has been running since 1978, was held at NUI Galway on 23–24 May. The conference had 35 participants and featured seven talks of speakers from Ireland, UK, continental Europe and USA. The following wide range of topics were covered: homology of linear groups, algebraic K-theory, hyperbolic manifolds, block algebras and fusion systems, algebras of essential relations, and Moonshine. The speakers and titles were:

- **Philippe Elbaz-Vincent** (Université de Grenoble):
  The group $K_8(\mathbb{Z})$ is trivial
- **Herbert Gangl** (Durham University):
  On the homology of linear groups over imaginary quadratic fields
- **Michael Tuite** (NUI Galway):
  A brief history of Moonshine.
- **Markus Linckelmann** (City University London):
  A characterisation of nilpotent blocks
- **Radha Kessar** (City University London):
  On transitive block fusion systems
- **Kevin Hutchinson** (UCD):
  Hilbert's third problem and scissors congruence groups
- **Grant Lakeland** (University of Illinois at Urbana-Champaign):
  Systoles and Dehn surgery for hyperbolic 3-manifolds
- **Jacques Thévenaz** (EPF Lausanne):
  From finite sets to group algebras

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Besides talks, there was also a poster competition for students and young researchers, and prizes were awarded according to conference participants' vote. Further details of the program, as well as some photographs from the event, can be found at

`http://www.maths.nuigalway.ie/conferences/gig14/`

The organizers, Sejong Park and Alexander Rahm, are grateful to NUI Galway (Registrar's Office and The Millennium Research Fund) and the Irish Mathematical Society for financial support of the conference.

Report by Sejong Park (NUI Galway)
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Women in Mathematics Day Ireland (WIMDI)
1 May 2014, NUIG

The National University of Ireland, Galway, in conjunction with the NCE-MSTL (UL) and MACSI (UL), hosted the 5th Women in Mathematics Day, Ireland. This free event took place on the 1st May 2014 in the School of Education, NUI Galway. The day included presentations and discussions by women active in mathematics, mathematics education and industry, and at a variety of career stages. The aim of the event was to bring together those passionate about mathematics and to demonstrate the variety of opportunities available to those engaged in mathematics/mathematics education. It was an opportunity to hear some inspirational talks and to informally chat to others at different career stages. While this is an occasion particularly for women active in mathematics to get together, men are certainly not excluded from this event!

Keynote speakers on the day includeDr. Patricia Eaton (Stranmillis University College, Belfast) and Dr. Fiona Blighe (Science Foundation Ireland). Dr. Eaton challenged us with deep questions such as what makes a mathematician, a question she struggled with herself when linking her two areas of passion pure mathematics research and mathematics education research. Her resolution emphasized that all of us involved in mathematics have a responsibility of passing on what is vitally important to the next generation and a responsibility

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The organizers were grateful for financial support from the School of Education, NUI Galway, the National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL, UL), the Mathematics Applications Consortium for Science and Industry (MACSI, UL), and the Irish Mathematical Society whose funding was used to provide travel bursaries for postgraduate students.

For further information please see
http://www.conference.ie/Conferences/index.asp?Conference=371

Report by Máire Ní Ríordáin, NUI, Galway
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Participants at the Linear Algebra and Matrix Theory Meeting

Second Irish Linear Algebra and Matrix Theory Meeting
29 August 2014, UCD

The 1st Irish Linear Algebra and Matrix Theory Meeting was held at NUI Galway in December 2012 and was organized by Rachel Quinlan and Niall Madden. Following the success of the first meeting, the hope was that the meetings will continue to bring together and stimulate Linear Algebra and Matrix Theory community.

The 2nd Irish Linear Algebra and Matrix Theory Meeting was held at University College Dublin on August 29th 2014, and was organized by Helena Šmigoc. The meeting was held in an intimate environment of Ardmore house, that allowed participants to meet and discuss during coffee and lunch breaks. Ten talks were given at this one day conference, with topics ranging from Numerical Linear Algebra to Education Issues in Linear Algebra: *Paul Barry*, Structural elements of the Riordan group of matrices, *Niall Madden*, Linear solvers for a singularly perturbed problem, *Thai Anh Nahn*, Analyses of preconditioners for problems with boundary layers, *James Mc Tigue*, Maximal nonsingular partial matrices, *Rachel Quinlan*, A proof evaluation exercise in an introductory linear algebra course, *Anthony Cronin*, Using an online adaptive learning tool to enhance the student's learning experience of Linear Algebra, *Helena Šmigoc*, Some motivational applications of linear algebra, *Oliver Mason*, Positive in the extreme: extremal norms for positive linear inclusions, *Richard Ellard*, The Symmetric nonnegative inverse eigenvalue problem, *Thomas Laffey*, Comparing various ranks of a nonnegative matrix. Abstracts of the talks are available at the conference webpage: http://mathsci.ucd.ie/conferences/lamt14/.

The meeting was sponsored by the Irish Mathematical Society and Science Foundation Ireland.

Report by Helena Šmigoc, UCD
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CONFORMAL FIELD THEORIES ON RIEMANN SURFACES OF GENUS $g \geq 1$

MARIANNE LEITNER

This is an abstract of the PhD thesis *CFTs on Riemann Surfaces of genus* $g \geq 1$ in mathematics, written by Dr. Marianne Leitner under the supervision of Prof. Dmitri Zaitsev at the School of Mathematics, TCD, and submitted in August 2013.

The purpose of this thesis is to argue that N-point functions of holomorphic fields in rational conformal field theories can be calculated by methods from algebraic geometry. We establish explicit formulae for the 2-point function of the Virasoro field on hyperelliptic Riemann surfaces of genus $g \geq 1$. N-point functions for higher N are obtained inductively, and we show that they have a nice graphical representation. We discuss the Virasoro 3-point function with application to the Virasoro $(2, 5)$ minimal model.

The formulae involve a finite number of parameters, notably the 0-point function and the Virasoro 1-point function, which depend on the moduli of the surface and can be calculated by differential equations. We propose an algebraic geometric approach that applies to any hyperelliptic Riemann surface. Our discussion includes a demonstration of our methods to the case $g = 1$.

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TEACHER CHANGE AND PROJECT MATHS: IMPLICATIONS AND LESSONS LEARNED

SHANNON GUERRERO

Abstract. Because of the similar intents, practices, and expected outcomes of Ireland's Project Maths and the United States' Common Core State Standards for Mathematics, insights into promoting successful adoption of Common Core in the U.S. can be gained by looking at factors that have supported and/or hindered Irish teacher adoption of Project Maths-related content, pedagogy, and assessment. In this paper, a U.S. mathematics educator summarizes her experiences with and insights into Project Math based on a variety of observations, interviews and interactions with various aspects and players involved in Project Maths. This paper highlights observed strengths and limiting structural and personal factors related to teacher change and adoption of Project Maths. The paper then discusses an integrated framework for considering teacher change and concludes with suggested next steps for continued growth and development of Irish math teachers under Project Maths, as well as implications for professional development of teachers adopting Common Core in the United States.

1. Introduction

In the early part of 2013, I was hosted by the Center for the Advancement of Science and Mathematics Teaching and Learning (CASTeL) as part of my 3-month sabbatical visit to Ireland. As a mathematics educator and university faculty member engaged in professional development related to Arizona's recent adoption of the Common Core State Standards for Mathematics, I was interested in exploring the professional development of Ireland's post-primary mathematics teachers as part of Project Maths. While my initial insights into Project Maths were provided by an American colleague and predecessor [15], my interactions, observations and interviews with various players in Project Maths provided me with several

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lenses through which to view curriculum, assessment, instruction, and, most importantly for my visit, professional development related to Project Maths.

I approached my experiences in the Irish mathematics education system as an educator who was hoping to garner a few "lessons learned" with respect to continued professional development and teacher change within the context of Project Maths. More specifically, I was hoping to investigate the factors that have both helped and hindered Irish teachers' adoption of the content and pedagogy of Project Maths. While I brought my own research-based and professional experiences to bear in my investigations into Project Maths, I remained cognizant of my own limitations as a visitor within the Irish educational landscape and tried to approach my inquiries with a critical but non-evaluative eye. It is within this context that I hope to contribute to the national dialogue on Project Maths as an outside voice that can provide a fresh perspective on the progress and professional development of Project Maths.

1.1. **Context of My Visit.** Unlike the centralized governance and funding of schooling in Ireland, the history of education in the United States is one of decentralized governance that places primary authority for schooling on states and individual school districts. Since the 1980's and 1990's, states have taken a more regulatory role over schools by raising education standards, emphasizing state-mandated curriculum requirements, and requiring more frequent standardized testing. Similarly, the federal government has played a larger role in promoting test-driven accountability through acts like No Child Left Behind [23]. Some argue that this decentralized approach to determining individual states' curricula, along with an increased focus on state-wide, nationally mandated testing, has promoted a "national" mathematics curriculum that is unfocused, incoherent and driven by rote computational competence rather than conceptual understanding.

In an effort to build on the foundation laid by individual state standards and create a more focused and coherent curriculum, the National Governors Association Center for Best Practices and the Council of Chief State School Officers released the Common Core State Standards for Mathematics (CCSS-M) in the United States in 2010 [2]. The new content standards called for more depth and less breadth; increased rigor; student centred problem solving; and investigative learning [3]. In addition, the Common Core included Standards for Mathematical Practice, a set of process-based standards of mathematical expertise that stressed the practices, dispositions and processes that all mathematics educators should strive to develop within their students (e.g. problem solving, perseverance, reasoning, modeling, making and testing conjectures, etc.). Though not necessarily a national curriculum, the Common Core State Standards for Mathematics were adopted by 45 states and the District of Columbia and is supported by U.S. Department of Education funding initiatives like Race to the Top.

The state of Arizona adopted a modified version of the Common Core State Standards for Mathematics by basing a majority of its mathematics curriculum on the Standards. Since its adoption of Common Core in 2010, Arizona has been holding various types of professional development through state and regional offices to familiarize teachers with the new content, pedagogy and assessment associated with Common Core. Schools were required to adopt the new standards in kindergarten and first grade during the 2011-2012 and 2012-2013 school years, with full implementation at all grade levels by the 2013-2014 academic school year.

Though my role as a professional developer is not directly linked to the statewide efforts to help teachers gain knowledge, application, and integration skills associated with Common Core, I regularly provide grant-based professional development to primary and post-primary mathematics teachers to support increased content knowledge and to help teachers understand and adapt to the content shifts and instructional implications of Common Core at a classroom level. It is in this role that I became aware of Project Maths in early 2012.

As teachers and educators across Arizona were familiarizing themselves with the new content and pedagogical shifts associated with Common Core, I began investigating the instructional and content-based shifts occurring in the Irish mathematics education system due to Project Maths. Though the contexts in which these two reforms were taking place were decidedly different, I was struck by apparent similarities in intent with respect to the teaching and learning of mathematics in Project Maths and as part of Common Core. Like Common Core, Project Maths included shifts in content, pedagogy and assessment that emphasized student understanding of math concepts; increased use of contexts and application; connections to everyday experiences; mathematical investigations; and problem solving [20]. However, unlike Common Core, Project Maths was a nationally developed and mandated curriculum with a meticulously developed and implemented roll-out schedule that included state funding to support curriculum development, project dissemination, and continued professional development of post-primary mathematics teachers in both content and mathematics pedagogy.

Although many facets of Project Maths were intriguing from this side of the pond, I was especially drawn to the continued professional development of Irish mathematics teachers and wondered what type of impact that training was having on classroom practice. Because of the fact that Project Maths was rolling out on a timeline that was three years ahead of Common Core, I was particularly intrigued with the possibility of examining lessons learned from Irish math teachers that could be used to encourage classroom level change for American teachers engaged with Common Core.

2. Data Collection and Analysis

Due to my limited time in Ireland, I was unable to implement a large-scale classroom-based study of Project Maths. However, I was able to systematically investigate several facets of Project Maths through a variety of observations, interviews, and interactions with various aspects and players involved in Project Maths. Although I will be the first to admit that this is in no way an empirical study, I do believe it represents a systematic exploration of various aspects of Project Maths.

I began my investigations into Project Maths by reading several reports and papers that provided the impetus and foundation for the development and implementation of Project Maths. Just prior to my arrival, several more documents were released that provided valuable updates on the progress of Project Maths, teacher insights into teaching and learning under Project Maths, and the impact of Project Maths on student learning and motivation. Taken together, these documents provided me with a solid foundation with which to begin thinking about Project Maths and its impact on teacher practice through continued professional development efforts.

Once I arrived, I began meeting with and interviewing mathematics and mathematics education university faculty throughout Ireland. These meetings served to further my emerging understanding of Project Maths and to help me become familiar with the educational landscape and mathematics education system in Ireland. I attended the one-day *Why Math Matters* conference (8^{th} March 2013) organized by the Higher Education Authority, hosted at the University of Limerick, and designed to take stock and address challenges of practice and policy of maths education in Ireland. I then began meeting with national stakeholders from the National Council for Curriculum and Assessment (NCCA), the Educational Research Center (ERC), the Department of Education and Skills (DES), the National Center for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL), and the Center for the Advancement of Science and Mathematics Teaching and Learning (CASTeL). Finally, I met with and interviewed the Director of Project Maths; a regional development officer (RDO) for Project Maths; a former Project Maths pilot school teacher and new member of the Project Maths Development Team; and a Project Maths facilitator and Implementation Support Group member.

In order to build on the emerging insights provided by my interviews, conversations, and readings, I then attended two different sessions of Project Maths Workshop 8, which focused on developing student understanding, problem solving, and connections in Calculus. My role at these workshops was that of participant-observer. I sat amongst the participating teachers, collaborating on and completing various activities. I informed the teachers around me of the purpose of my visit and talked informally with several of them on their experiences with and insights into Project Maths. The RDO of these workshops then helped me gain access to a large, representative, interdenominational, mixed gender, socioeconomically diverse national roll-out school in suburban Dublin. I spent 3 days at this school and was able to observe 7 mathematics teachers, some more than once, teach in Project Maths courses ranging from 1^{st} through 6^{th} year and foundation to higher level. At the conclusion of my observations, I interviewed three of the observed teachers on their experiences with Project Maths, its impact on their classroom practice, and factors that have helped or hindered their adoption of the content and pedagogy of Project Maths. As a last foray into the maths education landscape of Ireland, I was able to attend a leaving cert higher level grind course during one of my last weekends in Ireland. Though this course did not necessarily lend any insights into Project Maths, it did provide an interesting lens into the high stakes leaving cert culture of mathematics education in Ireland.

Conclusions from my investigations have been based on triangulating "data" from my many varied experiences and interactions with Project Maths. After compiling notes from interviews, observations, informal conversations, readings, and reflections, I looked for emerging common themes and then coded my notes based on those themes. In order to protect the identity of participants and contributors, I am limiting the amount of information provided on leaders, teachers, administrators, faculty and schools that were a part of my investigations.

3. Strengths of Project Maths

As a visitor to Ireland and Project Maths, I was struck by the sheer magnitude of resources being leveraged to support the development, implementation, and evaluation of Project Maths. Coming from a country with a decentralized educational establishment, I was impressed with the collaboration and coherence of Project Maths from its inception to its execution. While I am sure there are many more strengths of Project Maths than I am able to cover in a few pages, below are the elements of the project that stood out to me as particularly notable.

3.1. **Collaborative Planning of Project Maths.** As has been previously documented [15], Project Maths incorporated an unprecedented partnership and collaborative approach between various divisions of the Department of Education and Skills (DES), including the National Council for Curriculum and Assessment (NCCA), the Teacher Education Section (TES), the Maths Inspectorate and the State Exams Commission (SEC). Through an iterative process of content development (by the NCCA), exam modification (by the SEC) and professional development and planning (by the Project Maths Development Team within the TES), Project Maths evolved as a centralized and well-coordinated program that has been exceptionally and singularly focused on the same set of goals and on moving the country in the same direction in terms of content, instruction, and student learning. Despite backlash from popular media and some academic and teaching circles, all sectors of the DES involved in Project Maths have remained steadfast in their support, recognition, evaluation and revision of all aspects of Project Maths.

3.2. **Coordinated Implementation of Project Maths.** The intentional consequence of such an integrated approach to the development and implementation of Project Maths is that each arm of the DES is delivering a similar message and backing up the products developed by the other arms. In developing course syllabuses, the NCCA worked with SEC, TES, and the math inspectorate to make sure the content and pedagogy of the new syllabuses were understood and endorsed by all parties. The SEC, in turn, developed a new set of exams that incorporated more problem solving, context, and application in order to support the content shifts incorporated in course syllabuses. In developing teacher workshops and in order to deliver a coherent package of professional development in line with the intent of the syllabuses and exams, the Project Maths Development team regularly collaborated with and incorporated suggestions from TES, SEC and NCCA. While one may argue about whether or not these intended shifts have yet been realized, it is apparent that teachers and students are receiving the same message from curriculum, professional development, and exams regarding the content and pedagogy associated with Project Maths.

Such a coordinated and collaborative approach to the development and implementation of Project Maths has allowed for a remarkable coordination and leveraging of resources. Coming from an educational landscape in the United States where innovative programs are often well considered but (seemingly) randomly selected, implemented, and funded based on localized, decentralized decision making, the coordinated effort to fund and support all aspects of Project Maths is especially impressive.

3.3. **Continued Profession Development of Teachers.** Perhaps one of the strongest elements of Project Maths is its commitment to a long-term professional development model that promotes and encourages teacher change over the course of several years of continued professional development. As part of its development and implementation, Project Maths includes 10 one-day workshops, with two workshops per year over the course of five years, designed to introduce teachers to each content strand through pedagogical modeling and hands on explorations within the context of new syllabuses. In addition, teachers are able to attend regional evening content courses to upskill their own content knowledge in specific areas related to the phased in content strands. These courses, though lasting only one day each, are intensely focused on the application of specific strands of content through the modeling of pedagogy associated with the aims of Project Maths.

Considering the scope of content and methodology needing to be addressed in these workshops, the sheer number of participants, and the geographical, logistical and organizational aspects of such a wide scale undertaking, the success of these workshops is laudable. The rate of attendance and satisfaction with the professional development courses, as reported by the Project Maths Development team, speaks for itself. At the time of my visit, over 80% of Irish maths teachers had completed the first eight pedagogical day courses, with the final two courses being offered during the 2013-2014 academic year. Over 4500 teachers had attended optional evening content courses. Teacher satisfaction rates with both course options remained steady at 99% approval. In addition to face-to-face course options, teachers have been encouraged to make use of a seemingly endless supply of physical and online resources (cd's, workbooks, sample activities, readings, videos, syllabuses, etc.) through the Project Maths website and other sources of online support.

More recently, additional resources have been allotted to upskill out-of-field teachers through the Professional Diploma in Mathematics for Teaching (http://www.ul.ie/graduateschool/node/347). This program relies on a national consortium of higher education institutions and regional education centers, led by the NCE-MSTL at the University of Limerick, to provide focused subject matter and pedagogical instruction relevant to Project Maths to the approximately 48% of out-of-field teachers teaching mathematics at post-primary level [22]. Over 350 teachers began this free program in autumn 2012, and it was expected that an additional 400 teachers would be supported to begin the program in autumn 2013.

Content and pedagogical shifts associated with Project Maths are asking teachers to make significant changes in the way they think about teaching and learning mathematics. The widespread, well supported, well attended efforts of the Project Maths Development Team and the Professional Diploma offered through NCE-MSTL are a significant step in the direction of supporting teachers in achieving educational change in mathematics at the post-primary level.

4. Factors to Consider

Despite fairly widespread systemic support for Project Maths, classroom level instructional change has lagged. According to published studies [12], Inspectorate reports, and my own classroom observations, a high proportion of teachers are still engaged in traditional approaches to mathematics teaching and learning. Teachers, by and large, still rely on a didactic approach that includes direct instruction and modeling followed by student imitation of computational procedures with very little variation into problem solving or real world applications. While some teachers, where Project Maths supports their educational philosophies and established educational practices, are making significant progress in adapting to the content and pedagogical shifts associated with Project Maths, post-primary mathematics teaching in Ireland remains largely unchanged, with teaching to the Junior and Leaving Certificate exams a primary focus at all grade levels.

At first glance, the lack of any noticeable change in instruction could be used by detractors to support claims that Project Maths is having no significant impact on mathematics teaching and learning in Ireland. However, in talking with many teachers, it is clear that they generally understand the mission of Project Maths and are desirous of making instructional change, but are hindered in their efforts to change by a variety of internal and external factors. For many teachers, the pedagogy of Project Maths is being adopted in a manner that aligns with their largely traditional approaches with few shifts in fundamental conceptual frameworks about what it means to teach, learn and do mathematics.

> *Teachers see isolated bits of what they can do but are not truly changing their practices. They are confused. They are clinging to practices that have served them in the past. It's hard for them to blend the old with the new. –Megan, Project Maths leadership team*

This type of change-without-change has been described as "first order change" [17] to describe teacher adaptation of innovation to fit established norms and practices. First order change forces the innovation to fit into teachers' instructional status quo resulting in little to no true change in instruction. This is quite different to second order change where instruction changes to fit the innovation. Until teachers are given an impetus or motivation to change, innovative practice remains largely unrealized because teachers continue to teach the way they have always taught. While Project Maths is providing that initial impetus through its national dialogue and professional development, until other factors like assessment, curriculum, and national confidence in the system kick in, teacher change will remain sluggish.

This instructional stalemate can be attributed to a number of factors inherent in the sheer scope and scale of Project Maths. Educational change takes time. Some models for change argue that a shift in one year of instruction in one content strand can take over three years to achieve significant instructional change since it can take that long for early concerns to resolve and later ones to emerge [14]. Project Maths is attempting to shift the content and pedagogy of five years of post-primary mathematics instruction across five content strands (i.e. statistics & probability, geometry & trigonometry, number, algebra, and functions) with three development levels (i.e. foundation, ordinary, and higher). By the very nature of the enormity of change expected as part of Project Maths, it can be surmised that it may very well be ten to fifteen years before large scale significant instructional change is observed. As one Project Maths team leader acknowledged, *"This is going to take 10 years and teachers are going to have to keep at it."*

The difficulties with the roll-out schedule that incorporated shifts in 1^{st} and 5^{th} year at the onset of Project Maths have been widely publicized and detailed in several other publications [4, 9, 15]. While such a roll-out schedule undoubtedly created difficulties and confusion for both teachers and students, there is no assurance that other roll-out paradigms would have alleviated these problems. Any roll-out paradigm would have been disruptive and cause for considerable change for both teachers and students. An in depth discussion on the pros and cons of the chosen versus alternative roll-out plans is beyond the scope of this paper. However, it should be acknowledged that in addition to impacting teaching and learning, the phased in approach and associated professional development plan resulted in a gulf between the immediate instructional and planning needs of many teachers and the professional development services provided. For many teachers, if the content strand focus of each workshop was not immediately applicable, the training resources ended up on their shelves with very little immediate use or applicability. By their own admission, it has been hard for many teachers to see the "big picture" of Project Maths, with the many and varied connections between content, pedagogy, training, and classroom practice, while in the midst of their five years of workshop-based professional development. For many Phase I teachers involved with the pilot implementation of Project Maths, it was not until they had completed all 5 years of workshops and training that they were able to "see" the more holistic intent, cohesiveness, and methodology of Project Maths. As such, it will take time for national roll-out teachers to understand, experience, synthesize, and apply the connections between content strands, grade levels, and instructional shifts needed for Project Maths.

Having acknowledged that very little change has occurred to date, it would be hasty to assume Project Maths will have no long term impact on mathematics teaching and learning. The twice-a-year pedagogical workshops, evening content courses, and Teacher Professional Diploma program are all laying a foundation upon which future change and further growth will occur. Having talked with many teachers, participated in Project Maths workshops, and witnessed the singularly supported and focused Project Maths commitment by various arms of the TES, I fully expect Project Maths to have a long term impact on teacher content knowledge, classroom practice, and student engagement with mathematics in Ireland. However, as Project Maths moves from its second phase focused on intensive professional development at a national level to a third phase focused on supporting sustained instructional shifts, there are several factors, both internal and external, personal and structural, that must be considered and addressed.

These factors are not criticisms of Irish math teachers or of the structure of the Irish mathematics education system. They are factors that I discussed with various players in Project Maths, read about in various articles and reports, and observed through my own interactions with various facets of Project Maths. In discussing these factors, I realize that some may be directly addressed and/or rectified, while others may just need to be considered as factors that are influencing teachers' adoption of Project Maths. This discussion is not necessarily a call to change, but may be seen as an outsider's perspective of external (or structural) and internal (or personal) barriers that need to be acknowledged and considered as Project Maths moves forward into its next phase of implementation. (It must be noted that the following discussion is, by no means, exhaustive. Rather, it represents the prevalent issues I witnessed and experienced during my short visit and immersion in Project Maths.)

4.1. **Structural (External) Factors.** There are three main categories of external factors that I see playing a role in the lag between the aims and professional development of Project Maths and actual classroom level instructional change. These "structural" factors are inherent in the educational landscape and exist beyond the teacher, yet they influence teacher practice and ability or willingness to change. None of these factors, in and of themselves, will prevent the success of Project Maths, but each should be considered a structural element of the mathematics education landscape in Ireland that is influencing Project Maths' ability to promote instructional change.

4.1.1. *Time.* Anyone who has taught or talked with teachers knows that lack of time is the universal quandary of a teacher's life. In the course of a normal teacher's day, there never seems to be enough time to teach, plan, assess, grade, collaborate, or reflect. When being asked to participate in professional development and then devote time to new planning and teaching associated with instructional change, time becomes even more of an issue. For teachers of Project Maths, issues related to time are especially challenging when considered with respect to timetables, time for reflective practice, and time for dedicated departmental interactions.

The very nature of schooling in Ireland, with an emphasis on a wide variety of content areas, has resulted in relatively short periods that limit teachers' abilities to promote student-centred approaches to learning. As has been discussed in other reports [15, 20], current timetabling does not necessarily support the instructional and learning needs of teachers or students. With many periods lasting only 30 to 40 minutes [15, 20], most teachers admit to having only

15 to 20 minutes of instructional time once logistical elements of instruction like attendance and homework have been attended to. Left with such a short amount of time in which to teach a concept, many teachers feel forced to resort to the ever-efficient method of direct instruction. Add to that the effect of sometimes meeting only four of the five instructional days of the week, and some teachers see a given class for less than a total of two and a half hours per week . In contrast, students in the United States typically spend 45 to 55 minutes in a single period, meeting for math every day of the week, for a total of over 4 hours of math instruction. When faced with a seemingly longer syllabus as a result of content shifts in Project Maths, teachers simply do not feel they have the time to focus on student-centred approaches that promote conceptual understanding, problem solving or real world application. [4, 20]

> *There's not enough time to get through the syllabus. Not enough time to teach all the content of the course, especially when you take into account the new pedagogy that takes more time in the teaching. -Cian, maths teacher & department chair*

While short instructional periods for maths has an obviously detrimental impact on teacher instruction and student engagement with content, there is an even more alarming and indirect consequence of timetabling on teacher practice and reflection. Planning for effective student-centred instruction takes a substantial amount of time to reflect, arrange, prepare, and envision. Teachers that I observed in Ireland taught up to 8 or 9 different classes in a single day, within and outside of mathematics, and never repeated the same course or grade level from one period to the next. Though a bit extreme, it is my understanding that this arrangement of teacher instructional scheduling is not outside the realm of normal practice at most schools. [19] Even with adequate resources and a well-defined and supported curriculum, it would be nearly impossible for teachers to effectively plan for 9 different classes 5 days a week. The result is the type of first order change previously discussed where teachers must adapt Project Maths to their existing and largely traditional teaching style and resources since there is no conceivable way to effectively contemplate or plan for that many different classes in a single week. As one teacher put it, *"It is amazingly difficult to plan* *and prep for nine student-centred, problem solving based lessons per day."*

Because of timetabling practices that sometimes focus on grade level, content strand, or other such normalized factors, teachers often end up teaching a different grade level and a different content strand every period without ever repeating a single course or grade level during the day. In contrast, when I was a secondary teacher in the United States, I taught (as an example) Algebra I for three periods and Geometry for two periods every day of the week. Not only did I teach fewer, longer periods, but after teaching my first period of Algebra, I was able to quickly reflect on what went well in terms of both content and pedagogy, and what needed to be modified for the next class. The teachers I observed in Ireland were allowed no such time to reflect on practice, let alone make subtle changes for reteaching later in the day. They simply moved on to the next class with very little thought about the nuances of content or pedagogy just experienced by both them and their students. By the end of the day, teachers had taught up to 8 or 9 different classes and had neither the memory nor the capacity to reflect back on the planning, content, and instruction for any single period or class.

Reflective practice is an elemental part of good teaching, especially when asking teachers to engage in substantial shifts in the way they think about teaching and learning mathematics. As teachers try new approaches, there are going to be ups and downs, methods that work well, and others that need tweaking. Reflecting on planning, instructional shifts, and student learning experiences is an important part of the change process and one that is limited for Irish teachers by the very nature of timetabling. With far too many different classes to plan for and far too little time for immediate reflection or change, teachers are left in a frenzied limbo of instructional status quo. By the end of the day, teachers are so inundated with the pedagogical particulars of each class, each student, and each lesson that they are unable to remember or reflect on what they just taught let alone what they taught eight periods ago.

One school-based structure that could help ease individual teacher's burdens, in terms of collegial support, collaborative reflection, and systemic change, is the mathematics department unit. However, for many post-primary schools, there is very little dedicated time for content departments to meet and truly collaborate on content, planning and instruction [19]. The mathematics departments of most teachers I talked to met only a few times a year, if at all. Undoubtedly, secondary teachers are a busy group with their many and varied extracurricular activities, involvements, and commitments. Timetabling further complicates the matter with teachers on or off campus at various times during the day and week. However, as teachers engage in this change process, it is important for them to be part of a community of practice, where they can talk, reflect and share with other teachers engaged in the same change process.

> *What does a functioning maths department look like? It's not a gripe session. . . not a cover your ass session to get ready for the inspector. . . not time to talk about what chapter to cover for the common final. We need to spend time talking about content. . . how we could teach something. . . share ideas. But, there's no history or culture of that here. – Conor, Project Maths development team member & former maths teacher*

> *Individuals talk and share ideas but, as a department, we don't meet to talk about maths or pedagogy. Our meetings are usually about school issues and policies. –Cloe, maths teacher*

4.1.2. *Textbooks.* Because of the close interplay between curriculum and instruction, a considerable shift in instruction must be accompanied by curriculum that supports such change. While the Project Maths Development Team has done a commendable job of providing teachers with a plethora of activities and resources through their training and online/print resources, teachers inevitably rely on their textbooks as significant determiners of the content of their instruction. While analysis of textbooks was not part of my visit to math classrooms in Ireland, recent research [4, 9, 15, 18] and discussions with teachers highlight the disconnect between widely adopted texts and the intent of Project Maths.

> *Teachers teach from textbooks that are very predictable and serve their traditional approaches well. – Megan, Project Maths leadership team*

While textbooks remain largely traditional and procedural in their approach, teachers are being asked to make significant shifts toward problem solving and understanding in their teaching. As such, teachers are compelled to individually integrate and synthesize their traditional classroom texts with Project Maths syllabuses, Project Maths teaching and learning plans, Project Maths instructional resources, and their own ideas and experiences in mathematics teaching and learning. The result is that many teachers are trying isolated bits of Project Maths resources, but most teachers are not fully integrating Project Maths content or pedagogy into their teaching. Even where teachers are willing to incorporate a substantial amount of Project Maths materials, they still feel pressure to use texts because of the value "the system," especially students, places on texts. While effective teachers will use multiple sources in their planning and instruction, Irish teachers currently lack a single foundational resource from which to plan, assess, and create instructional experiences that align with the intent of Project Maths. [19]

> *Planning and resources are difficult. I try to use as much stuff from Project Maths that I can, but it's not super friendly for use in math classes. We're told that these materials are not classroom ready. Textbooks are. They're very traditional, though. –Sarah, maths teacher*

> *Am I not a good teacher for using the book? Sometimes I need to show them how to do something and have them practice. It's not the best, but it's all I've got. –Cloe, maths teacher*

4.1.3. *Certificate Examinations.* As with other American researchers who have visited Ireland before me [15], I was struck by the pervasiveness of the certificate exams and the hold they have on the entire educational system in Ireland. In almost every discussion, reading and observation I conducted, certificate exams, especially the Leaving Cert, held sway in driving content and pedagogy in the mathematics classroom. While a discussion on the merits of an exam-driven system is beyond the scope of this article, it must be acknowledged that the Leaving Cert Exam drives mathematics education in Ireland [1, 15]. As such, Project Maths is attempting to find a balance between the pervasiveness of "teaching to the test" and teaching mathematics for understanding and application. However, striking that balance in the midst of a heavily focused testing culture, and helping students and parents understand that each of these aims is not mutually exclusive, remains a difficulty.

> *I'm not sure if our exam driven culture fits with the philosophy of Project Maths. Kids aren't going to say thanks for teaching me, they're going to say thanks for helping me get an A. How can we not study for 3 to 4 months when we are reviewing for 2 or 3 years of content for the exam? – Cian, maths teacher & department chair*

> *From the students' perspective my job is to prepare them to do well on the exam. From the parents' perspective a teacher needs to be done so they can help their students review old exams and grading schemes. Their job may not necessarily look like this under Project Maths. -Megan, Project Maths development team*

While the SEC has done a commendable job being adaptive by responding to the content and pedagogical shifts of Project Maths, the Leaving Cert exam still remains a bit of a moving target for many teachers and students [4]. As each subsequent strand has rolled out over the years, the cert exams have blended old and new content and problem solving applications.

> *Exam papers are the worst of the whole lot. We haven't had enough of the new type of questions to help students prep. – Cloe, maths teacher*

With the final roll-out of Strand 5 during the 2012-2013 academic year and the Leaving Cert exam reflecting fully implemented Project Maths syllabuses by June 2014, teachers will move beyond transitional glimpses of Project Maths reflected in the Leaving Cert Exam toward a more holistic understanding of the application and assessment of Project Maths. It could be expected that, with time, teachers will begin to settle into the new approach to content and problem solving reflected in the exams. Parents, students, and teachers need to become more comfortable and experienced with questions and expected responses that focus more on meaning and explanation rather than a singular focus on a correct answer. Experience, exposure, and classroom modeling will similarly help address concerns related to mathematical literacy on the exams. Through the revision of transitional exams and problems, the SEC will also gain more experience in the writing and posing of items that truly assess student understanding of content in authentic ways. Changes such as reducing the predictability of content and ordering of exam questions, while controversial in bucking the established cultural norms of testing in Ireland, are similar improvements in promoting authentic problem solving that will become normalized as teachers and students become more familiar with Project Maths content and assessment.

A final external cultural aspect of testing in Ireland that needs to be contended with is the emphasis on revision. There is, in fact, a deeply engrained culture (and lucrative industry) that assumes the last 3 or 4 months of third and sixth year need to be spent in revision. Parents and students feel that teachers are not doing their job well if they have not finished teaching third and sixth year by February so that the revision process can begin.

> *The cultural influence is great. Teachers will be done teaching by February so that they can spend four months revising. There is tremendous pressure on teachers to finish early so they can spend time practicing exam papers. -Conor, Project Maths development team member & former maths teacher*

As such, a course that is expected to take two (Leaving Cert) or three (Junior Cert) years to teach, is expected to be taught in a fraction of that time. Even in courses where new instructional and learning approaches are utilized, as exams approach, teachers and students feel the need to revert to more traditional approaches for exam preparation and learning [19]. As Project Maths continues to evolve, a culture so singularly focused on exam success and preparation needs to be balanced with a focus on learning for understanding. In some respects, this becomes an issue of helping parents, students, and teachers understand that a pedagogical approach more focused on developing deep conceptual understanding and application of mathematics will reduce the need for procedural revision. Engendering confidence in the content and pedagogy of Project Maths to promote meaningful learning will take time, experience, and continual promotion of the benefits of Project Maths to all parties involved.

4.2. **Personal (Internal) Factors.** In addition to the external factors discussed above, there are three main categories of internal factors that I see as playing a role in hampering teachers' ability and willingness to adopt pedagogical shifts associated with Project Maths. Internal factors are deeply rooted personal beliefs about teaching and learning and are often more difficult to overcome than external, structural factors [8]. Helping teachers alter their internal beliefs and understanding of Project Maths, the nature of mathematics, and what it means to "do" mathematics will be crucial to the success of Project Maths. Based on my discussions, observations, and interviews, I feel that Irish teachers, by and large, lack confidence in their own content knowledge, in the pedagogy of Project Maths, and in the content and alignment of Project Maths with national assessments. For some teachers, increasing confidence in all areas may simply involve allowing them the time to experience and engage with Project Maths content and pedagogy; for others, a more concerted effort of continued content development and site based support may be the answer.

4.2.1. *Confidence in Content Knowledge.* With anywhere from 33% to 48% of Irish math teachers being considered "out of field" [4, 22], it is no wonder that a lack of confidence is a large impediment to authentic implementation of Project Maths.

> *If teachers don't understand the maths first, then they can't implement the pedagogy well. Teachers with weak background will hide behind the textbooks. You can't deliver a good lesson if you don't understand the mathematics. If teachers have confidence in their maths knowledge they'll ask the open ended questions, go there, and be able to explore. -Conal, Inspectorate*

For those who are qualified to teach mathematics and should easily adapt to content changes and additions, Project Maths is presenting them with content they have never seen or have not seen in many years.

> *Familiarity with the new content scares me the most.*
> *– Cloe, maths teacher*

Interpreting the syllabus is also an area of great concern for teachers. Shifts in detail and language remain problematic for some teachers, especially when it comes to making connections between the content of the syllabus and how that gets enacted in the classroom.

In addition to introducing unfamiliar content, Project Maths is expecting teachers to make more concerted connections across content strands and between grade levels. These types of connections require depth of content knowledge and proficiency in teaching mathematics for understanding that many Irish maths teachers are unfamiliar with at this time. While the evening Project Maths content courses and Professional Diploma are going a long way toward addressing teachers' lack of content knowledge, for many teachers the act of teaching will go a long way toward helping them become more familiar and confident with the content of Project Maths. As recent reports have argued [19], immersion in teaching under the revised syllabus will help teachers understand their content better, as well as understand connections between concepts.

4.2.2. *Confidence in Pedagogy of Project Maths.* Project Maths is asking teachers to completely rethink and confront their established norms, assumptions, and expectations regarding mathematics teaching and learning. But many teachers lack confidence in their ability to teach mathematics in a student-centred, investigative way [4]. This lack of confidence manifests itself in concerns about time – time to teach in a student-centred, investigative way; time to get through a (perceived) longer syllabus; and time to finish months early in order to begin revision.

> *Teachers race through the two years for leaving cert program in order to leave time for revision during the last four months. We need to get over this reliance on revision and teach for understanding over the whole two years. Teachers, parents and students need to buy into the system and trust the pedagogy a little more.*
> *–Liam, NCE-MSTL*

Besides not trusting the timing of Project Maths content and pedagogy, teachers do not really trust the pedagogy of Project Maths to teach students for long term understanding and success. While it may take more time than direct instruction, a problem-based approach to mathematics has the potential to connect content and promote conceptual understanding. Teachers, though, see a trade off in teaching for understanding versus the loss of being able to cover every aspect of the text.

> *It takes too much time to teach this way and then I fall behind where I need to be. -Sarah, maths teacher*

In addition, a student-centred approach runs counter to an instructional culture, reinforced by parents and students, where teaching mathematics means "telling how to do" mathematics. Project Maths is asking all players (i.e. parents, students and teachers) to deal with the dissonance of transforming long established norms and assumptions about what it means to do mathematics. Along with procedural competency, "doing maths" now involves interpretation, problem solving, understanding contexts, quantitative reasoning, and applying mathematical models in non-routine ways. As with building confidence in content knowledge, many teachers may simply need well-supported experience with and immersion in new pedagogical approaches.

> *We have a tendency to give up new methodologies too easily. We need practice teaching out of our comfort zone. –Cian, maths teacher & department chair*

Learning a new methodology is not easy. Understanding the nuances of when to question, how to pose a problem, when to allow students to discuss a problem, and when to pull them in for redirection takes practice. Through classroom-based experiences, collaboration, and reflective practice, teachers will be allowed to discover for themselves which pedagogical approaches are most appropriate for which situations and how their pedagogical decisions can be leveraged to promote connections between and within mathematics.

4.2.3. *Confidence in Content and Assessment.* While some would argue that Ireland needs to shift to an "exam led" culture over its current "exam driven" educational culture, it remains a fact that the Leaving Cert exam dictates what happens in the mathematics classroom, in terms of both content and pedagogy. The stakes for shifting long-established norms for content and pedagogy remain high. The result is that, in order to make sure their students have all the content they could possibly need for the exam, teachers admit that they try to teach all of the old content along with the new.

Rather than trusting a problem-solving approach to teach students how to deal with unfamiliar content and applications of mathematics in an exam-based situation, teachers are trying to prepare their students for every eventuality. This approach actually runs counter to the intent of shifts that have been made in both the Leaving Cert and Junior Cert that ask students to solve problems that, while supported by Project Maths syllabuses, may not have been explicitly linked to a specific learning outcome. We see the result of this disconnect in the annual backlash regarding wording, phrasing, and specific content of annual state certificate exams. Students and teachers are still adjusting to a problem solving based approach to assessment that removes all previously guaranteed elements of predictability and choice [4].

> *The predictability of the old papers is still causing problems since teachers are trying to predict new exams and teach everything. – Megan, Project Maths leadership team*

Because the exams have been seen as a bit of a moving target, the discomfort and lack of confidence in connections between Project Maths content and pedagogy and the exams themselves is understandable.

> *We're still caught up in these transition exams and are unsure how these will eventually look, what will be on them. -Cian, maths teacher & department chair*

It should be expected, too, that the SEC has needed time to adjust to new content and assessment-related norms and expectations in trying to create national exams and specific questions that delve deeply into student conceptual understanding, problem solving, and mathematical reasoning. Ambiguous questions, mathematically invalid solutions, and unclear expectations, though rare and unfortunate, are to be expected in this time of flux. Flexible rubrics that emphasize process over product are commendable. Ridding the country of exams where teachers and students universally know to avoid problem number six and know exactly which formula to use for problem number one is laudable. However, as with the content and pedagogy of Project Maths, teachers and students need time to adjust to these new exam-related norms, expectations, and problem situations. Now that all strands of Project Maths have rolled out, it can be assumed that the Junior Cert and Leaving Cert exams will more closely align with the content and problem solving approach of Project Maths. And, as frequently acknowledged throughout my many visits and interviews, once the exam becomes more predictable (in terms of methodology, if not content), then teachers will have the confidence in "the system" to begin making steps toward instructional change.

> *If assessment changes to reflect student understanding then that will force teachers to change. Exams will ultimately drive classroom practice. – Conor, Project Maths development team member & former maths teacher*

As can be seen from the above discussions, despite (or perhaps because of) a tremendous investment in continued professional development, teachers still have deeply entrenched concerns about Project Maths content, pedagogy, syllabuses, and assessment. The educational context in which these concerns arise has several structural factors further contributing to the difficulty of classroom level adoption of Project Maths. As Project Maths moves forward, it will need to address external, structural factors related to time, resources, and certs while continuing to support teachers to experience classroom-level success in order to help them overcome lack of confidence in their own content knowledge, the content and pedagogy of Project Maths, and national assessment paradigms.

4.3. **Addressing Factors Limiting Change.** Overlying each of the factors discussed above is the cyclical nature of change and an understanding that huge shifts in culture and long-established practice will take time. In some sense, external (or structural) factors may be easier to address than internal (or personal) factors.

External factors, such as textbooks and certs, are already being addressed as a natural extension of time and experience with Project Maths. Issues related to the structure and content of textbooks were raised by researchers [18] and a national dialogue has already begun around this issue. With the final rollout of Strand 5, questions about the content and structure of the Junior and Leaving Certs are expected to wane as exam writers become more experienced at developing fair and accurate assessments and students and teachers become more familiar with exam structures. With time and continued support, teachers will be better able to incorporate content and pedagogy that will prepare students for these types of evaluative experiences. Similarly, teachers and students can focus on modeling and developing the mathematical literacy skills needed to succeed on these exam questions. Perhaps the most pervasive and difficult external factor to address will be time. Structural issues related to short periods, limited instructional time, overtaxed teachers with too many periods and too many courses to prep for, and an educational configuration that systematically limits collegiality and reflective practice need to be addressed at national and local levels. While it is not realistic to overhaul the entire structure and timing of post-primary schooling and scheduling, a structured, systematic way to support embedded reflection, teacher collaboration, and focused planning must become part of the mathematics educational culture in Ireland if Project Maths is to ultimately change teaching and learning.

Internal factors related to teachers' lack of confidence in content, pedagogy, and assessment of Project Maths will largely be a matter of continued support and immersion in classroom contexts and reflective practice. This is not to say that teachers' comfort and confidence will simply increase with no further intervention. Rather, a continued focus on instructional growth with external support and resources, along with contextualized classroom-based experiences, will help teachers become more comfortable with and confident in the pedagogical and content-based shifts associated with Project Maths.

5. Contextualizing Teacher Change

While looking at barriers to teacher adoption of Project Maths may seem like a "deficit model" approach to teacher change, it is important to understand the context of mathematics teaching in Ireland and the factors that may be limiting change for some teachers. Obviously, teachers, teacher experiences, and teacher adoption of Project Maths vary from person to person and school to school. Throughout my entire Irish experience, teachers, by and large, exhibited thoughtfulness and effort in trying to understand the intent and approach of Project Maths and demonstrated genuine concern for the potential impact of Project Maths on their students. In trying to understand the various internal and external factors discussed above, this section will incorporate two frameworks, one related to the interaction of beliefs, practice and learning outcomes, and the other related to the developmental phases teachers go through as part of the change process. The use of such frameworks allows for examination of and informed decision making related to teacher change in Ireland in the context of documented themes and processes from existing research. Inherent in each of these frameworks is the understanding that continued professional development and support, aimed specifically at promoting authentic integration of Project Maths at an individual and classroom level, will be necessary in order to continue building upon the efforts of the national rollout and intensive professional development of Irish maths teachers to date.

As has been highlighted throughout this paper, changing teachers' beliefs about the nature of mathematics teaching and learning is a gradual and difficult process [10]. As they engage in the change process, it is expected that teachers will have various concerns and levels of engagement along the way. There are many paradigms, theories and frameworks with which to think about teacher change and with which to describe teachers' progress. Some [10] argue that shifts in teacher practice and student achievement need to occur before, and will ultimately result in, shifts in teachers' beliefs and attitudes. Simply put, "...significant change in teachers' attitudes and beliefs occurs primarily after they gain evidence of improvements in student learning." [10, p. 383] Other models, like the Concerns Based Adoption Model (CBAM) [11], are built upon an assumption that an innovation must first fit with individuals' beliefs and perceptions so that it can be incorporated into the operating principles of teachers. In other words, teachers must believe in the aims and philosophy of the instructional change model in order to act upon and internalize its basic tenets and instructional behaviors [17]. As a third option, teacher change models that focus on motivation contend that teachers must have dissatisfaction with their current practice and student outcomes in order to be motivated to engage in the change process. For these teachers, a desire to change stems directly from the belief that current practice is lacking and something needs to change in order to improve student learning.

An epistemological debate on the merits of various instructional change theories is beyond the scope of this paper. In fact, I believe none of these theories are mutually exclusive and that each may actually draw on the other. Though it seems a bit circular, change is a learning process in which teachers must engage in order to change. As such, changes in teachers' beliefs about the teaching and learning of mathematics influences their instructional practice which impacts student outcomes which, in turn, influences teachers' beliefs and promotes changes in practices (see figure 1). This more circular approach acknowledges that teachers' beliefs and practices are not static and continually influence the other.

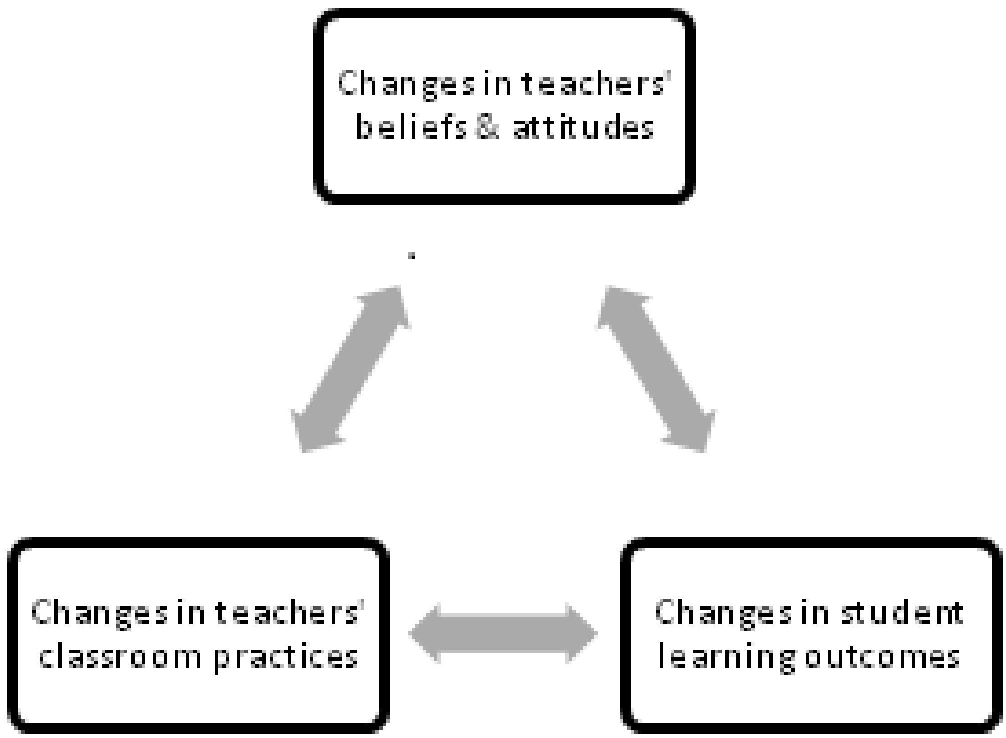

FIGURE 1. An integrated model of teacher change

Why is this important? Project Maths has invested a considerable amount of time and energy in continued professional development geared at changing teachers' classroom practices so that instruction will positively impact student learning outcomes, in both Leaving Certs and beyond. Though not explicitly addressed, it is assumed that teachers will likewise shift their beliefs by simply engaging in related continued professional development experiences and through immersion with Project Maths within the classroom. While true that some shift in attitudes and beliefs will occur as teachers begin to see a positive impact on student learning, classroom engagement, and performance on exams, it is apparent by the extensive lack of confidence in various elements of Project Maths and themselves (as discussed above) that teachers need much more than simple exposure or immersion to overcome some of these more internally driven concerns. Project Maths includes a much more fundamental shift in the way teachers think about the teaching and learning of mathematics than simply throwing in a hands-on activity here and there. Though teachers have been provided with initial skills and resources with which to begin making that shift, support throughout the next phase of adoption of Project Maths and change will be crucial.

Continued facilitation and encouraged collaboration at the school site level will provide teachers with a means of further refining their instructional practice through iterations of trying new approaches, sharing insights with colleagues, and trying again. Of course, the ironic potential consequence of this circular model of change is that teachers need to implement Project Maths with fidelity in order to see an impact on student learning that will drive changes in beliefs about the teaching and learning of math that will drive further changes in practice. But, if teachers are adopting Project Maths to fit into their largely traditional approaches then they will not see the expected impact on student learning and will not shift beliefs to align with a more conceptual, student-centred approach to mathematics teaching and learning, and will therefore not make authentic changes in belief or practice. In order to confront the potential for such a first order adoption of Project Maths, regional and site based leaders will need to play a continued role in supporting teachers throughout this change process.

Several models for change identify development phases individuals progress through as they become more aware of, engage in, accept and apply an educational innovation. These stages are fairly predictable, gradual, and common across models. For example, CBAM [11] identifies seven phases that include awareness, informational, personal, management, consequence, collaboration, and refocusing. Madinach and Cline [16] described a model where teachers progressed through stages of survival, mastery, impact and innovation. Duffy's [6] 9-point development model similarly progresses from early stages of confusion and trying out, to mid level stages of modeling and making sense, to later stages of creation and invention. When applied to the content and pedagogical shifts associated with Project Maths, I argue that we can analyze the changes Irish teachers are making, in the context of the integrated model of teacher change discussed above, according to three general phases that incorporate elements and general progressions outlined by these and other developmental phase change theories (see figure 2). In broadest terms, the first phase includes a focus on self whereby teachers are becoming aware of the innovation and thinking about its impact on their current practice. The second deals with classroom level impact, in terms of both content and pedagogy, and culminates with a focus on the impact of change on student learning. The final phase, which is realized several years down the road, is the result of authentic change and results in innovating practice and concerns related to broader educational impacts. By examining these three phases of developmental concern with an understanding that teachers are simultaneously and iteratively shifting practice, attitudes, and beliefs as they see impact of changes on student engagement, learning and achievement, then we can begin to contextualize the progress Irish teachers have made to date with adopting the pedagogical and content shifts associated with Project Maths.

Developmental Concern Three Phase Model	
Phase III	Impact on Broader Educational Community
	Original Innovation
Phase II	Impact on Student Achievement
	Impact of Project Maths Instruction on Student Learning Experiences
	Logistics of Classroom Implementation of Project Maths
Phase I	Impact of Project Maths on Established Teaching Practice
	Awareness and Interaction with Project Maths materials

FIGURE 2. A three phase developmental model of concern

Because of the intensive continued professional development efforts as part of the national rollout of Project Maths, teachers in Ireland are certainly aware of the scope and intent of Project Maths and have had ample opportunity to be exposed to and interact with the content and pedagogy of Project Maths. Most teachers, then, have progressed to the latter stages of Phase I or the initial stages of Phase II. Almost all of the teachers I interacted with were concerned with the impact Project Maths would have on their own practice (i.e. how they will incorporate it into their current practice) and the classroom level logistics of implementing instructional and content related shifts associated with Project Maths (i.e. how to rearrange timing of activities, progression of content, etc.). It should be noted that expressing concerns about various phases do not necessarily indicate action. Although teachers are thinking about the logistics of classroom implementation, they are still developing the skills with which to authentically implement and think holistically about Project Maths across grade levels and content areas. We see evidence of teachers' progression toward classroom related impacts and implementation via stated concerns about the personal (e.g. confidence in content knowledge, pedagogy, and content) and structural (e.g. time, textbooks, and certs) factors discussed previously. It is interesting to note that concerns about performance on Junior and Leaving Certs is included in concerns about established teaching practice rather than student achievement because teachers are focused on the potential impact of instruction on cert performance rather than any true indication of concern about longer term achievement, understanding, and learning in mathematics. This cannot be a surprise given the exam-driven educational culture in Ireland and the very high stakes involved for all players.

While teachers have progressed through stages related to instructional impact and implementation of Project Maths, they have not fully progressed into dealing with concerns related to the impact of instructional shifts on student engagement with mathematics and long term achievement. By and large, teachers are exploring various resources, scrutinizing new syllabuses, and dabbling in elements of Project Maths but are largely relying on established practice and resources. Very few teachers exhibited reflection on or understanding of the potential impact of instructional shifts on student learning and long term conceptual understanding of mathematics. True change and full implementation of Project Maths will occur when teachers have moved through the phases of development beyond concerns about national exams, textbooks, syllabuses, and impact on instruction, to concerns about the impact of their pedagogical decisions on student learning beyond achievement on exams. True shifts in beliefs and practice will be realized when teachers become committed to improving the student learning experience and believe this will occur through authentic adoption of Project Maths. Part of this shift will occur through continued exposure to and classroom experience with various elements of Project Maths. However, a true shift to impact and innovation will occur with continued support and resources aimed at classroom level implementation of Project Maths. Through sustained support and experience, new ideas and principles about mathematics will emerge when teachers begin seeing the positive shifts and results of Project Maths.

6. Next Step for Project Maths

The professional development efforts of Project Maths to date have laid the foundation for change and provided a common experience around which teachers can engage in professional dialogue, reflection and collaboration. Teachers have begun making shifts in their thinking regarding the teaching and learning of mathematics but remain largely concerned with details of the shift (e.g. syllabuses, exams, time, etc.) rather than focusing on the impact of instructional shifts on student learning. Fullan [7], in examining the effectiveness of several models for change, discusses several premises that must be met in order to support the change process. The most relevant for considering change in Ireland include capacity building with a focus on results, learning in context, and reflective action. Capacity building, or "...any strategy that increases the collective effectiveness of a group to raise the bar and close the gap of student learning," [7, p. 9] includes an emphasis on positive pressure that motivates and provides resources to support and encourage growth. By providing continued positive pressure for change and leveraging the resources and collective professionalism of Irish teachers through continued, localized support, national educational leaders can build capacity for Project Maths and promote and capitalize on internal accountability. Learning in context, a second premise, "...actually changes the very context itself," [7, p. 9]. By allowing teachers to engage in continuous and sustained learning in the classroom setting, the norms and structures of what is means to do mathematics and the culture of mathematics teaching and learning will change. Finally, reflective action includes purposeful thinking about what teachers do in the classroom and why they are doing it. Teachers will learn best and rethink their approaches to mathematics by "...doing, reflection, inquiry, evidence, more doing and so on."[7, p. 10]

When thinking about capacity building, learning in context, and reflective action as the "next steps" for Project Maths to further support and promote teacher change, there are three areas that stand out as particularly relevant. Though I am sure there are several more areas for growth that need to be considered by the Project Maths development team and other project leaders in determining where to go next with Project Maths, I was continually drawn to the need to clarify the message of Project Maths, provide continued and sustained site-based support, and stay the course.

6.1. **Clarify the Project Maths Message.** As part of capacity building and promoting ownership of Project Maths for students, parents, teachers, principals, and leaders alike, I feel Project Maths needs to clarify its message on two different fronts.

First, parents and students need a clearer message regarding the why of Project Maths [4, 5]. Although not a problem in the early days of Project Maths [15], there is now a small but loud faction of dissenters that is promoting "crisis rhetoric" that can potentially undermine current gains and inhibit future progress. Actively engaging in public dialogue about the research-based rationale behind Project Maths will help students, parents, and the general public better understand the intent and potential outcomes of Project Maths. Messages on project websites and research-based publications have not been sufficient to help inform parents and students of the various aspects of and connections between Project Maths, national assessments, and lifelong learning. Note that this is not encouragement to provide fodder for dissenters or engage in public debates to no avail. Rather, it is an opportunity to build upon a message that has been started but not fully developed or widely distributed.

> *In hindsight, we should have explained to parents that teaching under Project Maths was going to be different. There would be some disruption. Explain to them why the changes are taking place, what it will look like on their side of things. Engage with them and students to explain why, what and how to take the pressure off teachers. Give them rationale and understanding so they know teachers are being supported through these changes. –Michael, Project Maths program leader*

Second, teachers need to understand that Project Maths is promoting both pedagogical balance and incremental change. Although the Project Maths professional development team has done a very good job of sending a message of balanced pedagogy to Irish teachers through their day and evening courses, teachers largely view Project Maths as a huge and insurmountable pedagogical leap from their current practices. They interpret Project Maths as a complete departure from their more traditional approaches, rather than a "combined approach" [9] that incorporates reform-oriented approaches to mathematics along with appropriate aspects of more traditional instruction.

> *Project Maths is very good at sending the message of using appropriate strategies for your students but teachers leave the training sessions with the message that all math needs to be hands-on exploration. They see the alternative approaches and get a message that this is the way our teaching should always be. Teachers can be their own worst enemies. If a mix needs to be stipulated then we should tell them, even though teachers should know at the end of the day that mixed methodology is most appropriate. -Cian, maths teacher & department chair*

Likewise, teachers need to understand that change takes time. Though Project Maths may be promoting large shifts in instruction, it will take a series of small changes to get there. This is one message that was seldom heard during my time in Ireland.

> *Teachers see a huge gap of what Project Maths wants and what teachers are doing. A message of incremental change is not getting out. – Andrew, Project Maths program leader*

Through a concerted effort by Project Maths to continue sending a message of balanced pedagogical approaches and instructional decision making, along with continued support that promotes reflective action at the school-level and incremental change, teachers will begin to understand and act upon the take-away message of Project Maths:

> *There is a balance between skills and procedures and application. We've historically been skills and procedures in this country. We are trying to find that balance. We need to help teachers rethink what we're teaching and how we're teaching it. But, I think teachers are thinking that we're going too far in the other direction than we're really going. It's all about finding that balance. – Megan, Project Maths leadership team*

6.2. **School-Level Support.** As has been mentioned previously, Project Maths professional development efforts up to this point have laid the foundation for teachers to strengthen their content knowledge and begin thinking of teaching and learning mathematics in different ways. However, direct transfer and impact on teacher practice has been minimal. When asked which factors will promote actual classroom change, the overwhelming responses from teachers, leaders, administrators and Project Maths team members were: (1) school-level teamwork, collaboration, and communication and (2) strong local leadership interested in driving change. Interestingly enough, these are school-level factors that cannot be taught in a workshop but must be cultivated in context through reflective practice, collaboration, and access to additional resources, skills, and knowledge. This recommendation echoes findings of previous researchers in highlighting the importance of contextualized, resource-rich continued professional development [4, 5].

Two examples of self-sustaining initiatives that could be used effectively at a school-level in order to promote reflective action and learning in context are Lesson Study and Communities of Practice. Communities of practice [24] include small groups of teachers who are committed to continuous improvement of their craft by engaging in collective inquiry, collaboration and reflection into best practice. Communities of practice often focus on student learning, rather than instruction, to collectively undertake activities and reflection in order to improve student performance. Lesson study [13], a more refined focus on the impact of a single lesson on student learning, is a professional development strategy that can be used in conjunction with communities of practice or on its own. The process involves a small group of teachers examining their practice in depth and in the context of student learning by collaborating on, implementing, revising, and collectively reflecting on a jointly planned research lesson. Both of these approaches to promoting authentic teacher engagement in their own change have been proven successful at pilot schools in Ireland and provide a context for national schools to put Project Maths theory, content, and pedagogy into action. The skills and capacity needed to successfully implement these initiatives can be initiated and supported through the existing national structure of regional centers, regional development officers, and the Irish Maths Teachers Association. While an initial influx of funding will be necessary to train teachers in the use of such tools, professional development can be done within the national structure for support already in place. A primary example of building the infrastructure to support these self-sustaining initiatives is the recent "Math Counts: Insights into Lesson Study" conference held in Maynooth in November 2013 (http://projectmaths.ie/conferences/maths-counts.asp).

Along with new tools that promote classroom level collaboration and reflective practice, whole school shifts in prioritizing communication around practice need to occur. This is where strong leadership will play a role in determining the scope and direction of school wide change.

> *School leaders play a big part in helping teachers adapt. Principals need to be flexible to deal with the messiness of things like timetables, flexibility in scheduling and the like. Schools where principals are not as supportive have struggled. – Michael, Project Maths program leader*

As one inspectorate put it after observing many national roll-out schools, *"Schools with strong leadership that are supporting a culture shift are doing better than those lacking leadership."* Schools where the principal supports change and encourages the type of reflective practice that enables teachers to act as change agents will be more successful at truly adopting Project Maths. It is the school principal that will set the tone for change at a school and provide the structure and support, through explicit meeting times and positive pressure with a focus on results, for teachers and departments to spend dedicated time, energy, and resources in content area groups.

> *We have informal meetings at grade level, but there are very few organized maths department meetings. Whole school meetings focus on policies and procedures with pedagogy at the bottom. But, that should be switched so*

> *that pedagogy is discussed more and just tell us the new restroom policy. – Cian, maths teacher & department chair*

Timetabling and variations in contracts and hours often inhibit this, but schools need to find a way, perhaps through the use of Croke Park hours or other redistribution of contract hours, to enable teachers to meet as professionals and engage in the dialogue of their craft.

6.3. **Stay the Course.** Long term, meaningful change must be cultivated over time. It is often bumpy, messy and nonlinear in its progression. A strong resolve to stay the course with flexibility is paramount. "Failure to keep going in the face of inevitable barriers achieves nothing," [7, p. 11]. Project Maths is a research-validated approach to teaching and learning mathematics that needs further time and classroom-based support in order to realize its intended impact on student achievement and understanding in mathematics. If there is no follow up at a localized level to support teachers through contextualized learning, reflective action, and positive pressure that motivates, teachers will simply hold strong to their established practices. Teachers need school-based support to move past their initial self-focused concerns and worries into thinking about impact on student learning (beyond cert performance).

> *We need to hold the line. We need to keep moving forward with continued professional development. We need to make sure new teachers are coming in and moving up. We need to keep a focus on training our out-of-field teachers. We need to keep the RDO structure in place. We need a self-sustaining structure that might be supported by communities of practice. The departmental structure at secondary schools is not strong; we need to improve this. We can't just fall off into nothing. –Andrew, Project Maths program leader*

The teachers of Ireland care about their students and their craft. They are trying to engage in Project Maths in authentic ways, but need continued school-based support in order to deal with the structural barriers and internal factors that are hindering their continued growth. They need a structure in which to engage with one another in professional dialogue and reflective practice. They need to understand that small change is better than no change at all and stumbling along the way is an inevitable part of the change process. They need the support of their colleagues to try things, fail at them, share their experiences and try again.

> *The content is the content, and now we're being asked to rethink the pedagogy. I've been trying to do this all along. I think we're heading in the right direction but many teachers just need help in getting there. – Cian, maths teacher & department chair*

In the end, *"...you can't change teachers, you have to get them engaged in their own change,"* (Megan, Project Maths leadership team).

7. Implications for Common Core CPD

It is hard to compare the professional development efforts in Ireland with any past or current programs in the United States. While we in the US are certainly edging toward a near-national curriculum with Common Core, we are nowhere near having a centrally developed, funded, or implemented professional development plan to support implementation of Common Core. Helping teachers adapt to and adopt the content and pedagogy of Common Core is left to states and largely dependent upon state and local funding and existing professional development initiatives and structures. As with Ireland, there is a general expectation that once statewide assessments tied to Common Core change to reflect content and pedagogical shifts associated with Common Core, teachers will be left with no alternative other than to adopt the instruction and problem solving approaches inherent in Common Core. While the stakes associated with testing in the United States are directed more towards teachers and schools than students themselves, graduation from secondary school is often tied to students' performance and scores on such exams. In the meantime, teachers need support in interpreting and applying the new problem-solving, student-centred pedagogical and content-based approach of Common Core.

The prevailing model of professional development associated with most intensive Common Core-related projects is similar to that used by Project Maths. Professional development focuses on improving teachers' content knowledge through engagement in an intensive professional development program that includes one or two weeks of instruction during the summer and several weekends of follow up instruction during the academic year. Instruction is based largely on increasing teachers' content knowledge through modeling and engagement with content, explicit discussions centred around pedagogical decision making, and additional instruction in areas of assessment, student engagement, differentiation, and other related skills and knowledge. Communities of practice have long played a role in professional development, but one of the lessons learned from my time in Ireland is the absolute importance of longer term support at the school-level. While this one year intensive approach to Common Core appears to be successful, at least in the short term, there is no true measure of longer term gains related to changing classroom instruction and sustained teacher growth.

Like Project Maths, my work values the intensive content-based professional development workshops in laying the foundation for change. This initial experience provides teachers with the content and context around which to begin thinking about change. In and of itself, however, it has very little impact on teacher practice or student learning experiences. Like teachers in Ireland, teachers in Arizona struggle with issues of change and the resulting lack of confidence in their own content knowledge, the pedagogical shifts involved in a more student-centred problem based approach to mathematics instruction, and how those changes will be reflected in statewide assessments. True change will occur with longer term support that is focused on the context of the classroom and provides teachers with explicit experiences and expectations to engage in collaborative reflective practice. "Change is primarily an experientially based learning process for teachers..." [10, p. 384].

As Guskey noted, "Of all aspects of professional development, sustaining change is perhaps the most neglected," [10, p. 388]. Since change occurs after, during and throughout implementation, support coupled with positive pressure is needed for continued educational improvement. If Project Maths, or Common Core, is to be implemented well by teachers, the shifts in content and pedagogy must become part of teachers' instructional repertoire and used out of habit rather than conscious thought. Because change is a process rather than a single event, long term commitment to change and continued support are essential. "Support allows those engaged in the difficult process of implementation to tolerate the anxiety of occasional failure," [10, p. 388].

Project Maths and Common Core are both initiatives that are focused on changing teaching, changing learning, and changing the mathematics culture of students, parents, and teachers alike. As Fullan [7] indicated, such wide scale reform is not just putting into place new policy, but changing the culture of classrooms, schools, districts and, in the case of Project Maths, a country.

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IRELAND'S PARTICIPATION IN THE 55TH INTERNATIONAL MATHEMATICAL OLYMPIAD

BERND KREUSSLER

From 3rd until 13th July 2014, the 55th International Mathematical Olympiad took place in Cape Town (South Africa). A total of 560 students (56 of whom were girls) participated from 101 countries. This was the first IMO on the African continent. The four African countries Burkina Faso, Gambia, Ghana and Tanzania participated for the first time in the IMO. Moreover Botswana, Madagascar and Myanmar were represented by an observer in order to prepare their participation in IMO 2015.

The Irish delegation consisted of six students (see Table 1), the

Name	School	Year
Luke Gardiner	Gonzaga College, Ranelagh, Dublin 6	5th
Oisín Faust	The High School, Rathgar, Dublin 6	6th
Seoirse Murray	Maynooth Post Primary School, Co. Kildare	6th
Ivan Lobaskin	St Benildus College, Kilmacud, Dublin 14	6th
Karen Briscoe	Kinsale Community School, Co Cork	6th
Oisín Flynn-Connolly	Home-schooled, Ballyjamesduff, Co Cavan	5th

Table 1. The Irish contestants at the 55th IMO

Team Leader, Bernd Kreussler (MIC Limerick) and the Deputy Leader, Gordon Lessells (UL).

1. Team selection and preparation

Each year in November, the Irish Mathematical Olympiad starts with Round 1, a contest that is held in schools during a regular class period. In 2013 more than $10,000$ students, mostly in their senior cycle, from about 240 second level schools participated in Round 1. Teachers were encouraged to hand out invitations to mathematics enrichment classes to their best performing students.

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At five different locations all over Ireland (UCC, UCD, NUIG, UL and NUIM), mathematical enrichment programmes are offered to mathematically talented students, usually in their senior cycle of secondary school. These classes run each year from December/January until April and are offered by volunteer academic mathematicians from these universities or nearby third-level institutions.

Rarely, students who participate for the first time in the mathematics enrichment programme qualify for the Irish IMO team. Usually, those who make it to the team come back after their first enrichment year to get more advanced training. In order to activate the full potential of these returning students, last year an Irish Maths Olympiad Squad was formed. It consisted of the 11 best performing students at IrMO 2013, who were eligible to participate in IMO 2014. Between IrMO and the restart of the enrichment classes, for this group of students the following extra training activities were offered: two training camps (one in June and one at the end of August), a remote training which runs from September to December and participation in Round 1 of the British Mathematical Olympiad (November).

The centrally organised remote training was offered for the first time in 2013. At the beginning of each of the four months from September to December, two sets of three problems were emailed to the participating students. They sent back their solutions before the end of the month by email or traditional mail to the sender of the problems, who gave feedback on their attempts as soon as possible. The eight trainers involved were: Mark Flanagan, Eugene Gath, Norbert Hoffmann, Bernd Kreussler, Gordon Lessells, John Murray, Anca Mustaţa and Andrei Mustaţa.

An important component of the training for mathematical olympiads is to expose the students to olympiad-type exams. In recent years it became an established tradition in all five enrichment centres to hold a local contest in February or March. In addition, this year a number of students from Ireland was invited to participate in the British Mathematical Olympiad Round 1 (29 November 2013) and Round 2 (30 January 2014). I would like to thank UKMT, in particular Geoff Smith, for giving our students this opportunity.

The selection contest for the Irish IMO team is the Irish Mathematical Olympiad (IrMO), which was held for the 27^{th} time on Saturday, 10^{th} May, 2014. The IrMO contest consists of two 3-hour papers on one day with five problems on each paper. The participants of the IrMO, who normally also attend the enrichment classes, sat the exam simultaneously in one of the five centres. This year, a total of 104 students took part in the IrMO. The top performer is awarded the Fergus Gaines cup; this year this was Luke Gardiner. The best six students (listed in order in Table 1) were invited to represent Ireland at the IMO in Cape Town.

During the past 12 months four training camps were organised at various locations. During these mathematically intense 3 to 5 day long events, students have the opportunity to socialise with their enthusiastic peers and to increase motivation for their work throughout the year.

A kick-start camp for the remote training was organised in Cork for the wider squad from 28 August until 1 September 2013. From 4 to 6 June 2014 the wider squad for IMO 2015 was invited to a training camp that took place at MIC Limerick. As the LC Examinations started on these days, four of the six team members could not participate. A training camp for the six members of the Irish IMO team was held at the University of Limerick from 24 to 26 June 2014. The camps were organised by Anca Mustaţa, Bernd Kreussler and Gordon Lessells. The sessions with the students at these camps were directed by Mark Burke, Mark Flanagan, Eugene Gath, Norbert Hoffmann, Claus Koestler, Bernd Kreussler, Jim Leahy, Gordon Lessells, Anca Mustaţa, Andrei Mustaţa and last year's team members Adam Connolly and Jessica Weitbrecht as well as this year's team member Luke Gardiner.

Immediately before the IMO a five-day joint training camp with the team from Trinidad and Tobago was held at the University of Cape Town. The sessions were conducted by the two Deputy Leaders Gordon Lessells and Jagdesh Ramnanan. The students seem to have enjoyed this setting very much. The basis for the success of the joint camp was that the members of both teams have comparable ability levels.

A very special feature of this year's preparation was that Luke Gardiner was invited to participate in two training camps for the British IMO squad. These were the Hungary/UK IMO camp in Hungary (27/12/13 – 3/1/14) and the IMO training camp at Trinity College, Cambridge (3–7 April 2014). As a result, he considerably improved his performance compared with last year and was only one point short of a Bronze medal in South Africa. I would like to express my sincere thanks to UKMT, in particular to Geoff Smith, for their support of Luke's talent.

2. The days in Cape Town

The team (including Leader and Deputy Leader) arrived around noon on Tuesday, the 1st of July, at Little Scotia, a beautiful guest house within walking distance of the University of Cape Town (UCT).

From Tuesday afternoon until Saturday, the team was engaged in an intense training camp. On Wednesday, the team from Trinidad and Tobago arrived at Little Scotia. The joint dinner on that day was an excellent opportunity to get to know each other. From Thursday until Saturday the two teams spent many hours together to work on a mix of interesting problems. Thanks to the support of John Webb we were able to use a seminar room at UCT's Maths Department for the training sessions. As UCT lies at the foot of the Table Mountain, the daily walk to the training venue was a healthy morning exercise.

On Wednesday morning a ten-minute drive took me to the Hotel Garden Court, half way between UCT and the City Centre of Cape Town. As usual, the Jury was kept separated from the contestants until after the end of the second exam.

The Jury of the IMO, which is composed of the Team Leaders of the participating countries and a Chairperson who is appointed by the organisers, is the prime decision making body for all IMO matters. Its most important task is choosing the six contest problems out of a shortlist of 30 problems provided by a problem selection committee, also appointed by the host country. This year's Chairperson of the Jury was Prof. Sizwe Mabizela. In his serene manner, he led the Jury meetings in a pleasant but purposeful and efficient way.

Like last year, this year's Jury made an effort to have one problem from each of the four areas (algebra, combinatorics, geometry and number theory) included in problems 1, 2, 4 and 5. There was, however, a shortage in good easy and medium problems on the shortlist and many leaders felt that the shortlist was slightly biased towards combinatorics. For example, contest problem 5 was shortlisted as number theory, but it has a definite combinatorial flavour. After 15 hours of Jury meetings with intense discussion, the six contest problems were chosen and all translations and marking schemes approved.

On Sunday 6 July, the Irish team moved from Little Scotia to student accommodation on the campus of UCT and the IMO got under way. The opening ceremony took place in Jameson Hall of UCT on Monday afternoon. During the traditional parade, the teams appeared in order of the first participation of their countries at the IMO. Halfway, just after the appearance of the host team, the procession was broken up with an entertaining and skillful circus performance.

The two exams took place on the 8^{th} and 9^{th} of July, starting at 9 o'clock each morning. On each day, $4\frac{1}{2}$ hours were available to solve three problems. During the first 30 minutes, the students were allowed to ask questions if they had difficulties in understanding the formulation of a contest problem. The Q&A session on the first day of the contest, where 59 questions were asked, was not completed until 11.30 am, because scanning at the exam venue was very slow. On the second day, all the 105 questions were answered before 11.00 am.

Some contestants complained about the low temperature in the exam hall – during these winter days in Cape Town night-time temperatures were close to freezing – but the Irish students went well equipped with sufficiently many layers of clothing to the exams.

Thanks to the small distance between the leader's hotel and the exam venue, the student's scripts were available before 10 pm on the evening of the first exam day. Skimming through their work I quickly got the impression that this year's team had performed very well. Three of the solutions to Problem 1 were easily seen to be worth 7 points, whereas those of Ivan and Karen needed more detailed inspection. At the end of the third meeting with the coordinators, after explaining their solutions in detail, it was agreed that these were worth full marks as well.

After joining the contestants at UCT, Gordon and I went into the detailed study of our student's scripts. With five complete solutions to Problem 1 and three full solutions to Problem 4 (geometry) this was a pleasant experience.

On one of the coordination days, the students were entertained with an excursion to Cape Point, the Ocean View township and Boulder's Beach with its Penguins. On the other day they were given the opportunity to attend 'Celebrity Lectures' delivered by John Barrow, Peter Sarnak and Günter Ziegler. Our students seem to enjoy such activities as they feel that it gives them a first view into mathematical research beyond olympiad related mathematics. As a special treat, in the evening after the end of the final Jury meeting, Po-Shen Loh, leader of the USA team and specialist in combinatorics, explained to a large audience of interested students how a constant $c > 1$ can be achieved in Problem 6.

The final Jury meeting, at which the medal cut-offs were decided, took place on the evening of Friday, 11^{th} July. The closing ceremony followed by a Farewell Party was held on Saturday on the UCT campus. The journey back home started for our team on Sunday evening.

3. The problems

First Day (8^{th} July)

Problem 1. Let $a_0 < a_1 < a_2 < \cdots$ be an infinite sequence of positive integers. Prove that there exists a unique integer $n \geq 1$ such that

$$a_n < \frac{a_0 + a_1 + \cdots + a_n}{n} \leq a_{n+1}.$$

(Austria)

Problem 2. Let $n \geq 2$ be an integer. Consider an $n \times n$ chessboard consisting of n^2 unit squares. A configuration of n rooks on this board is *peaceful* if every row and every column contains exactly one rook. Find the greatest positive integer k such that, for each peaceful configuration of n rooks, there is a $k \times k$ square which does not contain a rook on any of its k^2 unit squares. *(Croatia)*

Problem 3. Convex quadrilateral $ABCD$ has $\angle ABC = \angle CDA = 90^\circ$. Point H is the foot of the perpendicular from A to BD. Points S and T lie on sides AB and AD, respectively, such that H lies inside triangle SCT and

$$\angle CHS - \angle CSB = 90^\circ, \quad \angle THC - \angle DTC = 90^\circ.$$

Prove that line BD is tangent to the circumcircle of triangle TSH.

(Iran)

Second Day (9^{th} July)

Problem 4. Points P and Q lie on side BC of acute-angled triangle ABC so that $\angle PAB = \angle BCA$ and $\angle CAQ = \angle ABC$. Points M and N lie on lines AP and AQ, respectively, such that P is the midpoint of AM, and Q is the midpoint of AN. Prove that lines BM and CN intersect on the circumcircle of triangle ABC.

(Georgia)

Problem 5. For each positive integer n, the Bank of Cape Town issues coins of denomination $\frac{1}{n}$. Given a finite collection of such coins (of not necessarily different denominations) with total value at most $99 + \frac{1}{2}$, prove that it is possible to split this collection into 100 or fewer groups, such that each group has a total value at most 1. *(Luxembourg)*

Problem 6. A set of lines in the plane is in *general position* if no two are parallel and not three pass through the same point. A set of lines in general position cuts the plane into regions, some of which have finite area; we call these its *finite regions*. Prove that for all sufficiently large n, in any set of n lines in general position it is possible to colour at least $\sqrt{n}$ of the lines blue in such a way that none of its finite regions has a completely blue boundary.

Note: Results with $\sqrt{n}$ replaced by $c\sqrt{n}$ will be awarded points depending on the value of the constant c. *(Austria)*

4. The results

The Jury tries to choose the problems in such a way that Problems 1 and 4 are easier than Problems 2 and 5. Problems 3 and 6 are usually designed to be the hardest problems. That this goal was met this year is reflected in the scores achieved by the contestants on the problems (see Table 2).

The medal cut-offs were as follows: 29 points needed for a Gold medal (49 students), 22 for Silver (113 students) and 16 for Bronze (133 students). A further 151 students received an Honourable Mention.

Overall, 38.2 % of the possible points were scored by the contestants, which is one point more that last year and the highest since 2004.

	P1	P2	P3	P4	P5	P6
0	75	240	479	24	301	514
1	23	32	43	103	60	7
2	14	25	1	28	83	7
3	22	17	2	16	10	11
4	15	14	3	5	8	0
5	18	39	0	3	3	5
6	23	71	4	3	11	1
7	370	122	28	378	84	15
average	5.348	2.971	0.505	5.189	1.709	0.296

TABLE 2. For each problem, how many contestants achieved how many points

Name	P1	P2	P3	P4	P5	P6	total	ranking
Luke Gardiner	7	1	0	7	0	0	15	296
Oisín Faust	7	0	0	7	0	0	14	321
Seoirse Murray	2	3	0	7	0	0	12	367
Oisín Flynn-Connolly	7	2	0	1	0	0	10	387
Karen Briscoe	7	0	0	0	1	0	8	421
Ivan Lobaskin	7	0	0	1	0	0	8	421

TABLE 3. The results of the Irish contestants

Table 3 shows the results of the Irish contestants. Writing a complete solution to a problem during the exam is a difficult task at a competition of this level, and is rewarded by the award of an Honourable Mention. All our students managed to achieve a complete solution of at least one of the problems. This is the first time ever that all students on the Irish team returned home with an award.

The figures in Table 4 have the following meaning. The first figure after the problem number indicates the percentage of all points scored out of the maximum possible. The second number is the same for the Irish team and the last column indicates the Irish average score as a percentage of the overall average. This table shows that our students performed above average on Problem 1 (algebra). Never before has an Irish team scored 37 or more points on a single problem. Their performance on Problem 4 (geometry) shows improvement in this subject area compared with the past.

Although the IMO is a competition for individuals only, it is interesting to compare the total scores of the participating countries.

Problem	topic	all countries	Ireland	relative
1	algebra	76.4	88.1	115.3
2	combinatorics	42.4	14.3	33.7
3	geometry	7.2	0.0	0.0
4	geometry	74.1	54.8	73.9
5	number theory	24.4	2.4	9.8
6	combinatorics	4.8	0.0	0.0
all		38.2	26.6	69.5

TABLE 4. Relative results of the Irish team for each problem

This year's top teams were from China (201 points), USA (193 points) and Taiwan (192 points). Ireland, with 67 points in total, shared the 64^{th} place with the host country South Africa. This is the second highest result an Irish team has ever achieved.

This year, three student achieved the perfect score of 42 points: Alexander Gunning (Australia), Po-Sheng Wu (Taiwan) and Jiyang Gao (China). The detailed results can be found on the official IMO website `http://www.imo-official.org`.

5. Outlook

The next countries to host the IMO will be

2015	Thailand	4–16 July
2016	Hong Kong	6–16 July
2017	Brazil	
2018	Romania	
2019	United Kingdom	

6. Conclusions

This year's Irish IMO team was one of the strongest in the 27 years of Irish participation at the IMO. Never before have all six students on the team been up to IMO standard in the sense that they could solve at least one of the problems completely. This is the first time ever that all six students on the Irish team returned home with an Honourable Mention.

When comparing Ireland with other countries, it is more meaningful to consider relative ranks than looking at absolute ranks, because the number of participating countries has increased over the years.

This year, 37% of the participating teams scored less than the Irish team; this is the second best performance ever.

Since Ireland's first participation in 1988, the Irish teams won eight medals and 31 Honourable Mentions, ten of these in 2013 and 2014. This underscores the increased ability level of the current students.

It is tempting to attribute the notable improvement over the past three years to the increased training activities. However, such a conclusion might be premature and it seems more prudent to wait a few more years to see if this improvement can be sustained. There is no doubt, however, that all the new training activities helped our students to develop their full potential.

In this context it should be mentioned that Ireland's involvement in the European Girl's Mathematical Olympiad (EGMO) certainly had a positive impact on the training and performance of the IMO team members. On the one hand, Karen's two participations at EGMO gave her a lot of useful experience. On the other hand, the extra training opportunities, geared towards the early date of EGMO in April, were offered to girls and boys alike.

Considering Table 4, we see that our students performed above average on Problem 1 (algebra). Even though their performance on Problem 4 (geometry) shows improvement in this subject area, it is clear that there is still some work to be done until our students reach an average level score on an easy geometry problem. To sustain the achievement level of this year's team, it seems necessary to increase the ability and confidence of our students to solve an easy IMO problem in all four subject areas (algebra, combinatorics, geometry and number theory).

In light of this year's success at the IMO, it is natural to continue to offer extra training opportunities for an Irish Maths Olympiad Squad that consists of those 10 to 15 best performing students at IrMO 2014, who are eligible to participate in next year's IMO. These activities help to keep the students motivated and working on olympiad related material.

Even tough the overall results of the Irish Team have improved during recent years, it is obvious that our students still have less experience in problem solving than the majority of the contestants from other nations. In addition to the enrichment classes, where the basic material is taught, it is important that potential team members gain experience in solving challenging problems on their own in limited time. Important contributions to this aspect of the training are the local contests at the enrichment centres and the involvement in Rounds 1 and 2 of the British Mathematical Olympiad. Including practice exams in the schedule of the training camps should become the norm in the future.

A number of other IMO teams regularly organise joint training camps that take place immediately before the start of the IMO. Joint sessions with other teams strengthen international relationships among mathematically gifted youth and enrich the training of all participating teams. The joint training in Cape Town with the team from Trinidad and Tobago was very successful and everybody agreed that similar camps should be held in future years as well, provided that sufficient funding is available. Prior to the IMO in Thailand 2015, such a camp could help the Irish contestants to adjust to the different time zone and the tropical climate.

To be able to fund such camps and to send a full team of six students and possibly also an Official Observer to any of the next IMOs, efforts have to be increased to get sufficient funding.

A generally established fact, based on the experience of many countries at the IMO, is that the earlier students are exposed to olympiad type problems the higher they are able to achieve in competitions and the more profoundly their problem solving skills are developed. Therefore, it seems to be imperative to get more students in their Junior Cycle, or even in Primary School, involved in mathematical problem solving activities. A few years ago, our colleagues at UCC initiated Maths Circles for Junior Cycle students in second level schools in the Cork area. As a follow-up to these, the training centre at UCC now runs Junior Maths Enrichment classes for students in second and third year. This initiative shows how it is possible to widen the scope of support of talent and interest in mathematics in Ireland.

Two initiatives should be mentioned here which aim at the involvement of the majority of pupils in mathematical problem solving. One is the PRISM (Problem Solving for Post-Primary Schools) competition which is organised since 2006 by mathematicians from NUI Galway and which takes place in October every year during

Maths Week. This multiple choice contest has a paper for Junior Cycle students and one for Senior Cycle students. It normally attracts about 2 500 participants. The other is the International Mathematical Kangaroo Contest which, thanks to the hard work of Michael Cotter and Mark Flanagan, took place for the first time in Ireland in April 2014. This contest consists of a multiple choice test with problems designed for the general student, not only for students with a particular talent in mathematics. With six different age levels this contest is available for all students, from first class in primary school to sixth year in secondary school.

7. Acknowledgements

Ireland could not participate in the International Mathematical Olympiad without the continued financial support of the Department of Education and Skills, which is gratefully acknowledged. Thanks to its Minister, Mr Ruairí Quinn TD, and the members of his department, especially Mary Whelan, for their continuing help and support.

Also, thanks to the Royal Irish Academy, its officers, the Committee for Mathematical Sciences, and especially Rebecca Farrell, for support in obtaining funding.

Also instrumental to funding the Irish IMO participation this year was the generous donation received by the Irish Mathematical Trust from Eoghan Flanagan, who was himself a member of the Irish IMO team in 1993 and 1994. Only this sponsorship, together with the funding by the DES, enabled Ireland to send a full team of six students with leader and deputy leader to the IMO. The pre-IMO training camp in Cape Town would not have been possible without Eoghan's generous sponsorship.

The foundation for the success of the contestants is the work with the students done in the enrichment programmes at the five universities. This work is carried out for free by volunteers in their spare time. Thanks go to this year's trainers at the five Irish centres:

At UCC: Tom Carroll, Sorcha Gillroy, David Goulding, Martin Killian, Clauss Koestler, Patrick McCarthy, Desmond McHale, Anca Mustaţa, Andrei Mustaţa and Jonathan Peters.

At UCD: Mark Flanagan, Marius Ghergu, Kevin Hutchinson, Eugene Kashdan, Adam Keilthy, Tom Laffey, Rupert Levene, Gary

McGuire, Lan Nguyen, Helena Smigoc, Nina Snigireva and Masha Vlasenko.

At NUIG: John Burns, James Cruickshank, Artur Gower, Niall Madden, James McTigue, Rachel Quinlan and Jerome Sheahan.

At UL: Mark Burke, Ronan Flatley, Eugene Gath, Norbert Hoffmann, Bernd Kreussler, Jim Leahy and Gordon Lessells.

At NUIM: Sonia Balagopalan, Stefan Bechtluft-Sachs, Stephen Buckley, Katarina Domijan, Peter Clifford, David Malone, John Murray, Anthony O'Farrell, James O'Shea, Lars Pforte, Adam Ralph, David Redmond and Richard Watson.

Thanks also to the above named universities for permitting the use of their facilities in the delivery of the enrichment programme, and especially to University College Cork, Mary Immaculate College, Limerick and to the University of Limerick for their continued support and hosting of the pre-olympiad training camps.

It should not go unnoticed that this was the twentieth time that Gordon acted as Deputy Leader of an Irish IMO team. A huge Thank You to him from all the Leaders and team members who enjoyed his skillful dealing with all the situations he has encountered over the years.

Finally, thanks to the hosts for organising this year's IMO in Cape Town and especially to the team guide in Cape Town, Eniola Kazeem. Special thanks go to John Webb, the Director of IMO 2014.

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MATTHEW O'BRIEN: AN INVENTOR OF VECTOR ANALYSIS

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Abstract. In the mid-nineteenth century, several mathematicians were engaged in a search for a new symbolism and methodology to express and solve physical problems in three dimensions. Hamilton's quaternions promised much but ultimately proved unequal to the task. Around 1880, the work of Gibbs and Heaviside led to the modern formulation of vector analysis. But decades earlier an essentially equivalent approach was formulated by Matthew O'Brien and applied to problems in geometry, statics, dynamics and optics. Although his work deserved greater attention, it was ignored by his contemporaries. One reason was that he was overshadowed by the towering and influential figure of Hamilton.

1. Introduction

Open any modern textbook on mechanics and bold letters will announce vector quantities. Vectors now play a central role in dynamics, fluid mechanics, elasticity and electromagnetism. Their popularity grew rapidly following the publication in the 1880s of Josiah Willard Gibbs' *Elements of Vector Analysis*, but there were many precursors of this work. The quaternions of William Rowan Hamilton promised much but were ill-suited to most problems in physics. In addition to Hamilton, another Irish mathematician contributed in a substantial way to vector analysis, namely Matthew O'Brien, the subject of this note.

2. Matthew O'Brien

Rev. Matthew O'Brien was one of several Irish mathematicians who flourished in Cambridge in the nineteenth century. Little is known about his personal life. He was born in Ennis, Co. Clare in 1814, where his father, also Matthew, was a medical doctor. He

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FIGURE 1. Matthew O'Brien (1814-1855). Illustration from Craik, 2008.

was ordained a clergyman of the Church of England. He had a son, Arthur Evanson O'Brien, who also graduated from Cambridge (in 1871) and who also took holy orders. Matthew O'Brien died in Jersey in 1855, while on a visit to recuperate from an illness.

In 1830 O'Brien entered Trinity College Dublin, aged just sixteen, and remained there until 1834. During his time in Trinity, he must have come into contact with William Rowan Hamilton, and was probably taught by him. Later, he became acquainted with Hamilton's work on quaternions, which generated widespread interest. Hamilton, whose attention from 1843 onwards was focussed almost exclusively on quaternions, made only the most fleeting reference to O'Brien in his published work. O'Brien made reference to Hamilton's work, most notably in his major paper of 1852. However, it is unclear just how much influence Hamilton's work had on O'Brien.

O'Brien was admitted to Gonville and Caius College Cambridge in 1834, aged 20. He was a pupil of the enormously effective tutor William Hopkins, who taught many of the leading mathematicians in Cambridge (Craik, 2008). Hopkins coached nearly 200 wranglers, of whom 17 were placed first in the entire university in the Mathematical Tripos, the final honours mathematics examinations, earning him the nick-name 'Senior Wrangler Maker'. In 1838 O'Brien was Third Wrangler, that is third place in the university. He was elected a Fellow of Caius College in 1840, and graduated as an MA the following year.

In 1844, O'Brien was appointed Professor of Natural Philosophy and Astronomy at King's College, London. From 1849 he also lectured in mathematics at the Royal Military Academy in Woolwich. In 1854 he resigned from King's to take up an appointment as Professor of Mathematics in Woolwich. Other applicants for the position at the Royal Military Academy included G. G. Stokes and J. J. Sylvester, the latter of whom was disgusted that he was not appointed. But Sylvester did not have long to wait. O'Brien died less than a year later, while recuperating in Jersey, and was succeeded at Woolwich by Sylvester.

3. Vector Analysis

O'Brien was the author of about twenty papers on mathematics. In addition, he published several books, including elementary texts on differential calculus and plane coordinate geometry and a treatise on mathematical geography. These books were good examples of expository writing and were moderately successful.

O'Brien's most notable contribution to mathematics was on the theory and application of vector methods. Hamilton's quaternions were cumbersome and were not convenient for applications. In quaternion algebra, if $q_1 = \mathbf{V}_1$ and $q_2 = \mathbf{V}_2$ are two quaternions with vanishing scalar parts, their product is

$$q_1 q_2 = -\mathbf{V}_1 \cdot \mathbf{V}_2 + \mathbf{V}_1 \times \mathbf{V}_2.$$

Thus, the scalar and vector products are inherent components of Hamilton's quaternions, but they are conflated into a single operation. Simpler, more suitable, methods were widely sought. Vectors have two multiplication operators, the vector and scalar products, which make them eminently suitable for application to a wide range of problems in physics. Ultimately, vectors proved invaluable in giving concise expression to the equations of fluid mechanics, electromagnetism and elasticity.

O'Brien's evolving ideas on vector formulations of mechanics were published in a series of seven papers between 1847 and 1852. A full list of relevant publications is given in Crowe (1967, p. 108, Note 92). O'Brien anticipated many of the results that appeared later in Gibbs' *Vector Analysis*. His treatment of vectors was remarkably complete. O'Brien defined the scalar and vector products for vector quantities. His notation can be confusing: he wrote $u \times v$ for the scalar (or dot) product and $u \cdot v$ or $Du \cdot v$ for the vector (or cross) product, essentially the reverse of the modern convention. He showed that the vector product is non-commutative and he derived several vector identities that are now standard.

O'Brien also defined the gradient operator; Hamilton had introduced this operator earlier and, presumably, O'Brien was aware of this although he did not explicitly acknowledge Hamilton in this context. His papers include identities such as

$$\text{curl curl } \mathbf{V} = \text{grad div } \mathbf{V} - \text{div grad } \mathbf{V}$$

although, of course, his notation was quite different. Indeed, O'Brien derived almost all the substantive results that appear in the initial chapters of Gibbs' *Vector Analysis*. He expressed the basic equations of mechanics in vector form and applied them to problems such as computing the Earth's precession and nutation, deriving an expression for the annual precession of the polar axis.

The history of vector analysis is recounted in considerable detail by Crowe (1967). One thing that O'Brien missed was the lack of associativity of the vector product:

$$(\mathbf{V}_1 \times \mathbf{V}_2) \times \mathbf{V}_3 \neq \mathbf{V}_1 \times (\mathbf{V}_2 \times \mathbf{V}_3)$$

Crowe put special emphasis on this omission. However, it is hard to doubt that O'Brien was well aware of it as, in the case where $\mathbf{V}_1 = \mathbf{V}_2$, the left hand side must vanish whereas the right hand side need not. It may be that O'Brien just took this for granted, much as we take for granted the lack of associativity for subtraction and division:

$$(a - b) - c \neq a - (b - c) \qquad \text{and} \qquad (a \div b) \div c \neq a \div (b \div c).$$

For example, O'Brien wrote the equivalent, in his notation, of $(\mathbf{U}\times)^2\mathbf{V}$ which, if interpreted as $(\mathbf{U}\times\mathbf{U})\times\mathbf{V}$ would vanish, but it is clear from the context that he intended $\mathbf{U}\times(\mathbf{U}\times\mathbf{V})$.

In his review of O'Brien's work on vectors, Crowe (1967) recognized him as a forerunner of Gibbs and Heaviside. In some of his papers, O'Brien back-tracked, reverting to "the ordinary notation of algebra". However, in the 1852 paper he returned again to his novel vector methods. Crowe paid most attention to O'Brien's last paper (O'Brien, 1852), making only passing mention of his earlier work on vectors. Smith (1982) rectified this, reconsidering the earlier work in more detail.

Crowe quoted C. G. Knott, an enthusiast of quaternions who, in 1892, wrote that "Gibbs and Heaviside had barely advanced beyond the stage reached by O'Brien [in 1852]". Crowe observed that "if Knott was correct, then O'Brien deserves great credit and must be called the father of modern vector analysis". As already stated, Crowe's discussion focussed on O'Brien's 1852 paper, but Smith's later analysis of the earlier papers suggests that O'Brien went farther than indicated by Crowe, coming very close to modern vector analysis.

4. The Rotating Earth

O'Brien presented an interesting application of his vector method in his 1852 paper. A year prior to its publication, Léon Foucault's pendulum experiment in Paris had generated widespread interest. O'Brien considered the equations of motion on the rotating Earth. He wrote down the expression for the rate of change of a vector quantity in the non-inertial Earth frame:

$$\left(\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t}\right)_{\mathrm{A}} = \left(\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t}\right)_{\mathrm{R}} + \ \boldsymbol{\Omega}\times\mathbf{A} \tag{1}$$

(we use modern notation but the expression is completely equivalent to O'Brien's equation). Here subscript A denotes the absolute frame and R the rotating frame, $\mathbf{A}$ is an arbitrary vector and $\boldsymbol{\Omega}$ is the angular velocity of Earth. This appears to be the first occurrence of this important relationship in vector form. As it has no name, and is so central in geophysics, it would seem appropriate to call it O'Brien's equation.

O'Brien used the relationship (1) in the usual way, first applying it to the position vector $\mathbf{R}$ to get the link between the relative and absolute velocity,

$$\mathbf{V}_{\mathrm{A}} = \mathbf{V}_{\mathrm{R}} + \boldsymbol{\Omega}\times\mathbf{R}$$

and then to the velocity to get the acceleration

$$\left(\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t}\right)_{\mathrm{A}} = \left(\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t}\right)_{\mathrm{R}} + 2\boldsymbol{\Omega} \times \mathbf{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{R})$$

O'Brien realized that the centrifugal acceleration, the last term in the above equation, could be combined with Newtonian gravitation, so that the final vector equation for motion relative to the Earth becomes

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = -2\boldsymbol{\Omega} \times \mathbf{V} + \mathbf{F}$$

(subscript R now omitted) where $\mathbf{F}$ is the sum of the applied forces (per unit mass). The term involving $\boldsymbol{\Omega}$ is what we now call the Coriolis term. O'Brien was the first person to express the Coriolis term in vector form. He described the term $-2\boldsymbol{\Omega} \times \mathbf{V}$ as "the force which must be supposed to act as a correction for the neglected rotation [in the non-inertial frame]".

O'Brien gave a complete solution for the motion of the pendulum on the rotating Earth, showing that the period is inversely proportional to the sine of latitude. He also gave an interesting interpretation of the motion as the super-position of two conical motions with slightly different frequencies. All in all, this was an impressive demonstration of the vector method in analysing a problem of great topical interest.

5. Overshadowed

O'Brien came quite close to constructing the system of vector algebra as it is used today. Yet, despite his innovative work, his ideas were almost completely ignored by his contemporaries, and it was several decades before Gibbs' *Vector Analysis* lit a bright flame. One of the reasons was that Hamilton was a figure of towering influence and he and his supporters worked indefatigably to promote the recognition and use of quaternions. O'Brien's work was completely overshadowed by this 'publicity campaign'.

O'Brien might have achieved much more had he had more leisure to pursue his research. But he was overburdened with teaching responsibilities, and his life was cut short at just forty-one years. While his formulation of vector analysis was incomplete, and imperfect in some respects, it merits recognition as a significant contribution. Designating any individual as the "father of modern vector analysis" is problematical. We must also recognise that at the time that O'Brien was beginning his work on vectors, Hermann Grassmann published his *Ausdehnungslehre* (Grassmann, 1844), a profound work of sweeping generality embracing many of the crucial concepts that underlie vector analysis.

We can conclude that O'Brien deserves more credit that he received in his short life. One means of rectifying this lack of recognition would be to give a new designation to the expression (1) for the rate of change of a vector quantity in a non-inertial frame. This equation plays a central role in geophysics, yet it has no accepted name. It is sometimes called Coriolis' Theorem, but this is inappropriate: while the physical content of the equation was well known to Coriolis, and the equation had been expressed in component form much earlier, O'Brien (1852, p. 193) was the first to formulate it as a vector equation. It would seem appropriate to call (1) O'Brien's equation.

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Mary Kelly and Charles Doherty editors: Music and the Stars, Four Courts Press, 2013. ISBN:978-1-84682-392-3, EUR 45, 288 pp.

REVIEWED BY ROD GOW

I have taught lecture courses in the History of Mathematics for several years. I like to discuss topics that are substantial, and preferably relate well to mathematics as we know it and practise it today. There is no shortage of material when following the history of Greek mathematics, from Thales until Diophantus and Pappus in the third/fourth century CE. Endless time can be devoted to the study of the work of Pythagoras, Euclid, Archimedes, Apollonius, and others, and this can be related to the philosophical ideas of Zeno, Plato and Aristotle.

It is generally accepted that the wonderful creativity of Greek mathematicians was gradually lost, especially from the first century BCE onwards. Nobody, for instance, was able to extend the techniques of Archimedes, which foresaw the infinitesimal arguments of integral calculus long before the revolution in mathematical thought brought about by Descartes, Fermat, Newton, and Leibniz, among others.

The Romans cared little for the achievements of Greek mathematicians, and seem to have had no idea of the full extent even of Euclid's *Elements*. The situation was different in the Eastern Roman Empire, centred on Constantinople, but nonetheless, while manuscripts were copied and circulated for use in the institutions of higher learning in the Eastern Empire, mathematicians and scholars tended to confine their creative skills to writing commentaries on the existing works, and added nothing much that was new or significant. Of course, in some cases, these commentaries have proved to be of basic importance, as, having miraculously survived for centuries, they provide us with the scarce information we have on ancient mathematics and

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its creators. A famous example is the commentary of Proclus (410-485 CE) on the first book of Euclid's *Elements*, which relates almost all of what we know about the history of Greek geometry.

Towards the end of the Western Roman Empire, a few elementary mathematical works were produced in Rome, and some of these continued to be used extensively in medieval Western Europe. An *Arithmetic* and a *Geometry*, written by the Roman aristocrat Boethius (*c.* 480-524 CE), are the most important examples of this kind, but they are considered to be poor synopses, without proofs, of more substantial work by Nicomachus and Euclid. Boethius was also the author of books on astronomy and music. (The book on astronomy is now lost, and it is not certain that the surviving fragments of a work on geometry attributed to Boethius are indeed written by him.) More importantly, he undertook to translate and explain as much as possible of the work of such Greek philosophers as Aristotle, and despite his shortcomings, his contribution to the continuity and history of scholarship has proved to be vital.

The four scientific subjects, arithmetic, geometry, astronomy and music, are said to form the *quadrivium*, a name possibly introduced by Boethius. Three less scientific subjects, grammar, rhetoric and logic, form the *trivium*. Taken together, the seven subjects of the trivium and quadrivium constituted the *Seven Liberal Arts* (*Artes Liberales*), which dominated much thinking on early medieval European education. The concept of the seven liberal arts was expounded by Martianus Capella, born in Africa in the early 5th century CE, in his work *De Nuptiis Philologiae et Mercurii* (On the Marriage of Philology and Mercury). It is said to be a convoluted allegorical encyclopedia of the liberal arts, of which numerous manuscript copies exist. To illustrate the remarkable longevity of this unusual work, it was even printed in Vicenza in 1499, and is still available today in reprints.

The Roman senator Cassiodorus (*c.* 485-585 CE) composed an influential work entitled *Institutiones Divinarum et Saecularium Litterarum*, between the 530's and the 550's, whose second section relates to the trivium and quadrivium. He considered the seven subjects useful for the study of divinity. Cassiodorus proved to be important for the transmission of learning during tempestuous times, as he systematized the copying of manuscripts in libraries.

We do not associate early medieval Western Europe with advanced mathematical thinking. It is always dangerous to categorize a given era in a few succinct words, but quoting from the paper *Seventh-century Ireland: the cradle of medieval science?*, p.46 in the book under review, by Immo Warntjes: "With the fall of the Roman empire, the secular institutions of education and learning lost their foundation. Learning shifted into the newly developing monasteries, and, with this, became decidedly Christian in character. The principal object of learning was to receive an understanding of God's creation, which manifested itself in two ways–the Holy Scripture and the Cosmos." Continuing on p.47, Warntjes writes: "Especially from the sixth to the eighth centuries, but also beyond the Carolingian age right up to the reception of Arabic science, computus was first the only, then the principal science within Christian learning."

Computus means *computation*, and is the science concerning the date of Easter. Much intellectual effort seems to have been expended on what one is tempted to regard nowadays as a fairly routine matter, or at least one that has been solved definitively. Of course, there were foundational problems, such as determining the dates of Christ's birth and death. There was also a problem about the lack of a number zero, so that there was no year zero for Christ's birth. Dionysius Exiguus (*c.* 470-544 CE), born in Scythia Minor, was the inventor of the *Anno Domini* system, used to number the years of the Gregorian and Julian calendar. In 525, Dionysius prepared a table of future dates of Easter, and a set of *arguments*, explaining their calculation (the *computus*), at the request of Pope John. In 725, the Venerable Bede incorporated the algorithms for dating Easter into his textbook on computistics, *De Temporum Ratione.*

Moritz Cantor, the German historian of mathematics, wrote as follows (in English translation) in Vol. 1 of his *Vorlesungen über Geschichte der Mathematik* (1892): "The computation of Easter-time, the real central point of time computation, is founded by Bede as by Cassiodorus and others, upon the coincidence, once in every nineteen years, of solar and lunar time, and makes no immoderate demands upon the arithmetical knowledge of the pupil who aims to solve simply this problem."

This brings me to focus on the book under review. As most mathematicians are probably not especially familiar with the intricacies of medieval mathematics and its principal aims, I thought it necessary to present the preceding introductory material, albeit somewhat incomplete and undetailed. An interest in computistics and *computus* has arisen in NUI Galway, and five international conferences on the subject have been held there between 2006 and 2014. I should point out, however, that the work under review is a volume of papers presented at a conference, *Music and the stars: mathematics in medieval Ireland*, held in Dublin in July 2012.

To give some understanding of why the study of *computus* relates to Ireland, I quote from Warntjes again, p.52: "The mathematical relation between the beginning of Lent and Easter Sunday had to be fixed, calendrical algorithms for establishing the right date and lunar age had to be invented. This was first done by Irish scholars.

"What Irish scholars of the seventh century achieved, therefore, was a comprehensive understanding of Easter reckoning, which was to become the unanimously accepted system for the calculation of Easter, from the ninth century onwards, for the rest of the Middle Ages and in the Orthodox Church to the present day. Their knowledge they put into their writing, computing, for the first time, comprehensive textbooks on the reckoning of time."

As might be anticipated from what I have written above, I found the article by Warntjes especially helpful for presenting me with the main ideas in computus and for explaining the role of Irish scholars in systematizing the subject. Other articles I found enjoyable were: *Music and the stars in Cashel, Bolton Library, MS 1*, by Charles Burnett; *Boethius in early Ireland: five centuries of study in the sciences*, by Pádraig Ó Néill; *Music and the stars in early Irish compositions*, by David Howlett. There is also *Saints, scholars and science in early medieval Ireland*, by Dáibhí Ó Cróinín. He has become a leading light in the recent study of computus, and has identified a number of Irish computus manuscripts, dating from the 7th century, in various European libraries. *Music and the Stars* also contains coloured photographic reproductions of various manuscripts described in the text, and these give a good idea of how surviving documents actually appear, and what it is that scholars actually have to study.

It is difficult for an outsider and non-specialist to do justice to topics that have only marginal connection to his own interests and knowledge. At first, I thought that the subject matter was too remote from me to attempt more than a feeble appraisal. In fact, I found several papers fascinating, especially that of Warntjes, which certainly served to give a very helpful overview of medieval mathematical objectives.

To conclude, I would like to mention a more recent example of how the problem of Easter calculation can inspire mathematical creativity. In his book *Choice and Chance* (first edition, 1867), the Reverend William Allen Whitworth (1840-1905), Vicar of All Saints, Margaret Street, London, proposed the following Question 115 (I quote from the 5th edition, 1901): "What is the chance that in a year named at random Easter should fall on April 25 of the Gregorian Kalendar?"

Now Easter may fall between March 22 at the earliest and April 25 at the latest. The answer given by Whitworth is

$$\frac{1}{19 \times 7} = \frac{1}{133}.$$

The 19 term arises from the 19 year cycle mentioned earlier, and the $1/7$ is just the probability that a certain phenomenon occurs on a Sunday, when it is equally likely on any day of the week. Whitworth notes that, on average, Easter will occur on April 25 about three times in four centuries. The last such occurrence was in 1943, and the next is in 2038. Since finding out this, I have always hoped that I would live long enough to appreciate the next late Easter (nobody currently alive is likely to see the next occurrence after that, which is in 2190).

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S. Louridas and M. Rassias: Problem-Solving and Selected Topics in Euclidean Geometry: In the Spirit of the Mathematical Olympiads, Springer, 2013. ISBN:978-1-4614-7272-8, EUR 42.79, 235 pp.

REVIEWED BY JIM LEAHY

The success of the International Mathematical Olympiad (IMO) has helped to revive interest in Euclidean geometry and to halt somewhat its decline during the second half of the twentieth century. Consequently there is a constant trickle of new publications on the subject of which the book under review is one. Both authors have connections with the IMO. Sotirios E. Louridas has been a coach of the Greek Mathematical Olympiad team while Michael Th. Rassias is a winner of a silver medal at the IMO 2003 in Tokyo and holds a Master of Advanced Study from the University of Cambridge.

The book has six chapters with a foreword by Fields Medalist Michael H. Freedman. Chapter 1, Introduction, is short with a little history of geometry and containing Euclid's axioms and postulates. Chapter 2 deals with the basic concepts of logic and covers methods of proof including proof by analysis, by synthesis, proof by contradiction and proof by induction with examples. The one induction example is more a problem in number theory than geometry having the theorem of Pythagoras as a starting point. There is no other problem in the book that uses proof by induction. Chapter 3 covers geometrical transformations, viz. translations, symmetry, rotations, homothety and inversion. These are illustrated with examples and some theorems with proofs. The section on inversion will be found particularly useful to students and teachers as it gives several examples of its power in solving certain types of problems. Some of the later IMO type problems in the book also use inversion, something not common in many publications on Euclidean geometry.

Chapter 4 is a collection of thirty-eight theorems some of which are proved. The selection of theorems is excellent. Knowledge of these theorems together with the theorems of Euclid would go a long way towards solving many a geometrical problem. The proof of

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Feuerbach's theorem, Theorem 4.21 in the book, contains an error and the proof of Morley's theorem, Theorem 4.11, is not correct. In the latter case if the word 'isosceles', used twice, is replaced by 'equilateral' the proof would be correct but incomplete. Interestingly the construction at the beginning of this proof is similar to the construction used in MacCool's proof of Morley's theorem [1].

Chapter 5 offers sixty-five problems divided into three categories, problems with basic theory, problems with more advanced theory and geometrical inequalities. There is little difference between the first two categories and many of the problems are of IMO standard. The solutions follow in chapter 6 forming the main body of the work. Reading through the solutions is not easy. In some cases parts of the solutions seem to have been omitted and a good deal has been left to the reader usually without any comment from the authors. The statement of problem 6.2.25 p.169 is false as the wrong angles are designated as being equal. The solution uses the correct two angles but if you were attempting to solve the problem without consulting the solution, which you would expect a reader to do, your work would be in vain. The problems are restated before each solution in chapter 6 and the error is repeated. The solution of problem 6.2.22 p.164 is also incorrect since it would require the side of an inscribed pentagon to also be a tangent to the circumscribing circle! Obviously there are serveral misprints in this solution. On the other hand some of the proofs are quite innovative and the solution of problem 6.2.15 p.151 is an excellent example of the use of inversion.

There is an appendix on the Golden Section which is a reprinting of an article by Dirk Jan Struik in [2]. I fail to see the point of this as it is a popular article containing all the usual material of such articles which can be found in many publications and on the internet. Besides the Golden Section is not mentioned anywhere in the main text of the book. There is also a useful index of symbols used in the text, a subject index and a list of references, ninety-nine in all, including a reference to Wiles' paper on the solution of Fermat's Last Theorem! Since there are no references in the text, apart from acknowledging authors of problems, the references should rightly be called a bibliography.

To summarise, this is not a book showing how to solve problems in geometry except in the sense of learning from seeing problems solved. This is not a criticism as much can be learned in this way particularly if a solution has been attempted beforehand. The book is beautifully produced, the quotions at the head of each chapter adding to the publication. However the work is unfortunately marred by poor editing and proofreading. I counted over sixty errors, omissions, typos or misprints, mostly the latter, which does not make for easy reading. In addition some solutions have no diagram. Woody Guthrie, the American folk singer, once said that he liked books with errors as it made them more human. I doubt he ever read a mathematics text.

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Desmond MacHale: The Life and Work of George Boole: A Prelude to the Digital Age, Cork University Press, 2014. ISBN:978-1-78205-004-9, EUR 19.95, 342+xxiii pp.

REVIEWED BY PETER LYNCH

University College Cork is currently celebrating the bicentenary of the birth of mathematician and logician George Boole. UCC has planned a wide range of events for the *Year of George Boole*,[1] which runs until November 2015. Boole's work has had a growing influence in recent decades, in computer science and digital technologies of all kinds.

A new issue of Desmond MacHale's biography of Boole has just been published by Cork University Press. This is a corrected version of the 1985 book published by Boole Press. The earlier edition had a foreward by the noted Irish mathematical physicist John L. Synge. The current book has a new foreward by Ian Stewart of the University of Warwick. The first edition was reviewed in this Bulletin by Ivor Grattan-Guinness.[2]

This is a very comprehensive biography, and an invaluable contribution to our knowledge and understanding of an important nineteenth century mathematician and logician. It provides a well drawn portrait of Boole, not just as a man of science, but as a social reformer, religious thinker and family man. He was quintessentially a philosopher. The book is clearly the result of extensive and thorough research. It is well-written, with the story of Boole's life and work unfolding systematically and with sustained interest.

Early Life

George was the eldest of four children, a shy but exceptionally bright youth. He had mastered Latin and Greek by his early teens and taught himself French, German and Italian. This later gave him direct access to mathematical developments on the continent,

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[1] `georgeboole.com`

[2] Vol. 14, Sept. 1985, `http://www.maths.tcd.ie/pub/ims/nl14/`

allowing him to go far beyond most of his compatriots. He had an elementary school teaching but, too poor to attend grammar school, he taught himself advanced mathematics by studying the original works of the leading mathematicians of the time.

Boole's serious study of mathematics began around the age of sixteen. Having mastered calculus, he studied the works of Newton, Lagrange, Laplace, Jacobi and Poisson. As an autodidact, Boole developed a fiercely independent approach to research. His initial interest in mathematics was for its applications to the solution of scientific problems but he soon came to appreciate pure mathematics as interesting and beautiful in its own right. He published papers on differential equations, integration, logic, probability, geometry and linear algebra.

In 1841, Boole's *Exposition of a General Theory of Linear Transformations* led to a new area of mathematics, Invariant Theory. Cayley and Sylvester brought the theory of invariants to an advanced stage of development, and both acknowledged Boole as the instigator and inspiration for their efforts.

His paper "On a General Method of Analysis" was published in 1844 in the *Philosophical Transactions of the Royal Society.* For this he won the society's Gold Medal, the first such award in the field of mathematics. The paper presented a general, systematic method of solving a broad class of differential and difference equations with variable coefficients.

Queen's College Cork

In October 1846 Boole applied for a position as Professor of Mathematics in one of the three Queen's Colleges, that were being established in Belfast, Galway and Cork. Boole's letter of application is amazing, containing the statement "I am not a member of any University and have never studied at a college." But his application included several very strong testimonials from leading mathematicians. After a long delay, he was offered a position in Cork and took up residence in October 1849 at a salary of £250 per annum.

The Queen's Colleges were multi-denominational, and were controversial from the outset, being described by the Catholic hierarchy as Godless Colleges. For a time, Boole managed to avoid direct involvement in the many religious conflicts, but he could not remain unaffected by them. Although a man of mild manners, he was drawn into a series of acrimonious college squabbles. Perhaps more details of these are given by MacHale than is really necessary, with long letters to the newspapers quoted *in extenso*.

During his early years in Cork, Boole was lonely and not very happy. Things changed when in 1855, aged 40, he married Mary Everest, then but 23 years old. Mary was the niece of the professor of Greek at Queen's College and also a niece of Sir George Everest, Surveyor General of India, after whom the world's highest mountain is named. The marriage was a happy if brief one, and they had five daughters, all extraordinary in different ways.

The Laws of Thought

In 1833, when he was eighteen, Boole had had a flash of inspiration, that logical relations could be expressed in symbolic form. This idea would later grow into his major contribution to science: to explain the process of human thought in precise mathematical terms. Efforts to make logic a precise science could be traced back to Aristotle, and Leibniz had gone some way to express logical relations in symbolic form bu, he lacked an adequate notation. In 1847, Boole wrote *The Mathematical Analysis of Logic*, which was described in a subtitle as "a calculus of deductive reasoning". This book marked the beginning of symbolic logic.

Boole's greatest work, *The Laws of Thought*, was written during his tenure at Queen's College. One of his profound insights was that mathematics is not confined to number and quantity alone but has a larger nature as universal reasoning expressed in symbolic form and conducted in accordance with definite rules. The objective of his work was the reduction of logical propositions to a symbolic form, so that logical conclusions become mathematical consequences of the initial assumptions. By considering classes rather than number as the focus of attention, it adumbrated set theory, now a central foundation of mathematics.

Boole was the first to perform algebraic operations on symbols that represented logical propositions, thereby giving a powerful impetus to the field of symbolic logic. Previously, no one had appreciated the mathematical nature of everyday language. The algebra devised by Boole is now the ideal tool for handling information and modern computers operate following its principles. Boolean algebra encompasses a vast array of topics including set theory, binary numbers, probability spaces, electronic circuit structures and computer technology. Many of Boole's ideas are today considered as self-evident and are found in elementary set theory and probability. They find wide applications in fields such as medical diagnosis, insurance and legal evidence.

Boole and Hamilton

Boole tried his hand at poetry but, as with his contemporary William Rowan Hamilton, the results were unremarkable. A full chapter is devoted to the surprising lack of contact between the two men. Boole was born ten years after Hamilton, and they died within a year of each other. They had common interests, in both mathematics and prosody, and had ample opportunity to collaborate or at least to interact. Yet the minimal contact between them suggests that there was some significant difficulty or disagreement between them, although no solid evidence of this has yet emerged. MacHale wrote in 1985 that the mystery of this lack of contact remains. The chapter is subtitled "Some unanswered questions". It is disappointing that, after a lapse of thirty years, nothing further is added in this edition. [MacHale has since told me that he has a new speculation about the possible rift, but we must wait for information on this to appear elsewhere.]

Family

The final chapter is interesting if somewhat peripheral. George and Mary Boole had five daughters, all interesting in different ways. Alicia made significant discoveries in 4-dimensional geometry. She coined the term *polytope* for the 4D equivalent of a polyhedron. G. I. Taylor, the leading British fluid dynamicist of the twentieth century, was a son of Boole's daughter Margaret. Another daughter, Ethyl Lilian (Voynich), had an adventurous life, and wrote a novel, *The Gadfly*, which was amazingly popular in Russia. Her life story is fascinating.

At the end of the penultimate chapter, The Final Years, MacHale writes of Boole: "His name will live on as long as the digital computers which depend so vitally on Boolean algebra continue to operate and as long as students of mathematics study ring theory, differential equations, probability theory, difference equations, invariant theory, operator theory, set theory and, of course, mathematical logic." He closes by noting how Boole would have been delighted to have known how all modern communication, whether of data, text or images, comprises long strings of the Boolean symbols 0 and 1.

Conclusion

The book is well produced and appears to be relatively error-free. One glitch that might perplex readers is the repeated reference to Lebesgue. The analyst Henri Lebesgue was not born until 1875, long after Boole's death. The reference must be to the number theorist Victor-Amédée (1791–1875) but the index lists Henri. A more serious problem is the omission of an appendix listing 27 "Additional References", which appeared in the first edition but has been omitted from the new version.

Some of the quotations are over-long, with a number of dubious digressions. For example, there are long pen-portraits of many religious thinkers contemporary with Boole who had an influence on him. The wisdom of including these is a matter of opinion, but I felt that they tended to interrupt the Boole narrative.

Bulletin readers may well feel that the level of mathematical detail is inadequate. Certainly, mathematical readers with a general idea of Boole's work, and who wish to deepen their knowledge, will not find this the ideal book for that purpose, and will have to revert to Boole's original publications. However, there is a quite complete and valuable list of these in the book.

Peter Lynch Peter Lynch is Professor of Meteorology at UCD. His interests include dynamic meteorology, numerical weather prediction, Hamiltonian mechanics and the history of meteorology. He writes an occasional mathematics column *That's Maths* in the *Irish Times*; see his blog at `http://thatsmaths.com`.

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Robin Harte: Spectral Mapping Theorems, A Bluffer's Guide, Springer, 2014. ISBN:978-3-319-05648-7 (ebook) 978-3-319-05647-0 (soft cover), GBP 24.11 (ebook) GBP 30.14 (softcover), 120+xiv pp.

REVIEWED BY ANTHONY W. WICKSTEAD

A bluffer's guide purports to pass on enough of the basics (and especially the jargon) of a subject that one can pass[1] as knowledgable about the subject without having to put in the effort that is required really to become knowledgable. Anyone who reads this book with such an aim is going to be disappointed. It is a serious mathematical monograph, albeit written in the rather eccentric style that we have come to expect of the author. The author's aim is to present enough spectral theory (for single elements, n-tuples or for infinite families) to reach spectral mapping theorems. He deliberately stops short of any notion of a functional calculus.

The book is divided into six chapters, the first two dealing with the algebraic and topological preliminaries that are required whilst the third brings these together to look at *Topological Algebra* including bounded operator theory. There follow chapters devoted to the spectral theory for the three cases of single elements, n-tuples and for infinite families. There is a bibliography of 294 items, of which over a hundred have Robin's name on the list of authors. The index is slightly disappointing, probably because of the lack of named theorems and definitions that I mention below. The table of contents is much more useful, as sections average just over two pages in length.

How, I can hear you asking, can he possibly do all this in a mere 120 pages? The answer, of course, is that proofs, and even precise statements in many cases, are conspicuous by their absence. The book is not set out in conventional Theorem-Proof format. I can, perhaps, best give a flavour of his style by comparing it to discussions that I had with Robin many years ago on beaches in West Cork when he attempted to give a very young mathematician a flavour of what

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[1]At least if not examined too carefully!

he was working on. He gives the salient definitions and formulae in a very conversational manner whilst hiding a lot of the hard work that is needed to understand everything completely. For example, §5.6 describes tensor products for vector spaces, normed spaces, algebras, Banach algebras and modules in a little over three pages!

So who is this book best suited for? I guess anyone who, for whatever reason, wants to learn more than the basics of spectral theory of one variable or to learn something of the spectral theory of more than one variable. This book on its own won't be adequate either to become an expert in the area or even to pretend that you are. Nor will it be a useful source of references to cite, if only because the unconventional format makes it virtually impossible to refer to it for major results. Unfortunately, the lack of citations within the text makes it difficult to follow up from the overview that this book gives and obtain more details. I certainly cannot recommend this for starting-out mathematicians to learn the subject on their own, in spite of the very basic starting point that is assumed. It would, however, be an excellent text for a group of young mathematicians to adopt as a starting point for a working seminar that filled in the details over the space of a year or so. All that being said, the right reader will find this a very useful way to get started out in this area. I certainly plan to add it to my own mathematical library.

Anthony W. Wickstead Tony Wickstead is Professor of Pure Mathematics at Queen's University Belfast, where he has worked since 1975 following short periods at the University of Ife in Nigeria and at University College, Cork. He obtained his PhD in 1973 from Chelsea College, London. His research is in the broad area of vector and Banach lattices and operators on them.

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PROBLEMS

IAN SHORT

Problems

I learned the first problem from Grahame Erskine of the Open University.

Problem 74.1. Given a positive integer A, let B be the number obtained by reversing the digits in the base n expansion of A. The integer A is called a *reverse divisor* in base n if it is a divisor of B that is not equal to B.

For example, using decimal expansions, if we reverse the digits of the integer 15, then we obtain 51. Since 15 is not a divisor of 51, the integer 15 is not a reverse divisor in base 10.

For which of the positive integers n between 2 and 16, inclusive, is there a two-digit reverse divisor in base n?

You may also wish to attempt the more difficult problem of classifying those positive integers n for which there is a two-digit reverse divisor in base n.

The second problem was proposed by Ángel Plaza of Universidad de Las Palmas de Gran Canaria, Spain.

Problem 74.2. Let $f : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ be an increasing, convex function with $f(1) = 1$, and let x, y, and z be positive real numbers. Prove that for any positive integer n,

$$\left(f\left(\frac{2x}{y+z}\right)\right)^n + \left(f\left(\frac{2y}{z+x}\right)\right)^n + \left(f\left(\frac{2z}{x+y}\right)\right)^n \geqslant 3.$$

The third problem was contributed by Finbarr Holland of University College Cork. To state this problem, we use the standard notation

$$f(n) \sim g(n) \quad \text{as} \quad n \to \infty,$$

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where f and g are positive functions, to mean that

$$\frac{f(n)}{g(n)} \to 1 \quad \text{as} \quad n \to \infty.$$

Problem 74.3. Prove that for $j = 0, 1, 2, \ldots$,

$$\sum_{k=0}^{n} k^j \binom{n}{k} \sim n^j 2^{n-j} \quad \text{as} \quad n \to \infty.$$

Solutions

Here are some solutions to the problems from *Bulletin* Number 72. The proposer sketched a method for solving Problem 72.1, but we have yet to receive a full solution. If we receive a full solution, then we'll publish it in a later issue.

The second problem was solved by the North Kildare Mathematics Problem Club and the proposer, Finbarr Holland. The solution that we give is equivalent to the submitted solutions.

Problem 72.2. Prove that the integral

$$\int_0^\infty \frac{x \sin x}{2 + 2\cos x - 2x \sin x + x^2}\, dx$$

exists as a Riemann integral, but not as a Lebesgue integral, and determine its value as a Riemann integral.

Solution 72.2. Let

$$f(x) = \frac{x \sin x}{2 + 2\cos x - 2x \sin x + x^2}.$$

The denominator is equal to $(1 + \cos x)^2 + (x - \sin x)^2$, so f is continuous on $[0, \infty)$, and hence it is Riemann integrable on any compact subdomain of $[0, \infty)$. Let

$$g(x) = \arctan\left(\frac{\cos x + 1}{\sin x - x}\right).$$

Then g is an antiderivative of f on $(0, \infty)$, so

$$\int_a^b f(x)\, dx = g(b) - g(a),$$

where $0 < a < b < \infty$. Since $g(a) \to -\pi/2$ as $a \to 0$, and $g(b) \to 0$ as $b \to \infty$, we deduce that, as a Riemann integral,

$$\int_0^\infty f(x)\,dx = \frac{\pi}{2}.$$

Next, to prove that f is not Lebesgue integrable on $(0, \infty)$, let

$$I_n = \int_\pi^{(2n+1)\pi} |f(x)|\,dx$$

for $n = 1, 2, \ldots$. Then

$$\begin{aligned} I_n &= \sum_{k=1}^n \int_{(2k-1)\pi}^{(2k+1)\pi} |f(x)|\,dx \\ &= \sum_{k=1}^n \int_0^{2\pi} |f(x + (2k-1)\pi)|\,dx \\ &= \sum_{k=1}^n \int_0^{2\pi} \frac{(x + (2k-1)\pi)|\sin x|}{(1+\cos x)^2 + ((2k-1)\pi + (x - \sin x))^2}\,dx \\ &\geqslant \sum_{k=1}^n (2k-1)\pi \int_0^{2\pi} \frac{|\sin x|}{(1+\cos x)^2 + ((2k-1)\pi + (x - \sin x))^2}\,dx. \end{aligned}$$

Let

$$a_k = (2k-1)\pi \int_0^{2\pi} \frac{|\sin x|}{(1+\cos x)^2 + ((2k-1)\pi + (x - \sin x))^2}\,dx.$$

Then

$$(2k-1)\pi a_k \to \int_0^{2\pi} |\sin x|\,dx = 4 \quad \text{as} \quad k \to \infty.$$

As the sum of the reciprocals of the odds numbers is infinite, we see that the sequence I_n is unbounded. Thus

$$\int_0^\infty |f(x)|\,dx = \infty,$$

so f is not Lebesgue integrable. □

The third problem was solved by the North Kildare Mathematics Problem Club and the proposer, Tom Moore of Bridgewater State University, USA. The two solutions are equivalent to the solution given below.

Problem 72.3. For $n = 1, 2, \ldots$, the triangular numbers T_n and square numbers S_n are given by the formulas

$$T_n = \frac{n(n+1)}{2} \quad \text{and} \quad S_n = n^2.$$

It is well known that every even perfect number is a triangular number. Prove that every even perfect number greater than 6 can be expressed as the sum of a triangular number and a square number.

Solution 72.3. The Euclid–Euler theorem says that every even perfect number can be expressed in the form $2^{p-1}(2^p - 1)$, where p and $2^p - 1$ are prime numbers, and every number of that form is an even perfect number. We use this representation of even perfect numbers to solve the problem.

After some basic algebraic manipulations, we can see that

$$T_n + S_{2n+1} = \tfrac{1}{2}(3n+1)(3n+2).$$

Let p be an odd prime number such that $2^p - 1$ is also a prime number. Notice that $2^p - 2$ is divisible by 3. Let $m = \frac{1}{3}(2^p - 2)$. Then with some further algebraic manipulations, we see that

$$T_m + S_{2m+1} = 2^{p-1}(2^p - 1).$$

This shows that every even perfect number other than the number 6 (which is given by $p = 2$) can be expressed as the sum of a triangular number and a square number. The number 6 cannot be expressed as the sum of a triangular number and a square number, as you can easily check. □

We invite readers to submit problems and solutions. Please email submissions to `imsproblems@gmail.com`.

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