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Jeremy Gray: Henri Poincaré: A Scientific Biography, Princeton University Press, 2013, ISBN:978-0-691-15271-4

REVIEWED BY PETER LYNCH

Henri Poincaré was a mathematician of the highest calibre. He was also a leading physicist, adumbrating Einstein on a number of key points in relativity, a prominent philosopher of science, and an excellent expository writer. Gray's book is a comprehensive scientific biography of Poincaré. It embraces the broad scope of Poincaré's work, from his philosophical speculations to his popular writing, and gives a thorough overview of his extensive mathematical researches.

The book is very 'scholarly', by which I mean very heavy going in places. The opening Chapter, at 126 pages, is the longest in the book. From the title 'The Essayist', I was lulled into hoping for a gentle and readable account of Poincaré's popular writing, much of which is highly accessible, entertaining and thought-provoking. In fact, the chapter is laced with heavy, stolid philosophical discussion. It was a great struggle to wade through this chapter. I am of the view that it detracts from the book: it is misplaced, over-long and unlikely to appeal to most mathematicians. Fortunately, Chapters 2 to 18 are very different in style. Only in the final chapter does the author falter again, becoming immersed once more in philosophical cogitations.

Poincaré published about 500 papers and more than 30 books covering mathematics, theoretical physics and astronomy. In his mid-twenties, he introduced ideas that led to the transformation of several areas of mathematics: complex function theory, differential equations in the complex domain and non-Euclidian geometry. He did not stay working in a single field, deepening his knowledge of it. Nor did he flit from one field to another. Rather, he took up new interests while maintaining contact with earlier ones, so that his command of mathematics ultimately became vast in scope. As is well known, E. T. Bell described him as 'the last universalist'.

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In 1880, Poincaré submitted a 46 page essay for a prize competition, the goal of which was to advance the theory of ordinary differential equations. He amended and extended Fuchs' work, discovering in the process a connection between singularities in the complex plane and non-Euclidian geometry. The functions he considered now come under the rubric of automorphic functions, generalisations of elliptic functions that are invariant under groups of transformations, and that are of topical interest.

Poincaré had a life-long involvement with the dynamics of planetary motion. He made definitive contributions to the three-body problem, showing that what is now called chaos can make long-range prediction impossible. In 1885 King Oscar II of Sweden sponsored a competition calling for original mathematical contributions in one of four areas. First on the list was the problem of the long-term stability of the solar system.

Poincaré's submission was judged the winning entry. Weierstrass's conclusion that its publication 'will inaugurate a new era in the history of celestial mechanics' turned out to be prescient. Poincaré's competition entry was a profound work, and it led on to his three-volume *Les Méthodes Nouvelles de la Méchanique Céleste*, which remains of great influence even today.

Poincaré's study of the three-body problem led him to the question of small divisors. In the course of this, he systematised the rigorous treatment of divergent series. Asymptotic analysis has proved of inestimable value ever since.

Poincaré discovered many ideas that we now automatically associate with Einstein: how observers can compare measurements by exchanging light signals; how the speed of light is an impassable limit; the Lorentz group of transformations between frames; and the invariance of Maxwell's Equations under this group. All these discoveries were made by Poincaré independently of Einstein and mostly before him. Gray discusses the complex conjunction of circumstances that prevented Poincaré from taking more decisive steps to develop special relativity. He was nominated in 1910 for the Nobel Prize in Physics. But, although he won a 'handsome number of votes', he did not get the prize.

One of Poincaré's most profound contributions was to topology. He introduced the algebraic objects associated with a manifold, the homotopy and homology groups. His monumental *Analysis Situs* of

1895, together with five supplements published over the following decade, were of enormous influence. He used algebraic structures to distinguish non-homeomorphic spaces, inventing the subject of algebraic topology. The Poincaré Conjecture emerged from this work. In fact, Poincaré raised it as a question rather than a conjecture: Is a closed 3-manifold with trivial fundamental group necessarily homeomorphic to the 3-sphere? The conjecture was recently proved true by Grigori Perelman.

In relation to his mathematical inventiveness, Poincaré wrote several popular accounts of how he benefited from 'flashes of inspiration'. These came with 'brevity, suddenness and immediate certainty', but always following a period of intense dedication to a problem. In a similar vein, Felix Klein is quoted as describing how, sitting up late one night afflicted by asthma, the boundary circle theorem (Grenzkreis Theorem) appeared suddenly before him. Within a week, Klein had written it up and sent it off for publication.

Poincaré pioneered qualitative analysis of differential equations. One of his abiding principles was that a qualitative analysis of a problem must precede a quantitative treatment, 'for if it is by logic that one proves, it is by intuition that one invents'. He was keenly aware that understanding a proof is not the same thing as checking its logical consistency. But he was no slouch when it came to the hard graft of putting theory on sound foundations: 'In mathematics, rigour is not everything, but without it there is nothing'.

Gray's book is certainly a valuable addition to scholarship on the scientific work of Poincaré and will be of interest to many readers of this Bulletin. Although it is a monumental tome, there are some notable omissions. In particular, it is surprising that chaos theory is hardly discussed by Gray. This idea pervades modern science, and Poincaré's contributions were singularly original and profound. Yet the term 'chaos' does not even appear in the index. On that point, a subject index of four pages is quite inadequate for a book of this scope.

For decades, much of Poincaré's work was eclipsed, but more recent developments have changed that. There has been a revival of three-dimensional geometry, most notably with Thurston's pioneering work in the field of low-dimensional topology. Dynamical systems theory is now a topic of intense research. And chaos theory has had a profound affect on our world-view. As Gray writes, 'in each of the fields [in which] Poincaré worked, his achievements in mathematics, physics and philosophy are still alive and important today'.

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