25TH ANNUAL MEETING OF THE IRISH MATHEMATICAL SOCIETY 27–28 AUGUST 2012

The 25th Annual Scientific Meeting of the Irish Mathematical Society was held at the Institute of Technology Tallaght. The meeting was organised by Cora Stack, Martin Marjoram, Paul Robinson and Ciaran O'Sullivan, mathematics lecturers at IT Tallaght. The organisers would like to thank the Institute of Technology Tallaght for its support and funding as part of its 20th Anniversary celebrations and would also like to thank the IMS for its additional funding. The meeting brought together researchers from a wide range of mathematical disciplines and across Ireland and the United Kingdom. Below we give the meeting timetable and abstracts of the talks. We also refer the reader to the meeting website `http://www.it-tallaght.ie/events/IMS2012/index.html`.

Timetable

Monday, 27 August

10:30-11:00	Registration; Tea & Coffee
	Chair: Paul Robinson
11:00–11:10	**Pat O'Donnell**, *Head of School of Engineering* Opening address
11:10–12:00	**David Applebaum**, *Sheffield University* Densities and traces on compact Lie groups
12:00–12:30	**Bernard Hanzon**, *University College Cork* 2-EPT Levy Processes with Applications in Finance
12:30–13:00	**Stephen Kirkland**, *Hamilton Institute, NUI Maynooth* Matrix theory, number theory and quantum spin chains
13:00–14:00	Lunch

	Chair: Cora Stack
14:00–15:00	**Peter Cameron**, *Queen Mary University, London* Permutation groups and transformation semigroups
15:00–15:30	**Brendan Guilfoyle**, *IT Tralee* Towards a causal topology of 4-manifolds
15:30–16:00	**James Cruickshank**, *NUI Galway* Frameworks
16:00–17:00	Tea & Coffee and meeting about Irish Maths Research Funding
	Chair: Sinead Breen
17:00–17:30	**Terry Maguire**, *IT Tallaght* Building Mathematical Competence—A Community Approach
17:30–18:00	**Julie O'Donovan**, *IT Cork and Cork Maths Centre* Mathematical enrichment programme (UCC) Maths Circles.
18:00–20:00	IMS Committee Meeting
20:00	Dinner

Tuesday, 28 August

	Chair: Ciaran O'Sullivan
9:00–9:30	**Aibhin Bray**, *Trinity College Dublin* A Framework for the Integration of Digital Technology into Mathematics Curricula
9:30–10:00	**Mícheál Mac an Airchinnigh**, *Trinity College Dublin* The Nature of Mathematical Modelling in 2012
10:00–11:00	**Duncan Lawson**, *Coventry University* Pre-University Mathematics Education in England and Some International Comparisons
11:00–11:20	Tea & Coffee

	Chair: Martin Marjoram
11:20–11:50	**Eimear Byrne**, *University College Dublin* Combinatorial Structures from Codes Over Rings
11:50–12:20	**Robin Harte**, *Trinity College Dublin* Spectral permanence
12:20–12:50	**Leo Creedon**, *IT Sligo* The Structure of Finite Group Rings
12:50–13:40	Lunch
	Chair: Stephen Wills
13:40–14:10	**Stephen Buckley**, *NUI Maynooth* Conditions implying ring commutativity
14:10–14:40	**Paul Barry**, *IT Waterford* The Hurwitz transform of sequences
14:40–15:10	**Oliver Mason**, *Hamilton Institute, NUI Maynooth* Delay Independent Stability and the Riccati Equation
15:10–15:40	**Shane O Rourke**, *Cork IT* Free affine actions on lines
15:40–16:00	Tea & Coffee
16:00	AGM of the IMS

Abstracts

David Applebaum, *Sheffield University*
Densities and traces on compact Lie groups
We use representation theory and weights to investigate the question of when a probability measure on a compact Lie group has a square integrable or smooth density with respect to Haar measure. We then turn our attention to convolution semigroups of measures (or Levy processes from the stochastic processes viewpoint). It turns out that having a square integrable density is equivalent to the associated semigroup of convolution operators being trace class in the L^2 space of Haar measure. There is a nice interplay between densities and traces which we use to prove an interesting new inequality and to investigate the short time asymptotics of the process. We

will also describe some applications to the statistical problem of deconvolution.

Bernard Hanzon, *University College Cork*

2-EPT Levy Processes with Applications in Finance

We consider a class of Levy processes that is characterised by the fact that at each point in time the process random variable has characteristic function that is a rational function or a real power of a rational function. A matrix calculus is available to deal with such processes. We consider how to deal with the requirement that the corresponding density functions have to be non-negative and we give conditions in terms of the poles and zeros of the characteristic function for it to be infinitely divisible (which implies that there is a corresponding Levy process). The Budan-Fourier type sequence of ExponentialPolynomial-Trigonometric functions plays an important role to verify non-negativity on a given finite interval. Application to non-Gaussian option pricing and hedging will be discussed. The 2-EPT Levy processes can be viewed as generalization of the variance-gamma Levy processes, which are well-known in financial mathematics. Further information and some software related to 2-EPT random variables and processes can be found on the website `http://www.2-ept.com`.

Stephen Kirkland, *Hamilton Institute, NUI Maynooth*

Matrix theory, number theory and quantum spin chains

The timeevolution of quantum spin systems models the dynamics of various nanodevices, including "quantum buses" for transferring information in quantum processors. One of the desirable tasks is to transfer the state of a particle into another one with maximum fidelity; when that fidelity is 1, we have perfect state transfer. In mathematical terms, this is equivalent to considering an undirected graph with adjacency matrix A, and asking for a particular entry of the matrix e^{itA} to have modulus 1 at some time t. It is known that for spin chains (where the graph in question is a path), perfect state transfer occurs between the end points of the chain only if the number of vertices is small. In view of that fact, we consider the following relaxation of perfect state transfer: if, for each $h > 0$, there is a $t > 0$ such that the fidelity at time t exceeds $1 - \varepsilon$, then we say that pretty good state transfer occurs. Using techniques from matrix theory and number theory, we show that a spin chain on n

vertices exhibits pretty good state transfer between the end points of the chain if and only if either $n+1$ is a power of 2, or for some prime p, we have $n+1=p$ or $n+1=2p$.

Peter Cameron, *Queen Mary University, London*
Permutation groups and transformation semigroups
In the last few years, the topics of group theory and semigroup theory have come closer together, partly because advances in group theory have provided us with new tools for studying semigroups. In particular, the Classification of Finite Simple Groups has led to a complete list of multiply-transitive permutation groups, and very strong information about primitive groups (which are completely known for degrees into the thousands). This has led to progress on transformation groups generated by a group and one extra element, or to semigroups normalised by an interesting group. I will describe some results (obtained in collaboration with João Araújo and others) on synchronizing semigroups and the Černý conjecture (one of the oldest conjectures in automata theory, and one with potential applications), and regular or idempotent-generated transformation semigroups. These applications are interesting not only to semigroup theorists; some new generalisations of multiply-transitive permutation groups arise naturally and seem worth further attention.

Brendan Guilfoyle, *IT Tralee*
Towards a causal topology of 4-manifolds
In this talk, tentative steps will be proposed for a new geometric classification of 4-manifolds, based on the causal structure determined by a metric with two time directions. We motivate this by the techniques developed in the proof of the Caratheodory conjecture and show the rich diversity of behaviour that can be accommodated by the new paradigms.

James Cruickshank, *NUI Galway*
Frameworks
A framework is a graph together with an embedding of the vertex set into some metric space—typically two or three dimensional Euclidean space. A flex of a framework is a motion of the embedded vertex set that preserves distances between adjacent vertices. This abstraction has obvious motivation from problems that arise in engineering, both of the static and kinematic varieties. In this talk, however, I will present a survey of some of the mathematical theory

of frameworks. There are many interesting mathematical questions associated to these objects and there are connections with graph theory, linear algebra, geometry and topology. Two basic questions that I will discuss are

- Given a framework, is there any nontrivial continuous flex of the framework?
- Given that there is such a flex, can we describe the space of all possible configurations of the framework?

Of course, in general there is little that one can say about these questions in the absence of further information about the specific framework. However, for many interesting and natural classes of frameworks there is much mathematical interest arising from these two questions.

Terry Maguire, *IT Tallaght*
Building Mathematical Competence—A Community Approach
This paper describes an innovative and novel approach to sowing the seeds of the recognition of the importance of mathematics competence in real life situations. Mathematical competence *is the ability to recognize, use and apply mathematical concepts in relevant contexts and situations which certainly is the predominant goal of the mathematical education for engineers*, (SEFI Mathematics Working Group 2011). Traditional methods of mathematics teaching and learning have resulted in a maturing population who do not appreciate the mathematics they use in their everyday lives. These 'everyday' mathematics skills often involve the use of complicated mathematical ideas and techniques. However, many people often consider the mathematics they can do as 'common sense' and the tasks they can't do as 'mathematics'. The paper describes a successful initiative 'Looking at Tallaght with Maths Eyes' that took place in June 2011 to coincide with the 18th International Conference Adult Learning mathematics (Mathematical Eyes: A Bridge between Adults, the World and Mathematics), hosted by the Institute of Technology Tallaght. Dublin, Ireland.

The initiative aimed to:

- Develop the maths eyes of the Tallaght community: (Every member of the community has maths eyes—they just need to be opened).

- Help the Tallaght community to make the link between mathematics and the real world. (A key focus was to encourage the community to use Maths Eyes when they think about their water usage and water conservation).
- Build people's confidence in their use of maths in their life.
- Empower people and build their confidence in their own maths knowledge and skills (empowered parents are more confident in supporting their children's learning; more confident citizens can make more informed evaluations of the information that bombards them every day and have a better understanding of the impact of their actions and decisions in their life, work and leisure).
- Build a positive image of maths.

The paper outlines the different approaches that were used to encourage participation from a range of stakeholders. These included a community wide 'curiosity' campaign; the development and piloting of a resource pack for educators called 'Developing Maths Eyes; An Innovative Approach to Building a Positive image of Mathematics' (2011); primary schools showcase; adult learners showcase; a maths poster exhibition and a curated photographic exhibition of instances of mathematics seen in the local area; the development of maths trails for the local parks and an audio maths 'I-walk' for Tallaght. In addition it describes how the initiative has since been adapted as a model for use both nationally and internationally.

Julie O'Donovan, *IT Cork and Cork Maths Centre*
Mathematical enrichment programme (UCC) Maths Circles.
Secondary school students with interest in mathematics need a means for their skills to be nourished and developed. The Maths Circles Initiative aims to do this with the cooperation of four key stakeholders: school students, school teachers, university maths students and university maths staff. In this talk we would like to describe the experience of these different stakeholders in the first year of the Maths Circles project.

Currently the Maths Enrichment centres in Universities across the country offer a great facility for senior cycle students. The Maths Circles initiative provides after school maths clubs for junior cycle secondary school students. These maths clubs or maths circles are run by third level maths students with the help of a maths teacher

from the school. In the past year there were twelve maths circles running in different schools in the Cork region. We ran two workshops for teachers and a maths circles open day where we brought students from the different maths circles together for a fun half-day event. A key part of the project was the involvement of university mathematics students. University students have the drive, enthusiasm, talents, and skills set necessary as well as being able to relate well to the secondary school students. The maths circles team tried to channel this energy in the maths circles project. The students were fundamental in the development of resources which are now available on `http://www.mathscircles.ie`. One of our key findings is that such a project can sustain an organic growth process due primarily to the enthusiasm of the stakeholders. We believe that there are many other positive outcomes of the project and we will discuss these in the talk.

Aibhin Bray, *Trinity College Dublin*

A Framework for the Integration of Digital Technology into Mathematics Curricula

Over the last three decades, a diverse range of technologies, and of resources accessed by them, has emerged. The development of a system of classification to ensure the appropriate use of these technologies, resources and associated pedagogies, will be beneficial for guiding the design of teaching, understanding learning, and improving mathematics education. In order to do this, the pedagogies themselves need to be investigated, as does the nature of the social interaction involved. The understanding that different technologies assist and benefit different learning experiences, will enable those who choose and implement systems to make informed judgements.

This research explores the current applications of digital technology in mathematics education and questions which of these make full use of their affordances to facilitate learning. In an attempt to answer this question, a theoretical framework is presented that classifies the types of technology as well as their pedagogical foundations. The potential of the technology to fundamentally alter how mathematics is experienced is investigated through the lenses of the following hierarchies: SAMR (substitution, augmentation, modification, redefinition) and FUIRE (familiarization, utilization, integration, reorientation, evolution).

Mícheál Mac an Airchinnigh, *Trinity College Dublin*

The Nature of Mathematical Modelling in 2012

Mathematica, and then subsequently WolframAlpha, changed my way of seeing how to teach mathematics (pure and applied) more effectively to the various students with whom I have come into contact over the past 20 years. As I write this, I am looking at my own diskette, Version 2.2 of Mathematica, "For the Macintosh, Enhanced Version", 1988-1993. When I did my University of London Mathematics degree, awarded January 1979 in the Royal Albert Hall, I recall the use of the slide rule (and I am looking at said object now) for serious computation (faster than the log tables). The slide rule I now know to be an Analogue Device. Very soon thereafter, perhaps one year, I acquired my personal Digital Device, a programmable HP Calculator. To recall such technology belies my, taken for granted, deeper engagement with punched cards, paper tape, and real magnificent computing machines, and the 14 or more programming languages I have acquired in my lifetime.

Who am I? I have belonged to the Computer Science Department since, umm 1980, and have spent most of my years there, teaching Mathematics in one form or another, with special emphasis on Computer Graphics. The Doctoral Dissertation, finished 3rd October 1990, is entitled "Conceptual Models and Computing", and having it before me now, I see that it is full of all sorts of practical/non-practical math such as Category Theory stuff. Today, I teach a Masters Course in Mathematics which is examined by open-book. There is just one text, Gershenfeld's "The Nature of Mathematical Modeling" (MIT & CUP 1999). Students answer questions via Mathematica Notebooks and are required to be knowledgeable in the editing and use of (Math-related) Wikipedia pages (in English and another chosen language). Expertise in the use of WolframAlpha (and MathWorld) is obligatory. Let me tell you my story? I believe much of it may be true.

Duncan Lawson, *Coventry University*

Pre-University Mathematics Education in England and Some International Comparisons

Pre-University Mathematics Education is a topic of great concern currently in England. Over the last two years a series of reports have been published by educational charities, professional bodies

and learned societies, quasi-governmental and governmental agencies which have highlighted aspects of mathematics education in which there are problems. A number of these reports have included international comparisons which show that practice in England often does not compare well with the rest of the world.

In this presentation, I will present a summary of some of the key findings from these reports. Where international comparisons have been made that explicitly include Ireland, these will be highlighted as well. The material from these reports will be supplemented with evidence from Coventry University which shows ways in which the mathematical competences of new undergraduates have changed over the last 20 years.

In 2004, a Government inquiry into post-14 mathematics education (the Smith Inquiry) concluded (amongst other things) that

> *Higher education has little option but to accommodate to the students emerging from the current GCE process.*

The evidence of the plethora of recent reports indicates that HE is still in the same position of having "to accommodate to the students emerging from the current GCE process". This presentation will conclude with a brief summary of ways in which HE is doing this, notably through a substantial increase in mathematics support provision.

Eimear Byrne, *University College Dublin*
Combinatorial Structures from Codes Over Rings
We give an overview of recent advances in the theory of ring-linear codes and their connections to other combinatorial objects such as strongly regular graphs, orthogonal arrays, authentication schemes and bent functions.

Robin Harte, *Dublin*
Spectral permanence
Spectral permanence, for C^* algebras, says that the spectrum of an element of a C^* algebra is the same whether it is taken relative to the subalgebra A or the whole algebra B: this discussion is sparked by the effort to prove that the same is true of a variant of spectral permanence in which the two-sided inverse, whose presence or not

defines "spectrum", is replaced by a *generalized inverse*. The argument involves a circuitous tour through "group inverses", "Koliha-Drazin inverses" and "Moore-Penrose inverses"; it turns out that the induced variants of spectral permanence are curiously inter-related.

Leo Creedon, *IT Sligo*
The Structure of Finite Group Rings
Group rings arise in many branches of mathematics and are well studied when the coefficient ring has characteristic 0. Less is known in the finite case where the characteristic is positive, especially when it divides the order of the group as Maschke's Theorem no longer applies. In this talk the author presents recent results on the structure and units of certain classes of group algebras FG, where F is a finite field and G is a finite group.

Stephen Buckley, *NUI Maynooth*
Conditions implying ring commutativity
It is well known that Boolean rings, characterized by the polynomial condition $x^2 = x$, are commutative. In this talk we discuss one-variable polynomial-type conditions that imply ring commutativity, beginning with the important work of N. Jacobson and I. N. Herstein.

Specifically, we will cover the following topics:

(1) Some elementary proofs of ring commutativity that avoid structure theory.
(2) A classification of all polynomials f such that a ring with unity R is necessarily commutative if $f(x) = 0$, or $f(x)$ is central, for all x in R. (The non-unital $f(x) = 0$ variant was previously characterized by Laffey and MacHale.)
(3) Variants of the above for semigroups of polynomials.

This is based on joint work with Des MacHale and with Yuliya Zelenyuk.

Paul Barry, *IT Waterford*
The Hurwitz transform of sequences
In recent years, the study of the Hankel transform of integer sequences has attracted much interest. These studies have benefited greatly from links with the theory of orthogonal polynomials and

of lattice paths, and with classical results on matrices and determinants. However, many open questions remain, and so any new perspectives on this simple to describe but hard to characterise transform is welcome. In this talk we shall look at one such perspective, the Hurwitz transform of sequences.

Oliver Mason, *Hamilton Institute, NUI Maynooth*

Delay Independent Stability and the Riccati Equation

The algebraic and differential Riccati equations play a key role in numerous areas of systems and control theory, ranging from robust stability analysis to optimal control. In this talk, I will consider a particular instance of the algebraic Riccati equation that arises in connection with delay-independent stability conditions for linear time-delay systems. I will describe a recent result providing necessary and sufficient conditions for the existence of diagonal solutions to the Riccati equation associated with a positive linear time-delay system and highlight the implications of this fact for the stability properties of such systems. I will also discuss several applications of this as well as related results concerning delay-independent stability for classes of nonlinear positive systems.

Shane O Rourke, *Cork IT*

Free affine actions on lines A line for us is a Λ-tree with no branch points, where Λ is a linearly ordered abelian group. Free isometric actions on Λ-trees have been studied by various authors; recently we have extended this study to (free) affine actions. The question of which groups admit a free isometric action on a line has a very easy answer, namely torsion-free abelian groups. The answer is less obvious in the case of free affine actions however. We will describe some classes of groups that admit such an action.