The IMS September Meeting 2006 at IT Tralee
Abstracts of Invited Lectures

Non-commutative geometry and K3 surfaces

Madeeha Khalid, Institute of Technology Tralee

K3 surfaces are two dimensional complex manifolds which are a natural generalisation of elliptic curves. An elliptic curve is diffeomorphic to $S^1 \times S^1$ and is given by $\mathbb{C}/\mathrm{L}$, where $\mathbb{C}$ is the complex plane and L is a rank two lattice in $\mathbb{C}$. The parameter space of elliptic curves is given by the upper half complex plane modulo an $\mathrm{SL}(2,\mathbb{Z})$ action. A similar result holds for K3 surfaces as well.

Any two K3 surfaces are diffeomorphic and there is a twenty dimensional complex space parameterizing all K3 surfaces. Physicists have exhibited a relation, called T-duality, between different string theories on an elliptic curve. This idea led to a search for dualities between K3 surfaces. Translating these relations in terms of geometric data has been an active area of research recently, especially in the context of non-commutative geometry. We will give an overview of the above mentioned results and a sketch of some of our recent results on dualities between certain non-commutative K3's.

Soap bubbles, integrable systems and singular perturbation theory

Mark Haskins, Imperial College, London

The mathematical idealization of static soap films and soap bubbles are minimal surfaces and constant mean curvature (CMC) surfaces, respectively, and have been studied by mathematicians since at least the 19th century. In the 1950s two fundamental results by Hopf and Alexandrov proved that under some conditions soap bubbles must be round and led to a conjecture that this was always the case (the Hopf conjecture).

In the mid 1980s Wente disproved the Hopf conjecture by constructing soap bubbles which are tori. Subsequently various mathematicians realized that all CMC tori could be understood using methods adapted from mathematical physics, namely (infinite-dimensional) completely integrable Hamiltonian systems.

Then in the early 1990s Kapouleas constructed many compact CMC surfaces of higher genus using a geometric/analytic method inspired by singular perturbation theory. This talk will try to explain the basic features of CMC surfaces, how integrable systems and singular perturbation theory arise in this geometric context and how they can be used to construct compact soap bubbles.

Optimality models in behavioural biology

JOHN MCNAMARA, UNIVERSITY OF BRISTOL

Natural selection tends to produce organisms that maximise an appropriate measure of fitness. This means that optimisation models can be used to both explain and predict behaviour. I will try to give a flavour of this area, reviewing ideas of fitness and illustrating a range of optimisation problems. In the simplest setting fitness can be taken to be just the mean number of offspring produced over an organism's lifetime, although this measure must be modified when individuals inherit some aspect of 'state'. In this setting, problems can usually be solved by standard techniques such as dynamic programming.

The fitness of an individual may also depend on the strategies adopted by other population members, so that rather than an optimisation problem the situation must be modelled as a game. I illustrate games between the sexes. In particular I describe a game in which each member of a population searches for a mate by inspecting a sequence of members of the opposite sex. Under this scenario the best acceptance rule for an individual depends on the acceptance rule used by members of the opposite sex. As I will show, the equilibrium rule leads to a 'class structure' in pairs formed, in which 'attractive' individuals are more choosy, and pairs are formed between males and females of similar levels of attractiveness.

When there are fluctuations in the environment as a whole affecting population members, counting the mean number of offspring is no longer appropriate. Instead fitness must be defined in terms of the geometric mean number of offspring, or some generalisation of

this. I will show how this fitness measure leads a situation in which individual members of a genotype cannot be considered in isolation, and to randomised strategies that are superior to any deterministic strategies.

Encouraging third level students to engage meaningfully with the concepts of analysis

Maria Meehan, University College Dublin

Many third level educators, who have taught introductory courses in topics such as Analysis and Abstract Algebra, will be acutely aware of the difficulties faced by students embarking on "advanced mathematics" courses. Students must learn to describe mathematical concepts formally by definitions, and to describe and explain relationships among concepts rigorously in the form of theorems and proofs. The transition to this type of thinking can prove too much for some students. Many of us will be familiar with complaints of "there is just too much theory", and we are all aware of students who out of desperation learn chunks of material off by heart in the hope of regurgitating it in the exam.

Much has been written in the mathematics education literature about the above topics over the past twenty years. In this talk we describe initiatives that have been adapted from the literature and introduced in a second year introductory Analysis module in UCD in order to address the following aims:

(1) To encourage students to think conceptually about abstract mathematical concepts and to intuitively understand the relationships between them;
(2) To encourage students to rigorously describe mathematical concepts and rigorously describe and explain the relationships between them.

Some results from a whole class survey at the start of the 2004/05 module and interviews with 13 students at the end of the module, will be presented.

Enhanced angular resolution from multiply scattered waves

Clifford Nolan, University of Limerick

Multiply scattered waves are often neglected in imaging methods; largely because of the inability of standard algorithms to deal with

the associated non-linear models. This paper shows that by incorporating a known environment into the background model, we can not only retain the benefits of imaging techniques based on linear models, but also obtain different views of the target scatterer. The net result is an enhanced angular resolution of the target to be imaged.

We carry out our analysis in the context of high-frequency radar imaging, in which a steerable beam from a moving platform is used to produce an image of a region on the earth's surface (the target scatterers being buildings, etc). We consider the case where the target we want to image is situated in the vicinity of an a-priori known reflecting wall. This is one of the simplest possible environments for the scatterer, and in the case when the illuminating beam is narrow enough to isolate different scattering paths, we will show that the imaging process achieves enhanced angular resolution. Although we carry out our analysis here in the context of radar, our technique is a general enough that it can be adapted to many imaging modalities, such as acoustics, ultrasound, elasticity, etc. The extension of the method to other more complicated environments is also possible.

Treelike objects and generalised metric spaces

SHANE O'ROURKE, CORK INSTITUTE OF TECHNOLOGY

Ultrametric spaces are metric spaces satisfying the following strong version of the triangle inequality: $d(x, y) \leq \max\{d(x, z), d(y, z)\}$ for all x, y, z. $\mathbb{R}$-trees may be characterised as geodesic 0-hyperbolic metric spaces in the sense of Gromov. We review recent work by B. Hughes where equivalences between (certain categories of) $\mathbb{R}$-trees and (certain categories of) ultrametric spaces are established. We consider various directions in which Hughes' results can be generalised.

Abstracts of Short Talks

Algebraic entropy

BRENDAN GOLDSMITH, DUBLIN INSTITUTE OF TECHNOLOGY

The theory of endomorphism rings of algebraic structures allows, in a natural way, a systematic approach based on the notion of entropy borrowed from dynamical systems theory. In this talk I want to introduce the notion in the context of commutative p-groups, paying particular attention to groups, all of whose endomorphisms have zero entropy.

New constructions of convex surfaces of constant width

Brendan Guilfoyle, Institute of Technology Tralee

A closed convex body in $\mathbb{R}^n$ has *constant width* if the distance between any pair of parallel tangent planes has a fixed distance. The simplest example of such a body is the $(n-1)$-sphere, but it has been known, at least since the time of Euler, that non-spherical bodies of constant width exist.

Minimising the volume of bodies of constant width is a natural and long-standing problem. In 2 dimensions, it was originally proved by Minkowski and Lebesgue that the Reuleaux triangle minimises the area amongst curves of constant width. For $n > 2$ the problem remains unsolved.

In 3 dimensions it is conjectured that the volume minimiser has tetrahedral symmetry. In this short talk we will give the background and known results for this problem. We will also present new constructions for surfaces of constant width, including an algorithm for generating tetrahedrally symmetric surfaces of constant width.

Using case studies in mathematics tutor training

Brien Nolan, Dublin City University

The article *Teaching Mathematics Graduate Students How to Teach*, Notices Amer. Math. Soc. **52** (2005), 842–847 by Prof. Solomon Friedberg of Boston College caught the eye of many people interested in mathematics education. In this article, Friedberg detailed his team's development and use of case studies for the training of teaching assistants and junior lecturing staff in mathematics. In this talk, I will describe our pilot use of Friedberg's materials in a mathematics tutor training programme in DCU, and a follow-up project in which we are developing cases that focus on the issues that arise in the Irish third level system.

Assessing the mathematical literacy of third-level students

Ann O'Shea, NUI Maynooth

Numerous recent reports have expressed concern with the mathematical competences of students as they come to the end of their secondary schooling and embark on third-level education. In particular, many students seem to rely on rote-learning in mathematics and display little ability to think independently. PISA 2003 measured the mathematical literacy of 15-year-olds across forty countries

by assessing each individual's ability to analyse, reason and communicate mathematical ideas. This study, which encouraged students to appreciate the role mathematics plays in everyday life through the formulation of, engagement with and solution of mathematical problems, ranked Ireland as twentieth (although our mean score was not significantly different to that of eight other countries).

In the past, many third-level institutions have tried to gauge the skills and proficiencies of their incoming students by means of a diagnostic test. However, these tests have tended to focus on procedural or computational skills rather than any understanding or appreciation of the mathematics involved. In order to assess, instead, the mathematical literacy of these students and identify their strengths and weaknesses, a PISA type test was administered to first year students in three institutions across the third-level sector. An analysis of the results aims to determine whether there is any improvement in students' literacy on completion of secondary schooling and whether the level of literacy achieved correlates to Leaving Certificate results obtained in mathematics. Moreover, information collected from the same students on their experiences of and attitudes to mathematics at second- and third-level is reported.