EXPERIENCES FROM A WEEKEND SPENT MODELLING

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While the title of this article may conjure up images of your favourite supermodel strutting down a Paris catwalk in the latest Autumn/Winter collection by Chanel, with a sophisticated, Armani-clad Kevin Jennings and Richard Murphy on either arm, we hope that you will not be disappointed to learn that we intend to relate to you the experiences of some UCD Mathematics students who spent a weekend modelling–modelling of a mathematical variety, that is.

1. Mathematical Contest in Modelling–an introduction

The Mathematical Contest in Modelling, better known as the MCM, is an international event, held annually, where teams of three undergraduates spend four days working on a real-life, open-ended problem. The brain-child of Dr Ben Fusaro from the United States, the contest was set up as an alternative to the Putnam Mathematical Competition in the US. Dr Fusaro noticed that it was difficult to drum-up enthusiasm among students for the Putnam exam. The emphasis on the "pure, formalistic approach, almost devoid of content" and the "reporting of a large percentage of low numerical scores" did little to encourage the practical-minded student to enter this exam and merely added to the "chilling effect." He also felt that one wouldn't know that computers even existed by examining a Putnam paper. Consequently, in 1983, he set about designing an "Applied Putnam."

His proposal was to have a competition that required students to clarify, analyse and propose a solution to an open-ended problem. The realistic problems would be chosen with the advice of mathematicians who worked in industry and government. Students would be able to draw on outside resources including computers, texts or any other inanimate source. The entire modelling process would be emphasized, and therefore substantial weight would also be given to how the solution was written up. In discussions with members of SIAM, the Society of Industrial and Applied Mathematics, one expert noted that to make the scenario realistic, students would need to be given a whole semester to work on the problem. However it was eventually decided that the contest should take place over four days. Finally a numerical score would not be awarded. Rather, successful teams would be categorized in ascending merit as Successful Participants, Honorary Mention, Meritorious and to the best six teams, the title of Outstanding Meritorious would be awarded and their papers published in a professional Mathematics Journal.

The first MCM was held in 1985, and the committee was delighted to discover that it was a resounding success, with 90 papers, representing 70 colleges, submitted. Since then, the MCM has grown from strength to strength. The 1998 MCM saw 472 teams participate, representing 246 institutions from 8 countries– Australia (2 teams), Canada (11), Finland (1), Hong Kong (2), Ireland (10), Lithuania (1), P.R. China (138) and the United States (307). The contest is not limited to third level institutions, and this year 13 high schools also participated.

2. The MCM in Ireland

The MCM arrived in Ireland along with Professor Pat Lambert from the University of Fairbanks, Alaska in 1991. While visiting University College Galway and Trinity College Dublin, in the 1990/1991 academic year, Professor Lambert encouraged staff and students in both universities to give the contest a go. As a student of Mathematics in UCG at the time, I attended a talk given by him, where he explained how the contest worked, described what a unique learning experience it was, and most importantly, what an enjoyable weekend could be had by all involved. Dr Ray Ryan and Dr Pat O'Leary in UCG put all their support behind the idea and thus 1991 saw the first Irish entries in the contest, from both

UCG and Trinity.

Ireland has been successfully represented by teams from various universities since then. UCD joined the participants for the first time this year after I managed to cajole/trick/blackmail/beg various students to offer up a weekend of their lives in the name of Mathematics. The preparations commenced in late October when I gave an introductory talk on the contest to a mixed group of students. Since UCD had not previously participated in the MCM, the students had only my word on what the contest entailed and it proved more difficult than I thought to convince them that their participation in this weekend would be a worthy investment in many ways. However, a group of about ten gullible students were sufficiently fooled and the training began.

We met usually twice a week, and with the help of Dr Ted Cox and Dr Peter Duffy, the candidates were introduced to some previous MCM problems and sample solutions. Since one can't cover in an hour, what is supposed to take four days, the most one can do is give an idea of how one might approach a particular problem. The Brain-Drug problem, Emergency Power-Restoration problem, Salt-Storage problem, Steiner-Tree problem and Velociraptor problem were just some that were discussed.

The next main task was to choose the teams. Many veteran team advisers have written on the ideal combination of students for a team. One suggestion is that with such a large emphasis placed on the exposition of the solution that the ideal team should consist of three English majors, one of whom is minoring in mathematics, although I would be extremely unwilling to try out that particular combination! Another coach suggests that each team requires one person who has a "*terrier personality*: someone who is jumpy, coming up with a new approach every thirty seconds or so," although he wisely points out that you can't put two people of that temperament together in a room for four days. Therefore the suggestion is that there has to be a team member "who can lend a degree of stability to the team–someone who can keep the terrier reined in, with a leash if necessary." Students of this type are easily recognized as "the ones who are in class every day with their homework done and know what they are going to be having for dinner two weeks from next Tuesday." The final team member should be a combination of both personalities. What is absolutely certain however is that one team member has to be able to program well.

One of our teams consisted of a fourth year Mathematics-Mathematical Physics student, a fourth year Electronic Engineering student and a first year Science student, while another team was comprised of a student from fourth year Mathematics, one from second year Science and one from first Medicine. Another team consisting entirely of first years decided to give the weekend a go, however due to the lack of a programmer on the team, they retired from the contest after a brave attempt.

With Friday 6 February quickly approaching, sponsorship was sought and found in the form of money, 4 large pizzas, 196 bottles of coke and 100 doughnuts. Four willing staff members handed over the keys to their respective offices along with their prized computers. Security was alerted to the fact that some students would be spending the weekend ensconced in the department and the head of security kindly set up an emergency "hot-line" to the offices should anything go wrong. One junk-food shopping spree later (although I did buy fruit but it wasn't eaten!) to stock up the Mathematics Department class room which doubled up as canteen for the weekend, and we were ready to roll.

The two MCM problems, from which each team had to choose one to work on, arrived on my desk a week prior to the contest, with the words **DO NOT OPEN UNTIL 12.01AM, FRIDAY 6, FEBRUARY** emblazoned menacingly across them. Despite my protests that a good night's sleep would be more beneficial to our participants, they opted to meet at a local watering hole on Thursday night, and at a minute past midnight, the contents of the envelopes became public knowledge (much to the astonishment of the other customers sitting in the hotel's foyer). One problem was entitled "Grade Inflation" while the other was on "MRI Scanners." Both of our teams eventually chose the latter problem. The problem explained that Magnetic Resonance Imagers (MRI) scan a three dimensional object such as a brain and deliver their results in the form of a three dimensional array of pixels. These scanners usually include facilities to picture on a screen any horizontal or vertical slice through the three dimensional array. The problem lies in finding an algorithm for picturing slices through oblique planes. It is pointed out that "current algorithms are limited in terms of the angles and parameter options available; are implemented only on heavily used dedicated workstations; lack input capabilities for marking points in the picture before slicing; and tend to blur and 'feather out' sharp boundaries between the original pixels."

In what follows, Kevin Jennings, a member of one of the teams, describes his experience of the first two days of the contest, while Richard Murphy, who was on the other team, gives an account of the two final days. Both Kevin and Richard are currently doing postgraduate work in Mathematics at UCD.

3. Friday, 6 February, 1998

"That's the lamest excuse I've ever heard from somebody not working in CIÉ" –my parents scowled as I left home that morning. I had just given them the less than credible explanation that I'd be spending the weekend in an office in the Mathematics Department in UCD with two other lads, working on a maths problem.

Stephen and Conor were already researching neuro-medical scanners when I arrived. We felt we had managed to foil the other team's early efforts, by scouring the library for the most relevant material and piling it on our desk. However they had exactly the same idea, and the ensuing battle taught us our first lesson: despite the advances in neuro-medical science, the human being is still quite a primitive beast. Maria brought us for lunch, where we remembered that we were in fact sophisticated students, and the daggers that we threw at the other team became sharpened with words of Latin origin. Subtle psychological techniques were applied to enable us to wean useful information out of our opponents. Of course we bluffed when they tried the same.

After lunch we gave the second problem on "Grade Inflation" some thought, and discussed the merits of a proposition that UCD should only award first class honours degrees, no matter how unworthy the scripts. This problem didn't catch the imagination quite as much as the thought of looking at the inside of a head. The fact also that we were already enthusiastic about the first problem and had a few ideas taking shape, meant that we promptly dismissed the second problem. In retrospect, we may have been too hasty in this decision, but time was passing, the adrenaline was flowing and a decision had to be made. Lesson number two–how to make a decision under time constraints.

Now that we had decided on the MRI Scanners problem, we set about organizing a strategy that would enable us to work independently on different aspects of the problem, while still complementing each other. Conor, our computer expert, set about retrieving information from the Internet and designing a computer program that would display the desired information. Stephen and I brainstormed for different mathematical models and read more about how the scanners actually worked.

A crucial part of the procedure was trying to decide what type of model to use. The standard approach seemed to be to treat it as an interpolation problem, and use various techniques of interpolation to minimize the inevitable discrepancies that would arise in going from a discrete to a continuous system. We eventually adopted an approach where we assumed that the data points represented spatial cubes, and thus filled space. Having adopted this approach, it was convenient to ignore the short-comings of this model and work on exploiting its advantages. Again, in retrospect, knowing the limits of our model would have offered greater insights as to its potential. This an important gem of knowledge which I collected from the experience –know your limits! Thus lesson number three was learned.

We didn't work too late on Friday night as Maria had suggested that we conserve our energy for the long weekend ahead, which she assured us would be frantic. My mother was pleased to see me when I arrived home, and just to convince her of my sanity, I told her that I wouldn't be home on Saturday or Sunday night as I'd been invited to a massive "Boogie Nights" party hosted by Guinness and TV3 newsreaders. She seemed much happier with this explanation and handed me a tenner. I spent a few hours reading about scanners that night, and next morning, Saturday, we started the hard work in earnest.

4. Saturday, 7 February, 1998

It quickly dawned on myself, Stephen and Conor that there was no ingenious solution to this problem strewn across Professor Laffey's desk, and those "trivial" calculations, that we had so glibly spoken about the previous day, would have to be detrivialized. We got down to the hard slog of trying to wrap our brains around three dimensional rotations and express our conclusions. [Note from Maria: I think I walked in on Kevin during this stage of the proceedings. He was lying flat on his back on the ground with an extremely fraught look on his face, and rotating a piece of cardboard over his head.] Conor then interpreted them on the computer and, of course, as there were "small" flaws in our method, we therefore set about looking for errant minus signs and varying constants. Finding these particular flaws became something of a personal challenge, and all sight of the problem was lost as I went about trying to repair a hose of unknown length, with an unknown number of holes, using only a finite number of bicycle-puncture repair kits (metaphorically speaking of course). The idea of the contest was that we could draw from established material, but it was so much more exciting to try and come up with the tools ourselves. Lesson number four: perhaps we didn't exploit this option to the full.

The other team was encountering similar frustrations, and when Maria arrived with enough junk-food to feed Elvis for a month, we congregated and secretly took pleasure in their contorted facial features and their freshly-formed little bald-patches where hair had been forcibly removed. The entire selection of crisps, doughnuts, chocolate, coffee, crunchy-nut cornflakes, morphine etc. was kept in a common room, and it was not unusual to find somebody banging his head off the blackboard when you entered looking for a coffee.

Confidence was high all round and we were all on the verge of announcing a proof of the Riemann hypothesis as a corollary to our work. It was at this point that Maria casually reminded us that ideally we should be starting the write-up by midday on

Sunday. The effect of these words on our brain-cells was akin to the effect throwing a stone at the queen would have on a bee-hive. Suddenly addition of 3×3 matrices required a formula and finding inverses became trivial. Soon every aspect of mathematics became trivial, except those aspects which we required for our model. If only we'd chosen a different model, the solution would have been obvious! We worked frantically until a late hour, and after sweating all the caffeine out of our systems, [Note from Dr Marjoram, who donated his office to this team: it took a full week, with all the windows open, to rid my office of the woeful smell!] we got some sleep before the final battle.

5. Sunday, 8 and Monday, 9 February, 1998

And thus we have reached the point where I take over from my esteemed colleague Mr Jennings and begin relating my experiences of the second half of the weekend. Sunday morning is the point at which I will start; an opening made convenient by our unanimous decision to return to our respective homes and have decent rests for the night. Unanimous, except of course for the indomitable Mr Jennings, who just *had* to bring in a sleeping bag, and by staying in college, snatch a few precious hours of sleep and get one over on everyone else. Not that we all had gone to bed early on Saturday night. I think it was sometime after four on Sunday morning when I got to the safety of my cosy bed, which meant that it was actually bordering on the afternoon when I arrived in.

Our team had also opted for the MRI problem. I had somehow contrived to spend almost two days trying to derive equations to transform three-dimensional coordinates into the two-dimensional position on the slice, taking an inordinately long time to figure out the various ways a plane can intersect a cuboid. Having finally achieved this, my next task was to surmount the problem posed by discreteness: when you intersect a pixel lattice with a plane, you don't get the nice neat collection of uniform squares one needs for computer display purposes (unless you're very lucky or just plain unadventurous). For the time being we were going to use crude rounding-off techniques, but something better was required. I set about my task with the same trademark vigour and efficiency

I had exhibited already. Meanwhile, David was busying himself with finding sample material with which we could test our finished program. To this end he trawled the Internet in search of layered cross-sections of brains from which we could construct our lattice. The actual program was being taken care of by Robert (our computer expert), who was glad at last to have some equational fodder to stick into the program, that he might eventually have some sort of working thing in place. Inevitably, I have been able to divulge more of my endeavours than those of my teammates, being as I am more familiar with them.

We certainly finished the contest a good deal fatter that when we began – I don't think I ever drank so much coke or ate so many doughnuts in such a short space of time. We got fed twice on Sunday to prepare us for the night of madness ahead. Unfortunately, not being renowned for a voluminous appetite, I could gorge myself little more by the time the Chinese take-away arrived. Still, too much food is a good complaint to have, and it was certainly better than my usual diet of bread and burnt cornflakes.

Being such caring folk, we were at great pains to regularly visit the others and see how they were doing. Indeed at times such was our altruistic enthusiasm that we would burst into their room unannounced, and accidentally see some of their work. On other occasions, we were more restrained and hovered undecidedly at the door wondering whether or not to go in, sometimes for minutes at a time. This too could lead to unfortunate overhearing. Not that we had a monopoly on concern, mind. The other team were just as, nay more, eager to see if we were doing okay.

Surprisingly, given my preceding successes, I failed to come up with any improvement on our rough rounding–off scheme over the course of the day. David's Internet-raiding scheme had also run aground on importation difficulties, but he decided to create a rough Euclidean approximation of the brain with which we could test our program. Robert was encountering teething difficulties with said program, not the least of which was the limitation imposed by memory, which meant that our lattice had to be considerably smaller than it was meant to be. He hoped to use the hard disk as an extension of memory.

As day became night, our tired and erratic minds found it increasingly difficult to grapple with the problems we were to overcome. After a while it became clear that we needed to emulate Poincaré in his work on Fuchsian Functions. Thus we played football for about half an hour, hoping the respite would bring fresh insight as in the case of the great man. Or maybe we just played football. Either way, we felt it beneficial, for we repeated the process throughout the night.

Around three or four on Monday morning, both teams began the writing-up process. This was the ultimate penance. Figuring out an approach to a problem and attempting to flesh out your ideas is an endeavour rich in dead ends, exciting insights and general activity. But having to elaborate extensively on a project with which you are intimately familiar, to the point of it all seeming quite obvious, is at four in the morning a labour of the damned. I was also quite distressed to discover, by the occasional cursory glance at our rivals' thesis, that ours was not quite so profuse in ξ's, η's or even $\aleph$'s, as theirs. However, I did manage to appease my pretentiousness by including a theorem (in the broadest possible sense) in an appendix, but it's just not the same.

In addition, the deficit of sleep over the previous few days, and complete lack of it on this night, was starting to take its toll. Exhaustion was becoming overtiredness and then delirium, which was of course for the more Dionysian among us its own reward. While it imbued the writing-up process with more than its fair share of hilarity, it made the compilation of any sort of coherent résumé of our efforts an uphill struggle, to say the least.

By the end we had actually got a working program, at least in the sense that it chose to work for any weird angle you cared to throw at it, but perversely rewarded you with complete garbage if you gave it a nice slice aligned with the axes. Towards the end we had the usual problem of everyone trying to get as much done before the 5pm deadline, which meant that the write-up was being constantly chopped and changed amid fears that an incorrect version might be submitted. Still, somehow, we managed to get everything in place, in time. It was quite a relief to have the thing finally finished, to put it out of our minds and relax. And then the Poincaréan insights came.

With the ordeal over, the person responsible for bringing the event to our college, the elegant, resourceful, talented, dynamic and ever-delightful Dr Meehan (Note from Maria: You never recovered from the delirium then Richard?) treated us to a couple of pints in the UCD bar. Except of course for the ever-eccentric Jennings who disappeared to wash or something. Never having the highest resistance to alcohol, I found my friend delirium making a welcome return, and I wound up complaining that people don't look at the world from the point of view of cows quite enough. What's wrong with the world these days? Having ruthlessly put paid to any form of decent social conversation with my proselytizing, our party soon broke up and we went our separate ways. By the time I got home, delirium had fallen from favour and was replaced in my affections by bed, which was never so dear. An exhausting but rewarding weekend all round.

References

[1] *MCM–the first ten years*, The UMAP Journal, 1995 COMAP, Inc.

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