CURRICULUM DEVELOPMENT IN SECONDARY SCHOOL MATHEMATICS, WITH SPECIAL REFERENCE TO GEOMETRY

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1. There is a substantial amount of published material on curriculum development in secondary school mathematics, particularly on the "New Mathematics" epoch since about 1950. On the latter, there is a progression from the objectives and syllabi of the pioneers, through the projects and project-evaluations, on to the text-books, to articles reviewing progress or non-progress, and to books. I am certainly no specialist in this field, and the aim of this article is to provide references to what I have encountered (without any expectation of completeness), to review briefly aspects of the international scene, and hopefully foster a consideration of the Irish experience in the light of this.

There is a very informative and readable book, Howson, Keitel and Kilpatrick [19], and surveys in the UNESCO publication *New trends in mathematics teaching*. Vol. III (1972) [42]. These will be my main references, but they and the others listed here contain a host of others. Other references specific to mathematical education are Cooper [8], Howard, Farmer and Blackman [18], and Servais and Vargo [37]. The UNESCO:IBE publication *Curriculum innovation at the second level of education* [11] provides a more general background. There are also chapters on mathematics in general books, e.g. in Tanner [39].

As a background, it would perhaps be well to mention briefly the variety internationally of the modes of control and innovation in education; [19] deals with this and in particular gives a reference (p. 58) to a grouping of European countries having similar administration organisations for education and consequently similar approaches to curriculum development (a first group being characterised as having little decentralisation but

some central government movement towards it, a second group as having little decentralisation but some grassroots movement towards it, and a third as having considerable decentralisation but some myth-making about local autonomy). There is also a reference (p. 78) to a distinction between profuse and confined systems rather than between centralised and decentralised ones (a profuse system containing a variety of development and dissemination agencies and a confined system containing a limited number).

2. To come now to the objectives of the pioneers of the 'new' mathematics, we refer to the College Entrance Examination Board: Commission on Mathematics: *Program for College Preparatory Mathematics* (New York, 1959) [6], the OEEC (Paris) publications *New Thinking in School Mathematics* (1961) [30] and *Synopses for Modern Secondary School Mathematics* (1961) [31], the introductions to Dieudonné [10] and Choquet [5], and others.

A common aim was to bring school mathematics more closely into line with university courses, by introducing topics that have emerged over the last century or so as being of basic importance, e.g. sets, functions, equivalence and order relations, the laws of algebra, vectors. There was a desire to have clear concepts and proper (instead of pseudo) reasoning. The existing material would have to be pruned to make room for the new, and the treatment of it should be efficient and informed with the new spirit. Progress was seen to lie not only in having new syllabi but also in adopting new pedagogical approaches to the presentation of material.

That much was largely common ground and has been carried into effect in the reforms in many countries. For comment on curriculum development strategies, projects, pedagogical approaches, and evaluations of these, we refer to [19].

There was a general expectation among the pioneers that pupils following the new courses would give an improved performance all-round, on the retained material of the old syllabus as well as on the new. For an analysis of the outcome of these expectations we refer to [19, Ch. 7] and to Pidgeon [34, Ch. 7]. From the pressure due to so much new material, and perhaps also because it was felt that the clarity of new concepts and the greater power of the new approaches would suffice, many of the new courses had much less time for and emphasis on practice at acquiring skills at manipulation, solving of problems, and application to other fields. To quote Dieudonné [10, p. 12]:

> "I have swept away all traditional considerations and allowed myself to be guided uniquely by my knowledge of what immediately *follows* a secondary education, namely, the first-year courses in universities (or in the polytechnics)."

For resistance to this trend we refer to Ahlfors *et al* [1] and Nevanlinna [29]. Some references have self-explanatory titles, e.g. Kline [21] *Why Johnny can't add: the failure of the New Math*, Thom [40] *'Modern' mathematics: an educational and philosophical error*, and Vogeli [44] *The rise and fall of the 'New Math'*. There has been continuing controversy over the decline in skills, and lack of application.

3. Having dealt with what was largely common ground, allowing for differences in emphasis and detail, we now turn to an area of great divergence, to wit geometry. Chapter 3 in the UNESCO survey [42] starts with the following:

> "The content for geometric study at the secondary school level has been one of the most controversial issues debated by mathematicians and educators for more than fifty years. Many conferences on this subject have led to two distinguishable positions: one, to preserve a large section of Euclidean synthetic axiomatic geometry; the other, to make a completely new approach to the study of space."

A clear focus to modern controversy on this can be given by quoting Choquet [5, p. 13]:

> "From the mathematician's point of view, the most elegant, mature and incisive method of defining a plane (or space) is as a two (or three)-dimensional vector space over R having an inner product, i.e. a symmetric bilinear from u.v such that u.u > 0 for all non-zero u."

and p. 14:

> ":we have a 'royal' road based on the concepts of 'vector space and inner product.'"

The UNESCO survey [42] went on to detail different basic positions on geometry which we re-summarise as follows.

4(i) The first broad approach we mention is the least integrated. It organises local areas of school mathematics, allows several approaches to a topic, and for pedagogical reasons avoids (resists, in fact) placing these in a globally organised or axiomatised framework.

Examples of this are found in England, with an emphasis on transformation geometry [see, e.g. 27, 36], and in the Netherlands with a course using axial symmetries as a major building block [13].

(ii) The second broad approach we mention has as focus a vector space with inner product, but is divided into three streams.

(a) A first stream envisages an initial geometrical familiarisation stage, with an informal treatment of vectors, plane transformations and geometrical figures.

There is then produced a synthetic axiomatic system which aims at a vector space with inner product as an ultimate goal. Examples of this are due to Papy [33] and Servais in Belgium. Choquet [5] and Queysanne, Revuz etc. [35] in France. The difference between the French and Belgians in this is that the French axiomatisations assume a knowledge of the real number system whereas the Belgian approach integrates a build-up of the real numbers with the geometry.

(b) A second stream also envisages an initial informal geometrical familiarisation stage. Axioms are then given for a vector space with inner product, and the geometry is extracted from this. Thus this type of course starts with a vector space. Examples are due to Dieudonné [10], and [26] from the Strasbourg area of France.

(c) A third stream is based on a familiarisation with the concept of vector space without any motivation from or reference to geometry, e.g. from groups, rings, integral domains and fields of numbers. Then from axioms for a vector space with inner product, the geometry is built up. For advocacy of this approach see Glaymann [16].

This approach (ii) is the most integrated of the three approaches, and makes the most extensive demand for the inclusion of abstract algebra. It involves an initial substantial stage of affine geometry, in which explicitly or implicitly there is distance along each line in a plane but the units of distance on the various lines are not co-ordinated so as to produce distance on the plane. Then at an appropriate stage the geometry is specialised to Euclidean geometry.

(iii) The third broad approach is intermediate between the other two in point of integration. The geometry is Euclidean from the start, synthetic and within a global framework that is or can be axiomatised. This broad approach offers the greatest continuity with the past. One treatment is based on congruence

more or less in the style of Euclid, as completed by Hilbert; examples of this occur in West Germany and the USA. There is also a combination of these two in SMSG axioms [22 or 41] in the USA, and there are axiomatisations based on distance, e.g. [23] and [15].

Within (ii) and (iii) there is also a division between courses which contain an axiomatic organisation from the start of secondary school (age 12), and those which proceed in two phases, an initial organisation of the experience and spatial intuition of pupils with local deductions (age 12-15) followed by a related global axiomatic organisation.

5. The specific geometrical aims of advocates of approach 4(ii) are perhaps most completely expressed by Dieudonné, although it can be seen that the other courses implement what is being articulated by him. Arguments based on congruence and similarity of triangles are to be omitted, and objects such as triangles and parallelograms are to be referred to as little as possible; instead, arguments based on linear algebra are to be used, and an emphasis placed on abstract concepts such as a geometric transformation regarded as a single entity. The trigonometry is to be of rotations rather than angles and we are to avoid [10, p. 11]:

> "those unbelievable complications and fallacies surrounding such a straightforward concept as that of 'angle' when regarded from the traditional point of view."

and further [10, p. 16],

> "As for the so-called 'measurement' of angles, it deservedly wallows in the general confusion which reigns in this sphere."

Approach 4(i) concerns itself with pedagogy as much as syllabus content, and stresses that pupils should be helped to discover mathematical facts and development for themselves, and not have the facts dictated to them. We refer to Freudenthal [14, p. 426].

Those who continue to support approach 4(iii) do so on the basis that to subjugate geometry to linear algebra leads to an impoverishment of geometry. They value the visual as a helpful rewarding method of reasoning, they are reluctant for pedagogic reasons to impose extra unnecessary layers of abstraction on the young, and they value how mathematics can arise naturally in the small in geometry, growing from simple to more complex situations, in contrast with having to deal from the start with a large, abstract, complex system. They query whether 4(ii) is in fact a 'royal road' to geometry, as for example the difficult topic of 'angle' is submerged in the topic of rotations. On this side and ranged mainly against approach 4(ii) we can refer to Nevanlinna [29], Thom [40], and two speakers Osserman [32] and Grunbaum [17] at the Fourth International Congress on Mathematical Education at Berkeley in 1980. To quote the latter briefly:

> "Disparaging the importance of the visual, instinctive - even tactile - aspects of geometry and urging their replacement by tool-oriented techniques certainly will not make the future role of geometry any easier. Such an attitude is inherently as absurd as the promotion of soundless music, or verbal rendering of paintings."

6. Bell [3] gives an idea of the amount of retraining of teachers necessary for a type 4(ii) approach; he says that in France teachers attended in-service training for one afternoon a week for a whole year.

7. What has been described applies to secondary schools. There is another stand which perhaps should be mentioned, although it appertains mainly to primary schools. The educational psychologist Jean Piaget has conducted a major series of experiments on how children learn mathematics and in particular what they are or are not able to assimilate at a given age. This has profound implications for the content and sequence of mathematical topics in primary school. An introduction to Piaget's work is given in Copeland [9]. There is also reference to Piaget's work in [18].

As recounted in Copeland [9, p.7]:

> "The Bourbaki group of mathematicians attempted to isolate the fundamental structures of all mathematics. They established three mother structures: an algebraic structure (the prototype of which is the notion of a group), a structure of ordering, and a topological structure. These were later modified to include the notion of categories.'
>
> At a meeting of mathematicians and psychologists in Paris, Piaget and Dieudonné on listening to each other found that there was a direct link between these basic mathematical structures and Piaget's three structures of children's operational thinking."

So if you encounter, on p. 3 of [19] a calm mention of the possibility of introducing category theory at primary school, this is the likely source.

8. Thus internationally there is great diversity in the treatment of geometry, with continuing controversy. In France and Belgium vector spaces predominate; in the Netherlands and England there is localisation, with an emphasis on transformation; congruence is a continuing component in West Germany; for the USSR we refer to [24]. The USA, the original home of the 'new' mathematics, is a country of great diversity. For one sombre

analysis consider Allen [2] in 1984:

> "Now, our mathematics programs, for all except the very best students, present algebra without structure, geometry without proof, and, worst of all, instruction that, some believe, has 'no established or widely accepted set of goals.'"

A look at the special issue on geometry of the *Mathematics Teacher* [25] in September 1985 shows no evidence of any geometrical approach other than one based on congruence.

Servais and Varga [37] gave syllabi for eight countries. Cooper [8] refers in great detail to Great Britain.

9. Turning now to the Irish scene, I have not encountered much published material. Perhaps it would be well to refer to a general work, Mulcahy [28], and to the fact that the Irish Association for Curriculum Development has published a bi-annual journal *Compass* [7] since 1972.

In the present syllabus for the Intermediate Certificate, which has been current for a generation, we can see the introduction of the mathematical topics which we referred to in Section 2 as being common ground internationally. I do not wish to publish at this stage a detailed analysis of its geometrical content, which to put it mildly is inadequate. Briefly it has emphasised transformations, but the appearance of a vector-space focus as in 4(ii) is misleading, as it is only a veneer; basically the course never departed from proofs by congruence, although it has trophies from other courses, such as equipollence from Papy and angle-measure from Birkhoff.

In the new syllabus, announced on 25 September 1986 and to be first examined in 1990, a traditional treatment of geometry, based on congruence, is being reverted to.

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