overemphasis on accuracy and 100% correct answers will lead to the production of graduates with a need to analyse every problem in depth before producing any answer.

The relative absence of a large research and development sector in the industry of this country means the absence of what is in other countries a traditional source of employment for mathematically trained graduates. However, the development of the scientific approach to management in this as well as in other countries also requires the analytical abilities which mathematically trained graduates possess. There is and will be an increasing demand from this sector for mathematicians who can model and analyse complicated management and industrial problems.

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COMPUTERS AND THE MATHEMATICS CURRICULUM

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INTRODUCTION

Information technology will have a radical and pervasive effect on education, affecting both the aims, content and teaching and learning methods of all subjects; it will also affect the organisation of education, enabling its wider dispersal, both in terms of location and age of pupils. Teaching and learning methods, assessment and the curriculum are all bound up together, but here attention will be focused on curricular matters, specifically on the impact of computers on the mathematics curriculum, concentrating on the senior cycle of second-level education. Many of the ideas, however, will have a broader application, and will apply to mathematics education in general.

COMPUTERS AND MATHEMATICS

Firstly, a few comments on this perennial topic for debate. Although there are many connections between computers and mathematics, for example, programming may be regarded as a branch of logic (see e.g. Murphy [9]), present opinion is almost unanimous in regarding the linking of mathematics and computer studies on the curriculum as undesirable. For example: "Their view was unanimous that computer studies should not be regarded as part of mathematics but should ideally exist within a separate department." (Cockroft Report [1], par. 397.) Reasons commonly given are the need for special training in order to teach computer studies, the need to prevent such a subject becoming an elitist, and in particular a sexist one, and the fact that the linking of computer studies with mathematics inhibits its development across subject boundaries.

In Ireland, computer studies is an optional section in the mathematics syllabus in the senior cycle; as a result, its hours either come out of the mathematics allocation, or are tacked on to the end of the normal timetable. Of those teachers involved with computing in schools, mathematics teachers form the largest group. The situation is similar in Britain. So we see that *de facto* there is a strong connection between computer studies and mathematics in the education system. That this is bad for computer studies is generally recognised. However, I believe that it is equally bad for mathematics, because the time and resources of both teachers and pupils are diverted from mathematics. The situation should improve at second level with the establishment of computer studies as a subject in its own right, as seems likely.

CALCULATORS

It is instructive to consider briefly the impact which calculators have had on pupils' learning of mathematics. The commonest objection to the introduction of calculators into the mathematics classroom was that their use would impair pupils' abilities to carry out pencil and paper calculations. In fact the opposite has proved to be the case. In a meta-analysis of 79 research reports in the USA, Hembree and Dessart [3] report: "At all grades but Grade 4, a use of calculators in concert with traditional mathematics instruction apparently improves the average student's basic skills with paper and pencil, both in working exercises and in problem solving." They also report that the use of calculators improves a student's attitude towards, and self-concept in, mathematics. The use of calculators has been allowed in the Leaving Certificate since 1986.

COMPUTER EDUCATION IN IRISH SECOND-LEVEL SCHOOLS

The situation as it evolved up to 1983 was described in a previous article by Moynihan [8]. Some developments since then will be briefly described. Since September 1980 computer studies has been an optional module in the mathematics syllabus in the senior cycle, requiring 35 hours of study, but not forming part of the Leaving Certificate. In September 1984 an optional computer studies module was introduced into the junior cycle. This programme is independent of mathematics, requires about 70 hours of study and is not examinable. A syllabus committee for this module was set up, and it completed its work in May 1985. Both syllabi are available in the Department of Education's Rules and Programme for Secondary Schools 1986/87. Since August 1986 the Curriculum and Examinations Board has taken over responsibility for syllabus committees. At the time of writing, the Board of Studies for Science, Technology and Mathematics to the Curriculum and Examinations Board is still preparing its report. It seems likely that it will recommend a computer studies module in the junior cycle, independent of mathematics, and the establishment of a full senior cycle subject called either computer studies, or, more generally, information technology. Some pilot projects are being supported by the Department of Education: topics include courseware development and control applications. The spread of information technology across subject boundaries has been slow. A national policy on information technology has often been called for, but is slow in coming. In particular, there seems to be no commitment to adequate pre-service and in-service teacher training. However several third-level institutions, including Trinity College, the Regional Technical College Waterford, and Thomond College/NIHE Limerick provide postgraduate courses in computer education for teachers. On the hardware side, the official commitment to Apple has been maintained, although Commodores, BBCs and Amstrads are also popular; however, the days of the 8-bit machine must be numbered. Languages being used include BASIC, COMAL and LOGO. An interesting feature of the junior cycle syllabus is the recommendation of PROLOG as a suitable descriptive language.

GENERAL TRENDS IN MATHEMATICS EDUCATION

The effects of the widespread availability of microcomputers on the mathematics curriculum are difficult to predict, but the following trends can be discerned:

(a) the increased use of numerical methods; where these are already in use, their earlier introduction, either before, or at the same time as, analytical methods.

(b) the increased use of graphical methods, including dynamic graphics.

(c) the use of symbolic manipulation systems (for example, to carry out algebraic manipulation, or differentiation). These systems are not new, but have not up until recently been available to teachers.

(d) an increased emphasis on algorithmic thinking, linked to programming, and in particular, on the development, rather than just the use, of algorithms.

(e) the displacement of low-level skills, such as solving a simple equation, by higher-level skills, such as interpreting and applying the solution.

(f) the possibility of giving a dynamic, as well as a static view of some mathematical topics.

(g) the use of more realistic numbers in applications of mathematics: this should lead to a reduction in the level of abstractions.

(h) the formal teaching of estimation skills: mental arithmetic, approximation and general 'number sense'.

(i) an increased emphasis on problem-solving methods such as guess and check or successive approximation.

(j) the possibility of more heuristic learning: the learning of mathematics through personal discovery can be facilitated through the use of 'tool-kits' or 'microworlds'.

These challenge the linear model of learning.

(k) an increased use of mathematical modelling and simulation.

SOME TOPICS AT SENIOR-CYCLE, SECOND-LEVEL

In a previous article, Seda [11] has discussed some topics which might form a basis for the discrete component in a better balanced curriculum for mathematics students. Many of these topics are also suitable for senior cycle pupils: some candidates for inclusion would be graph theory, mathematical logic, difference equations and the study of algorithms. For a description of an algorithm option in the A-level examination, see Kowszun [6].

In analysis, we often have a choice between continuous and numerical methods: the basic concepts of analysis, e.g. rate of change and area, can be implemented in either discrete or continuous ways: difference and sum, or derivative and integral. The discrete concepts are simpler, but their implementation generally involves much calculation. Calculus is limited in the range of functions it can deal with. Nevertheless, it is a powerful and beautiful theory. Continuous analysis is necessary for the theoretical justification of, and as a guide to the numerical methods (see Winkelmann [13] for a discussion of this topic). Numerical techniques may be introduced either before, or at the same time as, analytical ones. Symbolic differentiation and integration programmes will become available to teachers and pupils: less time will need to be spent on techniques or tricks-of-the-trade, leaving more time available for developing theoretical insight. Graphical techniques may be used to facilitate understanding of the concepts involved in analysis. Much useful graphics software already exists.

Symbolic manipulation systems (e.g. muMATH) will have an impact on the teaching of algebra similar to that of calculators on the teaching of arithmetic. Algebraic manipulations may now be carried out by machine. The implications of this are that less importance will attach to the ability to manipulate algebraic expressions, and more to the ability to formulate them. This will hopefully lead to a better understanding of their meaning. Facilities available in such systems generally include manipulation of polynomials, symbolic differentiation and integration, arbitrary precision arithmetic and simplification of algebraic expressions. Graphics will be available in similar programmes in the near future. General data types are increasingly important: besides numbers, other data types, such as strings, matrices, sets and Boolean variables, should be encountered. Spreadsheet programmes can facilitate the introduction of matrix algebra. The commercial programmes are quite useful, although a spreadsheet specially designed for mathematics would be an improvement. Incidentally, these programmes have many other uses in mathematics (see Hsiao [5] for some examples). Programmes are being developed which help the user to explore algebra: for example, Algebraland, being developed by the Xerox Palo Alto Research Center, keeps a record of the algebraic transformations applied during the solution of an equation, thus facilitating an examination of the solution path. The case has been made by several authors that the teaching of programming facilitates the understanding of many mathematical concepts, and in particular algebraic concepts (see e.g. Hart [2]). The concept of variable appears very early in programming (in most languages). Simple programming can give pupils concrete examples of the use of variables. Of course, a variable in programming is not exactly the same as a variable in mathematics: programming gives a more dynamic, changing view. Programming may also be used to introduce the ideas of operation on a variable, and function. Inverse and composite functions are also met in easily understandable ways; e.g. in LOGO, LT is the inverse of RT. The concept of function may also be compared and contrasted with that of procedure. Procedures commonly take an input, do something with it, and produce an output. More generally, the writing of programs involves the use of formal symbols, and of a particular syntax, and this in itself has analogies with algebra.

The availability of graphics makes certain areas of geometry, for example transformation geometry and three-dimensional geometry, much more easily accessible to pupils. The writing of programmes to produce certain graphic effects also provides a strong motivation for learning geometry. 'Tool-kits' are being produced which enable the user to explore some aspect of mathematics. The most commonly used one at the moment is Papert's Turtle Geometry incorporated in the language LOGO. Another example is the Geometric Supposer (see Schwartz [10]). This enables the user to experiment with geometric constructions, and hopefully to make and test hypotheses. Much work has been done on the impact of LOGO on mathematics education: for a discussion of this, see Hoyles and Noss [4]. Some effects on geometry are: a pupil's experience of angle may be enlarged; LOGO provides a procedural description of curves rather than a static one, opening the question of which curves should be emphasised (see Laski [7]); curves may be defined as limits, or using recursion: either of these techniques is commonly used when defining a circle in LOGO. Both the display devices and the programming languages used have their own intrinsic geometries; for example, BASIC generally uses absolute 2D cartesian co-ordinates, while LOGO uses relative 2D polar co-ordinates (see Oldknow's essay in the Ware conference report [12] for a discussion of this point).

CONCLUSION

There are several main effects of the increased availability of computers on the mathematics curriculum: firstly, there will be pressure for new topics to be introduced into the curriculum; secondly, existing topics can be taught and learnt in new ways: this can change the sequencing of topics, and, more fundamentally, can challenge the concept of a curriculum topic: the curriculum could follow the trend at primary level and become more pupil-centred; thirdly, some pressure is being put on the teaching of mathematics due to the present lack of adequate teacher training in computer studies.

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