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A CONNECTED TOPOLOGY WHICH IS NOT LOCALLY CONNECTED

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To demonstrate that a connected topological space may not be locally connected, authors of modern text-books on point-set topology usually employ an example, either of a geometrical nature in the real plane (such as the so-called "topologist's sine curve". and "infinite broom"), or of a number theoretical nature in the integers (such as the "relatively prime integer topology"), or of an analytical nature in the real line (such as the "indiscrete or pointed extensions of the reals" and the "one-point compactification of the rationals"; see [1]). The complete exposition of such an example tends to rely heavily on a knowledge of the various intrinsic properties of the supporting set. For the instructor who may, perhaps, prefer a more abstract and topologically succinct example, an alternative is readily available.

Let X be an infinite set containing distinct points x,y. A topology τ for X may be defined by declaring open, apart from $\emptyset$ and X itself, those subsets G of X for which $y \notin G$ *and* either $x \notin G$ or $X-G$ contains (at most) a finite number of points. Observe that $\tau = (\gamma \cup \varepsilon(x)) \cap \varepsilon(y)$, where γ denotes the well known cofinite topology for X and $\varepsilon(x)$, $\varepsilon(y)$ denote, respectively, the excluded point topologies $\{G \subseteq X : x \notin G\} \dot{\cup} \{X\}$ and $\{G \subseteq X : y \notin G\} \cup \{X\}$ (see [1]). That is, τ is the intersection of a Fort topology $\gamma \cup \varepsilon(x)$ and an excluded point topology $\varepsilon(y)$.

It is immediate that (X,τ) is a connected space (since $\tau \subseteq \varepsilon(y)$ and $(X,\varepsilon(y))$ is obviously a connected space). Let U be any proper τ-open neighbourhood of x. Thus $y \notin U$ and $X-U$ is finite. If $z \in U$, $z \neq x$, then $\{z\}$ and $U-\{z\}$ are each τ-open (since y belongs to neither, $x \notin \{z\}$ and $X-(U-\{z\}) = (X-U) \cup \{z\}$ is finite) so that U is not τ-connected. It follows that (X,τ) is not locally connected (at x).

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NONCOMMUTATIVE ANTICOMMUTATIVE RINGS

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An associative ring R is said to be *anticommutative* if $xy + yx = 0$ for all $x,y \in R$. If R has characteristic 2, then the concepts of commutativity and anticommutativity coincide, but Z_3, with the usual addition and trivial multiplication, shows that an anticommutative ring need not have characteristic 2.

If a ring R satisfies $x^2 = 0$ for all $x \in R$ then clearly R is anticommutative, but not conversely. However, if R is anticommutative it is easy to verify that R satisfies each of the following identities.

(i) $2x^2 = 0$ (ii) $(xy - yx)^2 = 0$ (iii) $x^2y - yx^2 = 0$.

Frequently, when looking at commutativity theorems for rings, one requires counterexamples to show that certain conditions are not sufficient for commutativity. For example, if $(xy)^2 = x^2y^2$ for all $x,y \in R$ and either of the following conditions holds then R is commutative:

(a) R has unity; (b) R has no non-zero nilpotent elements.

To show that some such additional condition is necessary, it is enough to produce a non-commutative ring in which $x^2 = 0$ for all $x \in R$. In this note, for finite rings, we pose the question, *"what is the order of a smallest noncommutative anti-commutative ring?"* and show that the answer is 27. Since this number is odd, we see that it is also the answer to the question, *"what is the order of a smallest noncommutative ring satisfying the identity $x^2 = 0$?"*.