

Problem Solving

Set 20

22 July 2012

1. Denote by S_n the group of permutations of the sequence $(1, 2, \dots, n)$. Suppose that G is a subgroup of S_n such that for every $\pi \in G \setminus \{e\}$ there exists a unique $k \in \{1, 2, \dots, n\}$ for which $\pi(k) = k$. (Here e is the unit element in the group S_n .) Show that this k is the same for all $\pi \in G \setminus \{e\}$.
2. If $n = 4444^{4444}$, a is the sum of all the digits of n , b is the sum of all the digits of a , and c is the sum of all the digits of b , find c .