

Problem Solving

Set 15

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1. Let p be a polynomial with integer coefficients and let $a_1 < a_2 < \dots < a_k$ be integers.
 - (a) Prove that there exists $a \in \mathbb{Z}$ such that $p(a_i)$ divides $p(a)$ for all $i = 1, 2, \dots, k$.
 - (b) Does there exist an $a \in \mathbb{Z}$ such that the product $p(a_1)p(a_2) \cdots p(a_k)$ divides $p(a)$?
2. S is a set of 15 integers, with the property that for every $s \in S$, there exists $a, b \in S$ such that $s = a + b$.
Prove that there exists a non-empty subset $T \subset S$ containing 7 or fewer elements whose sum is zero.