Problem Solving
Set 14

17 July 2012

1. Denote by V the real vector space of all real polynomials in one variable, and let $P : V \to \mathbb{R}$ be a linear map. Suppose that for all $f, g \in V$ with $P(fg) = 0$ we have $P(f) = 0$ or $P(g) = 0$. Prove that there exist real numbers x_0, c such that $P(f) = cf(x_0)$ for all $f \in V$.

2. The smallest number with four factors is $6 : 1, 2, 3, 6$. The smallest number with six factors is $12 : 1, 2, 3, 4, 6, 12$. What is the smallest number with 100 factors?