Problem Solving
Set 11

14 July 2012

1. Call a polynomial $P(x_1, \ldots, x_k)$ *good* if there exist 2×2 real matrices $A_1, \ldots, A_k$ such that

$$P(x_1, \ldots, x_k) = \det\left(\sum_{i=1}^{k} x_i A_i\right).$$

   Find all values of k for which all homogeneous polynomials with k variables of degree 2 are good.

   (A polynomial is homogeneous if each term has the same total degree.)

2. Show that the equation

$$2x^4 + 1 = y^2$$

   has no integer solution.