

# 18<sup>th</sup> IMC Competition

2011

A1. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function. A point  $x$  is called a shadow point if there exists a point  $y \in \mathbb{R}$  with  $y > x$  such that  $f(y) > f(x)$ . Let  $a < b$  be real numbers and suppose that

- all the points of the open interval  $I = (a, b)$  are shadow points;
- $a$  and  $b$  are not shadow points.

Prove that

- (a)  $f(x) \leq f(b)$  for all  $a < x < b$ ;
- (b)  $f(a) = f(b)$ .

A2. Does there exist a real  $3 \times 3$  matrix  $A$  such that  $\text{tr}(A) = 0$  and  $A^2 + A^t = I$ ? ( $\text{tr}(A)$  denotes the trace of  $A$ ,  $A^t$  is the transpose of  $A$ , and  $I$  is the identity matrix.)

**B3.**

B2.