1^{st} IMC Competition

Plovdiv, Bulgaria

July 29–30, 1994

A1. (a) Let A be a $n \times n$, $n \geq 2$, symmetric, invertible matrix with real positive elements. Show that $z_n \leq n^2 - 2n$, where z_n is the number of zero elements in A^{-1}.

(b) How many zero elements are there in the inverse of the $n \times n$ matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 2 & 2 & \dots & 2 \\ 1 & 2 & 1 & 1 & \dots & 1 \\ 1 & 2 & 1 & 2 & \dots & 2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 2 & 1 & 2 & \dots & \dots \end{pmatrix} ?$$

A2. Let $f \in C^1(a, b)$, $\lim_{x \to a+} f(x) = +\infty$, $\lim_{x \to b-} f(x) = -\infty$ and $f'(x) + f^2(x) \geq -1$ for $x \in (a, b)$. Prove that $b - a \geq \pi$ and give an example where $b - a = \pi$.

A3. Given a set S of $2n - 1$, $n \in \mathbb{N}$, different irrational numbers. Prove that there are n different elements $x_1, x_2, \dots, x_n \in S$ such that for all nonnegative rational numbers $a_1, a_2, \dots, a_n$ with $a_1 + a_2 + \dots + a_n > 0$ we have that $a_1 x_1 + a_2 x_2 + \dots + a_n x_n$ is an irrational number.