Exercise 6

In exercises 1–10 find the smallest simultaneous solution $n \geq 0$ of the given congruences, or else show that there is no such solution.

** 1. $n \equiv 1 \bmod 4,\ n \equiv 2 \bmod 7$

** 2. $n \equiv 2 \bmod 5,\ n \equiv 5 \bmod 8$

** 3. $n \equiv 2 \bmod 3,\ n \equiv 3 \bmod 4$

** 4. $n \equiv 2 \bmod 5,\ n \equiv 3 \bmod 7,\ n \equiv 1 \bmod 8$

** 5. $n \equiv 3 \bmod 4,\ n \equiv 5 \bmod 7,\ n \equiv 2 \bmod 9$

** 6. $n \equiv 1 \bmod 5,\ n \equiv 3 \bmod 6,\ n \equiv 2 \bmod 7$

** 7. $n \equiv 2 \bmod 4,\ n \equiv 4 \bmod 5,\ n \equiv 3 \bmod 7$

** 8. $n \equiv 2 \bmod 4,\ n \equiv 3 \bmod 6,\ n \equiv 4 \bmod 7$

** 9. $n \equiv 4 \bmod 7,\ n \equiv 6 \bmod 11,\ n \equiv 9 \bmod 11$

** 10. $n \equiv 1 \bmod 9,\ n \equiv 2 \bmod 10,\ n \equiv 3 \bmod 11$

*** 11. How many positive integers $x \leq 10,000$ are there such that the difference $2^x - x^2$ is not divisible by 7?

*** 12. Show that

$$\phi(n) \to \infty$$

as $n \to \infty$.

**** 13. Find an odd integer k such that $k \cdot 2^n - 1$ is composite for all $n \geq 1$.

**** 14. Is there a 9-digit number

$$N = d_1 d_2 \cdots d_9$$

with the following properties: the 9 digits are distinct, and for each $k \in [1, 9]$ the number

$$d_1 d_2 \ldots d_k$$

is divisible by k?