## Exercise 5

In Exercises 1–16 determine all solutions of the given congruence.

- \* 1.  $3x \equiv 1 \mod 23$
- \* 2.  $7x \equiv 1 \mod 47$
- \*\* 3.  $5x \equiv 2 \mod 210$
- \*\* 4.  $6x \equiv 7 \mod 25$
- \*\* 5.  $8x \equiv 5 \mod 31$
- \*\* 6.  $8x \equiv 12 \mod 32$
- \*\* 7.  $12x \equiv 6 \mod 21$
- \*\* 8.  $2x \equiv 2 \mod 16$
- \*\* 9.  $20x \equiv 8 \mod 24$
- \*\*\* 10.  $7x \equiv -3 \mod 2009$
- \*\* 11.  $x^2 \equiv 1 \mod 12$
- \*\* 12.  $x^2 \equiv -1 \mod 15$
- \*\* 13.  $x^2 + x + 1 \equiv 0 \mod 3$
- \*\* 14.  $x^2 2x + 3 \equiv 0 \mod 5$
- \*\* 15.  $x^2 2 \equiv 0 \mod 7$
- \*\*\* 16.  $x^4 + 2x^2 + x 2 \equiv 0 \mod 7$ 
  - \* 17. What is the order of 10 in the additive group  $\mathbb{Z}/(24)$ ?
- \*\* 18. Determine the orders of the elements 7, 11, 21 in the multiplicative group  $(\mathbb{Z}/36)^{\times}$ .
- \*\* 19. What is the order of the group  $(\mathbb{Z}/36)^{\times}$ ?
- \*\*\* 20. Is the group  $(\mathbb{Z}/36)^{\times}$  cyclic?

- \*\*\* 21. Is Christmas equally likely to take place on any day of the week?
- \*\*\*\* 22. Given integers  $x_1, x_2, \ldots, x_{11}$ , show that there exists a finite sequence  $a_1, \ldots, a_{11}$  of numbers from  $\{-1, 0, 1\}$  such that the sum

$$a_1x_1 + \ldots + a_{11}x_{11}$$

is divisible by 2009.

- \*\*\* 23. Construct the field containing 4 elements.
- \*\*\*\* 24. Show that there is no field containing 6 elements.
- \*\*\* 25. Determine the orders of all the elements in  $\mathbb{F}_{11}^{\times}$ ?
- \*\* 26. What is the order of the multiplicative group  $\mathbb{F}_q^{\times ?}$ ?
- \*\*\* 27. How many elements are there of order 4 in  $\mathbb{F}_{17}^{\times}$ ?
- \*\*\* 28. Prove that there is a multiple of 2009 which ends with the digits 000001.