Exercise 17

In exercises 1-10, determine the continued fraction of the given number.

** 1. $\frac{17}{5}$

** 2. $\frac{5}{17}$

** 3. $-\frac{7}{8}$

** 4. $\frac{1001}{10001}$

** 5. $\frac{2317}{2009}$

*** 6. $\sqrt{3}$

*** 7. $\sqrt{7}$

*** 8. $\sqrt{11}$

*** 9. $\frac{\sqrt{3}+1}{2}$

*** 10. $7\sqrt{3}$

*** 11. Suppose the quadratic surd

$$\alpha = [a_0, a_1, \ldots]$$

satisfies the equation

$$Ax^2 + 2Bx + c = 0.$$

where $A, B, C \in \mathbb{Z}$ with $\gcd(A, B, C) = 1$. If the corresponding equation for

$$\alpha_n = [a_n, a_{n+1}, \ldots]$$

is

$$A_n x^2 + 2B_n x + c_n = 0$$

show that

$$B^2 - AC = B_n^2 - A_n C_n.$$

*** 12. Find the first 5 convergents to π.

***** 13. Show that

$$e = [2, 1, 2, 1, 1, 4, 1, 1, 6, \ldots].$$