## Exercise 16

*** 1. Determine which rational primes $p$ split in the real number ring $\mathbb{Z}[\sqrt{3}]$.

In exercises 2-5, determine the prime factorisation of the given number in the ring $\mathbb{Z}[\sqrt{3}]$.

*** 2. $-2$

** 3. $3$

*** 4. $7$

*** 5. $1 + \sqrt{3}$

*** 6. Show that the real number ring $\mathbb{Z}[\sqrt{2}]$ is a Unique Factorisation Domain, and determine the primes in this ring.

In exercises 7-10, determine the prime factorisation of the given number in the ring $\mathbb{Z}[\sqrt{2}]$.

*** 7. $2$

*** 8. $7$

*** 9. $2 + \sqrt{2}$

*** 10. $3 + \sqrt{3}$

*** 11. Show that the ring $\mathbb{Z}[\sqrt{5}]$ is not a Unique Factorisation Domain. [Note: this is not the number ring associated to the field $\mathbb{Q}(\sqrt{5})$.]

*** 12. Show that the imaginary number ring $\mathbb{Z}[\omega]$ (where $\omega^3 = 1,\ \omega \neq 1$) is a Unique Factorisation Domain, and determine the primes in this ring.

In exercises 13-15, determine the prime factorisation of the given number in the ring $\mathbb{Z}[\omega]$.

*** 13. $1 - \omega$

*** 14. $2 + \omega$

*** 15. $2 - \omega$

*** 16. Show that the imaginary number ring $\mathbb{Z}[\sqrt{-5}]$ is not a Unique Factorisation Domain, by considering the factorisations of the number 6 in this ring, or in any other way.

**** 17. Determine if the imaginary number ring $\mathbb{Z}[\sqrt{-6}]$ is a Unique Factorisation Domain.

**** 18. Determine if the imaginary number ring $\mathbb{Z}[\sqrt{-7}]$ is a Unique Factorisation Domain.

**** 19. Show that the real number ring $\mathbb{Z}[\sqrt{6}]$ is a Unique Factorisation Domain.

**** 20. Show that the real number ring $\mathbb{Z}[\sqrt{7}]$ is a Unique Factorisation Domain.