Exercise 11

In exercises 1-5, find the value of the given Legendre symbol

** 1. $\left(\frac{13}{23}\right)$

** 2. $\left(\frac{23}{13}\right)$

** 3. $\left(\frac{40}{53}\right)$

** 4. $\left(\frac{36}{61}\right)$

** 5. $\left(\frac{2009}{2011}\right)$

In exercises 6-15, determine if the given congruence has a solution, and if it does find the smallest solution $x \geq 0$.

** 6. $x^2 \equiv 10 \bmod 36$

** 7. $x^2 + 12 \equiv 0 \bmod 75$

*** 8. $x^2 \equiv 8 \bmod 2009$

*** 9. $x^2 \equiv 56 \bmod 2317$

*** 10. $x^2 + 2x + 17 \equiv 0 \bmod 35$

*** 11. $x^2 + 3x + 1 \equiv 0 \bmod 13$

** 12. $x^3 \equiv -1 \bmod 105$

*** 13. $x^7 \equiv 3 \bmod 17$

*** 14. $x^3 + 2 \equiv 0 \bmod 27$

*** 15. $x^5 + 3x + 1 \equiv 0 \bmod 25$

**** 16. If $n > 0$ is an odd number, and $n = p_1 \dots p_r$, we define the Jacobi symbol $\left(\frac{a}{n}\right)$ by

$$\left(\frac{a}{n}\right) = \left(\frac{a}{p_1}\right) \dots \left(\frac{a}{p_r}\right).$$

Show that if $m, n > 0$ are both odd then

$$\left(\frac{m}{n}\right)\left(\frac{n}{m}\right) = \begin{cases} -1 \text{ if } m \equiv n \equiv -1 \bmod 4, \\ 1 \text{ otherwise }. \end{cases}$$

In exercises 21-25, find the value of the given Jacobi symbol

** 17. $\left(\frac{9}{15}\right)$

** 18. $\left(\frac{15}{9}\right)$

** 19. $\left(\frac{40}{49}\right)$

** 20. $\left(\frac{2317}{2009}\right)$

** 21. $\left(\frac{2009}{2317}\right)$

**** 22. Is there a power 7^n which ends with the digits 000011? If so, what is the smallest such n?

**** 23. Is there a power of 2009 which ends with the digits 2317?

**** 24. Is there a power of 2319 which ends with the digits 2009?

*** 25. Determine $\left(\frac{3}{p}\right)$ for an odd prime p without using Quadratic Reciprocity.