Exercise 10

In exercises 1-5, find the value of the given Legendre symbol

** 1. $\left(\dfrac{3}{5}\right)$

** 2. $\left(\dfrac{5}{3}\right)$

** 3. $\left(\dfrac{-1}{5}\right)$

** 4. $\left(\dfrac{5}{5}\right)$

** 5. $\left(\dfrac{5}{7}\right)$

In exercises 6-15, determine if the given congruence has a solution, and if it does find the smallest solution $x \geq 0$.

** 6. $x^2 \equiv 5 \bmod 10$

** 7. $x^2 \equiv 5 \bmod 11$

** 8. $x^2 \equiv 5 \bmod 12$

** 9. $x^2 \equiv 4 \bmod 15$

** 10. $x^2 \equiv -1 \bmod 105$

** 11. $x^2 + 3x + 1 \equiv 0 \bmod 13$

*** 12. $x^2 + 3x + 1 \equiv 0 \bmod 13$

*** 13. $x^2 \equiv 2 \bmod 27$

*** 14. $x^2 + 2 \equiv 0 \bmod 81$

*** 15. $x^2 \equiv 4 \bmod 25$

*** 16. Show that if p is a prime satisfying $p \equiv 1 \bmod 4$ then $x = ((p-1)/2)!$ satisfies

$$x^2 + 1 \equiv 0 \bmod p.$$