Exercise 5

In Exercises 1–16 determine all solutions of the given congruence.

* 1. $3x \equiv 1 \bmod 23$

* 2. $7x \equiv 1 \bmod 47$

** 3. $5x \equiv 2 \bmod 210$

** 4. $6x \equiv 7 \bmod 25$

** 5. $8x \equiv 5 \bmod 31$

** 6. $8x \equiv 12 \bmod 32$

** 7. $12x \equiv 6 \bmod 21$

** 8. $2x \equiv 2 \bmod 16$

** 9. $20x \equiv 8 \bmod 24$

*** 10. $7x \equiv -3 \bmod 2009$

** 11. $x^2 \equiv 1 \bmod 12$

** 12. $x^2 \equiv -1 \bmod 15$

** 13. $x^2 + x + 1 \equiv 0 \bmod 3$

** 14. $x^2 - 2x + 3 \equiv 0 \bmod 5$

** 15. $x^2 - 2 \equiv 0 \bmod 7$

*** 16. $x^4 + 2x^2 + x - 2 \equiv 0 \bmod 7$

* 17. What is the order of 10 in the additive group $\mathbb{Z}/(24)$?

** 18. Determine the orders of the elements $7, 11, 21$ in the multiplicative group $(\mathbb{Z}/36)^\times$.

** 19. What is the order of the group $(\mathbb{Z}/36)^\times$?

*** 20. Is the group $(\mathbb{Z}/36)^\times$ cyclic?

*** 21. Is Christmas equally likely to take place on any day of the week?

**** 22. Given integers $x_1, x_2, \ldots, x_{11}$, show that there exists a finite sequence $a_1, ..., a_{11}$ of numbers from $\{-1, 0, 1\}$ such that the sum

$$a_1 x_1 + ... + a_{11} x_{11}$$

is divisible by 2009.

*** 23. Construct the field containing 4 elements.

**** 24. Show that there is no field containing 6 elements.

*** 25. Determine the orders of all the elements in $\mathbb{F}_{11}^{\times}$?

** 26. What is the order of the multiplicative group $\mathbb{F}_q^{\times}$?

*** 27. How many elements are there of order 4 in $\mathbb{F}_{17}^{\times}$?

*** 28. Prove that there is a multiple of 2009 which ends with the digits 000001.