

Exercise 4

In Exercises 1–5 determine the additive order of the given element.

- * 1. $3 \bmod 5$
- * 2. $3 \bmod 6$
- * 3. $2 \bmod 7$
- * 4. $-13 \bmod 14$
- ** 5. $100000 \bmod 123456$

In Exercises 6–10 determine the multiplicative order of the given element.

- * 6. $3 \bmod 5$
- * 7. $7 \bmod 12$
- ** 8. $2 \bmod 31$
- ** 9. $-2 \bmod 31$
- *** 10. $2 \bmod 3^5$

In Exercises 11–15 determine the multiplicative inverse of the given element.

- * 11. $3 \bmod 5$
- * 12. $3 \bmod 13$
- * 13. $2 \bmod 111$
- ** 14. $137 \bmod 253$

In Exercises 16–20 determine the order of the given multiplicative group, and list its elements.

- * 15. $(\mathbb{Z}/2)^\times$
- * 16. $(\mathbb{Z}/6)^\times$
- * 17. $(\mathbb{Z}/8)^\times$
- * 18. $(\mathbb{Z}/12)^\times$
- * 19. $(\mathbb{Z}/15)^\times$
- * 20. Determine $\phi(45)$
- * 21. Determine $\phi(3^n)$
- * 22. Determine all positive integers n with $\phi(n) = n - 1$.
- ** 23. Determine all positive integers n with $\phi(n) = n - 2$.
- ** 24. What is the smallest value of $\phi(n)/n$?
- ** 25. Show that there is a field containing 4 elements.
- ** 26. Show that there is no field containing 6 elements.