Attempt 3 questions. (If you attempt more, only the best 3 will be counted.) All questions carry the same number of marks. Unless otherwise stated, all groups are compact (or finite), and all representations are of finite degree over $\mathbb{C}$.

1. Define a group representation.

What is meant by saying that a representation $\alpha$ is simple? Determine all simple representations of $S_3$. from first principles.

Determine the characters of $S_4$ induced by the simple characters of $S_3$. Hence or otherwise draw up the character table for $S_4$.

2. Show that the number of simple representations of a finite group $G$ is equal to the number $s$ of conjugacy classes in $G$.

Determine the conjugacy classes in $A_4$ (formed by the even permutations in $S_4$), and draw up its character table.

Determine also the representation-ring for this group, ie express the product $\alpha \beta$ of each pair of simple representation as a sum of simple representations.

More questions overleaf!
3. Determine all groups of order 8 (up to isomorphism); and for each such group $G$ determine as many simple representations (or characters) of $G$ as you can.

4. Determine the conjugacy classes in $\text{SU}(2)$; and prove that this group has just one simple representation of each dimension.

Define a covering homomorphism

$$\Theta : \text{SU}(2) \to \text{SO}(3);$$

and hence or otherwise show that $\text{SO}(3)$ has one simple representation of each odd dimension $1, 3, 5, \ldots$. 