Finite Fields

Exercises on Chapter 1

Exercise 1

In questions 1–5 find all solutions of the given equation in $\mathbb{F}_{13}$.

* 1. $2x = 7$.

** 2. $x^2 = 7$.

* 3. $3x = 4$.

*** 4. $x^{10} = 5$.

** 5. $x^2 + x + 3 = 0$.

** 6. Find the multiplicative order of each non-zero element of $\mathbb{F}_{17}$.

* 7. Find the additive order of each element of $\mathbb{F}_{13}$.

** 8. Show that the group $F_{23}^{\times}$ is cyclic.

*** 9. How many elements are there in the group $\mathbf{GL}(2, \mathbb{F}_7)$ (the group of non-singular 2×2-matrices over $\mathbb{F}_7$)?

**** 10. How many elements are there in the group $\mathbf{SL}(2, \mathbb{F}_{11})$ (the group of matrices over $\mathbb{F}_{11}$ with determinant 1)?