Finite Fields

Exercises on Chapter 3

Exercise 3

In questions 1–8, V is a vector space of dimension 3 over $\mathbb{F}_2$.

* 1. How many elements are there in V?

** 2. How many linear maps $\alpha : V \to V$ are there?

** 3. How many of these maps are surjective?

** 4. How many vector subspaces does V have?

** 5. Is there a linear map $\alpha : V \to V$ satisfying $\alpha^2 + I = 0$?

** 6. Which linear maps $\alpha : V \to V$ commute with every linear map $\beta : V \to V$?

** 7. How many linear maps $\alpha : V \to V$ have trace 0 and determinant 1?

*** 8. Are any two such linear maps similar?

*** 9. Show that the subsets of a set X form a ring of characteristic 2 if we set $U + V = (U \setminus V) \cup (V \setminus U)$ and $U \times V = U \cap V$. What are the zero and identity elements in this ring?

*** 10. Is this ring a field for any set X?