UNIVERSITY OF DUBLIN

1417

TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

JS Mathematics **Hillary Term 2000**
SS Mathematics

COURSE 373 PART A

Friday, March 10 Maxwell Theatre 16.15 — 17.45

Dr. T.G. Murphy

Attempt 5 questions. (If you attempt more, only the best 5 will be counted.) All questions carry the same number of marks.

1. Prove that the multiplicative group $F^\times = F - \{0\}$ of a finite field F is cyclic.

   Find all the generators of $(\mathbb{Z}/13)^\times$.

2. Define the *characteristic* of a field, and show that the characteristic of a finite field F is always a prime number.

   Show that a finite field F of characteristic p contains p^n elements for some n.

   Show also that any subfield of F contains p^m elements, where $m \mid n$

3. Listing the elements of $\mathbb{F}_8$ in any way you wish, draw up the addition and multiplication tables for this field.

4. Prove that two finite fields containing the same number of elements are necessarily isomorphic.

5. If F is a finite field of characteristic p show that the map

$$\Phi : x \mapsto x^p$$

is an automorphism of F; and show that every automorphism of F is of the form $x \mapsto \Phi^i(x)$ for some i.

6. Show that if there are $\Pi(n) = \Pi_p(n)$ prime polynomials of degree n over $\mathbb{F}_p$, then

$$\sum_{d|n} d\Pi(d) = p^n$$

for each positive integer n.

Determine all prime polynomials of degree 5 over $\mathbb{F}_2$.

7. Prove Wedderburn's Theorem, that every finite skew field is in fact commutative.