REMARKS ON THE REMAINDER OF THE CONSIDERATIONS RELATING TO FLUXIONS &c. THAT WAS PUBLISHED BY PHILALETHES CANTABRIGIENSIS IN THE REPUBLICK OF LETTERS FOR THE LAST MONTH TO WHICH IS ADDED BY DR. PEMBERTON A POSTSCRIPT OCCASIONED BY A PASSAGE IN THE SAID CONSIDERATIONS

By

Benjamin Robins

(The Present State of the Republick of Letters,
September 1736, Appendix, pp. 2–40)

Edited by David R. Wilkins

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NOTE ON THE TEXT

This article appeared in an appendix to *The Present State of the Republick of Letters* for September 1736.

The following spellings, differing from modern British English, are employed in the original 1736 text: expresly, vertue, stile, defense, bigottry, explicite, falshood, preceeding, inabled, intirely, reallity, imploy.

David R. Wilkins
Dublin, June 2002

AN APPENDIX TO THE Present State OF THE Republick of Letters

For the Month of *September* 1736.

BEING

REMARKS *on the* Remainder *of the* Considerations *relating to* Fluxions, &c. *that was published by* Philalethes Cantabrigiensis *in the* Republick of Letters *for the last Month. To which is added by Dr.* Pemberton *a* Postscript *occasioned by a* passage *in the said* Considerations.

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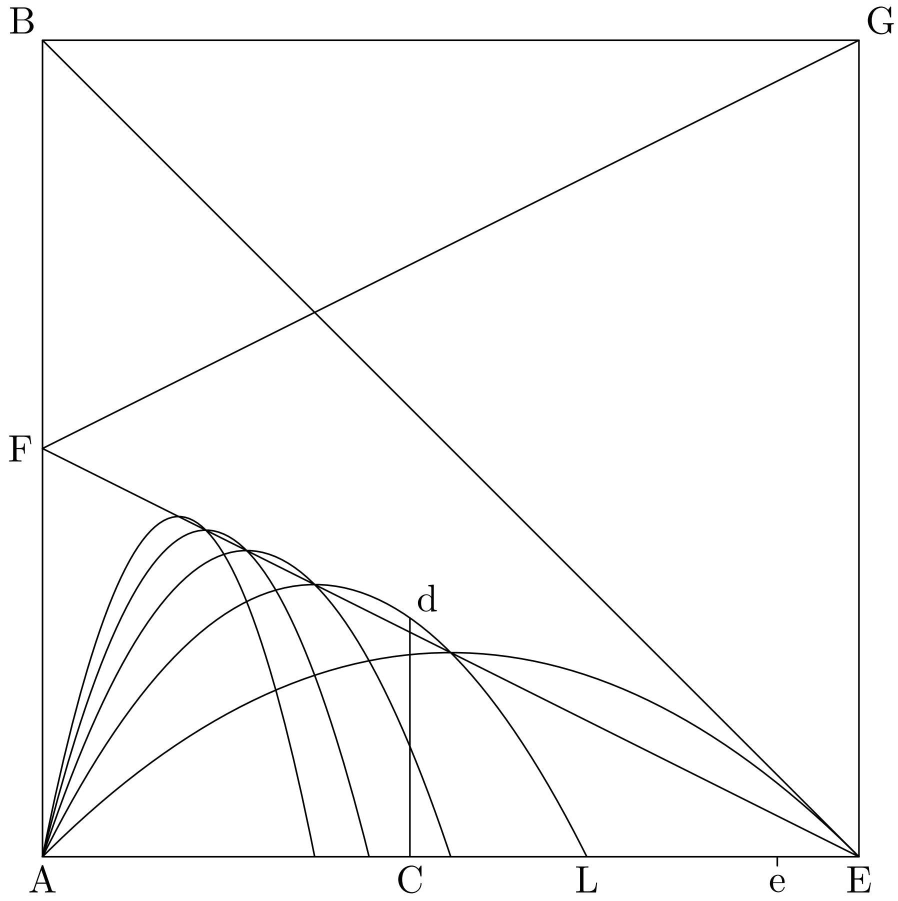
[*The Present State of the Republick of Letters*, September 1736, Appendix, pp. 2–40.]

Sect. XVII. Because Mr. *Robins* has suggested, that no kind of continued motion could be contrived, whereby to describe the parallelograms in question, which will not be involved and perplex'd; *Philalethes* has employed this whole section in attempting to represent to the imagination what he calls the actual equality, at which the inscribed and circumscribed figures will arrive with each other, and with the curvilinear figure, at the expiration of the finite time. But is this producing any continued motion for the purpose, Mr. *Robins* speaks of?

However, let us see how well *Philalethes* has executed even his own design. Two curves are imagined to be described, whose ordinates may express continually the proportion between the inscribed and circumscribed parallelograms in question; by the intersection or concurrence of which curves it may be found, when the inscribed and circumscribed figures become equal. But though each of the lines E d d F, and F D D G are drawn by *Philalethes* in his figure, as simple curves; yet in reality they are each compounded of an endless number of portions of as many different curves combined together. In his particular example at page 118 this equation $\mathrm{C}\,d = \dfrac{\overline{x+r} \times \overline{a-r}}{2a}$ is set down, as the equation for the continued curve (as it is there called) E d d F; whereas this equation is not confined to one curve, but comprehends a whole series, to find each of which the value r must be assigned. Because A e is always some multiple of A C, let A e be to A C as m to 1: then will r be $= mx - \overline{m-1} \times a$, and $\dfrac{2a}{m \times \overline{m+1}} \times \mathrm{C}\,d$ will be $= \overline{a-x} \times x - \left| \dfrac{m-1}{m+1} \right. a$. This equation shews each curve, which contributes to the forming the compound line E d d F, to be a parabola, the latus rectum of the axis being $\dfrac{2a}{m \times \overline{m+1}}$; so that for every different value of m we shall have a different parabola. All these parabola's pass through the point A, and the other intersection with A E is found by taking $\mathrm{E\,L} = \dfrac{m-1}{m+1}a$. Thus this *continued* curve E d d F is not to be described, but by an endless number of parabola's, as in the figure: insomuch that the continuation of this incurvated line as far as the point F, or of its partner to be drawn from G towards F is impossible. This is a specimen of *Philalethes*'s skill in the common algebra of curve lines, to give as an equation expressing the nature of a single curve, one which in reality includes an infinite series.

Sect. XVIII. Here Mr. *Robins* is positively and rudely charged with changing his opinion. Mr. *Robins* as positively avers the contrary.



Philalethes supposed a last form of these figures, which should be called equal to the curve. Now Mr. *Robins* has observed, that equality implies the things, which have that property, to be distinct from each other. For to say a thing is equal to itself is certainly no proper expression, and Mr. *Robins* in the passage here referred to has fully proved, there is no such last form distinct from the curve. For if these Parallelograms could actually arrive at a last form distinct from the curve, which is necessarily implied, when it is asserted, that they can attain such a last form, which shall be equal to the curve; it is certain, that such a last form must essentially differ from the curve, for the reason Mr. *Robins* has given. Therefore *Philalethes*, by granting at length that there is no such last form distinct from the curve, gives up the point. What then does *Philalethes* mean by charging Mr. *Robins* with changing his mind, when he has reduced *Philalethes* to a concession inconsistent with the opinion he first held, and still appears desirous to support? See the *Remark* on Sect. XXII. § 10.

Sect. XIX. The treatise, whose brevity Mr. *Robins* here makes mention of, is Sir *Isaac Newton*'s *Principia*, in which, not the doctrine of fluxions, but the doctrine of prime and ultimate ratio's is delivered. What then is the design of *Philalethes* in misrepresenting Mr. *Robins* by thus confounding things together, which Mr. *Robins* contends to be different? I question not, but *Philalethes* knows many persons, who believe, the understanding of this book is attended with difficulty, and I would advise *Philalethes* to acknowledge it to be so; since I think it evident, that *Philalethes* must employ still more careful attention, before he will understand the doctrine there delivered.

Sect. XX. Mr. *Robins* has not here said, that the method taken by Sir *Isaac Newton* is blameworthy; but if Sir *Isaac Newton* was not to blame in using these expressions, what are we to think of such, as have suffered themselves to be misled by them? However does *Philalethes* look on it as a ridiculous affectation to avoid impropriety? Or does he imagine, that Sir *Isaac Newton* was so bad a writer as to be incapable of avoiding what *Philalethes* (I suppose without ridiculous affectation) calls *unintelligibleism*, if he had confined himself to the exact propriety of language?

Sect. XXI. §. 1–8. Mr. *Robins* is falsely accused in the fourth paragraph of asserting, that the words *diminuuntur in infinitum* have either no meaning, or no clear meaning; for Mr. *Robins* has set down expresly what their meaning is in his translation of the passage, where they are used at page 316. line 11; they are there rendered, *diminished to nothing*; which interpretation *Philalethes* has fully justified by the passage he has quoted from Sir *Isaac Newton* in this very paragraph, where *in infinitum diminui* is explained by Sir *Isaac Newton* himself by the words *esse nihil*.

Suppose it be agreed, that Mr. *Robins*'s words, *the* ultima summa *there mentioned in strict propriety of speech has no meaning, for it is really infinite*, are defective, and that it would have been better to have said for the number of those parallelograms is really infinite; will that any more reconcile the expression to strict propriety of speech? For can any sum of a set of quantities, whose number is supposed infinite, in strict propriety of speech be called their last sum. However it seems Mr. *Robins* is here catched at a notable piece of prudence in omitting the words *parallelogrammorum evanescentium*. These words were not indeed set down by Mr. *Robins*, but evidently referred to by the words [*there mentioned*]. But what would *Philalethes* conclude from these words? What in strict propriety of speech is the sum of any number of quantities, when every one of them vanishes into nothing? For in *Philalethes*'s way supposing all these parallelograms to vanish at the end of an hour, can it be asserted, that any number of things, when they become nothing, can still fill up a space, or that an infinite number of nothings can indeed compose a finite quantity.

§ 9. Here Mr. *Robins* is misrepresented. He does not say, that the words *really infinite* in strict propriety of speech have no kind of meaning, but gives this character of the expression *the last sum of quantities, which are supposed infinite in number*.

§. 10, 11. Does not this *summa ultima* really imply an infinite number of terms? But what will all this trifling criticism about the propriety or impropriety of a few of Mr. *Robins*'s expressions contribute towards the point in controversy?

Sect. XXII. §. 1. By the parenthesis *Philalethes* appears not to know, that the word *curve* is promiscuously applied to denote either the line, or the space included by it. But is not Sir *Isaac Newton*'s treatise concerning the mensuration of such spaces entitled *Tractatus de Quadratura Curvarum*?

§. 2–4. Mr. *Robins* puts the impropriety of the expression upon the words *ultima summa* being applied to quantities infinite in number.

§. 5, 6. Has not *Philalethes* here shewn, that he does not understand the very *Axioms* of *Euclide*? Is not this axiom made use of in the fourth proposition of the first book to prove two triangles distinct from each other to be equal? Or will *Philalethes* say, that all the triangles, which throughout the *Elements* are concluded by vertue of this proposition to be equal, are not different triangles, but the same one with another? The meaning of this axiom is, that those quantities are equal, which can be made to coincide. Now if we allow coincidence to imply the quantities becoming the same, then the axiom will indeed mean, that all such quantities are equal, which are capable by such coincidence of being converted into the same; but not that, when they are become the same, they can still properly be called equal.

§. 7, 8. Are all spaces, that can be conceived to be laid one upon another, the same identical spaces? Can *Philalethes* be supposed really not to know Mr. *Robins*'s meaning in these words, *that those things are equal which have no difference*?

§. 9. I believe every indifferent reader from the stile of this discourse of *Philalethes* will find no difficulty in determining, whose judgement is most likely to be perverted by heat of controversy.

§. 10. Does not here *Philalethes* contradict himself? It is now asserted, *since* any collection of these inscribed or circumscribed parallelograms *is distinct from the curvilinear figure, equality may* properly subsist *between them, one may* in strict propriety of speech *affirm, that they may become equal to one another*: and before Sect. XVIII. §. 2. it was asserted, that *the idea of the figure, at which we conceive the inscribed or circumscribed figures at last actually to arrive, is no other than that of the curvilinear figure itself.*

§. 11. *Philalethes* had better prove himself untainted with indivisibles, than content himself with barely affirming it.

Sect. XXIII. How much does this answer come short of a contradiction to the paragraph upon which Mr. *Robins* ask'd the question here mentioned? Is not the actual equality, *Philalethes* contends for, assigned as necessary to the accuracy and geometric rigour of Sir *Isaac Newton*'s demonstrations? But now he acknowledges Mr. *Robins*'s demonstrations to be just, though he has not supposed this actual equality. Therefore if Sir *Isaac Newton* had not supposed this actual equality (which Mr. *Robins* believes he did not) yet still he would have been very far from contenting himself with approximations only.

Sect. XXIV. 1, 2. Here is no unfairness; for becoming actually equal, and becoming equal in a finite time are synonymous expressions. Is it unfairness in Mr. *Robins* not to animadvert upon all occasions on *Philalethes*'s incorrectness? Both these reasons are in effect no more than one; for one is proposed merely as the consequence of the other; so that what is here referred to as the second reason is only an intermediate step between the first and the conclusion.

§. 3–5. The only reason, why a telescope composed of an hundred glasses will not be equally diaphanous with one composed of two, is because no one of the glasses is perfectly diaphanous, and therefore the increase of the number increases the Defect. Now if every proposition in any demonstration by exhaustions be perfectly perspicuous, must not the whole be so? But this answer is besides altogether evasive; for in the passage of *Philalethes* referred to by Mr. *Robins* the perspicuity of Sir *Isaac Newton*'s demonstrations is thus distinguished from their brevity; *This method seems greatly to exceed the method of the Ancients in perspicuity as well as in the conciseness of the demonstrations, tho' this last alone*, &c.

To what purpose is this comparison between the direct form of demonstration, and that by *deductio ad absurdum.* The assertion here made concerning them is not universally true, but depends upon the nature of the propositions, to which they are applied. However the remark is of no use in this place; for is not Sir *Isaac Newton*'s fundamental proposition owned to be in this negative form? And does not *Philalethes* know, that in the method of exhaustions this form is necessary only in the last and conclusive proposition in each subject.

Sect. XXV. §. 1, 2. If *Philalethes* has indeed made this insinuation concerning Sir *Isaac Newton*'s demonstrations, of what consequence is it, where it was made?

§. 3–8. Where will *Philalethes* find demonstrations of the ancients, which do not shew, by what steps the truth undertaken to be proved is brought about? The instances here produced are not to his purpose; for to prove, that three equal cones are together equal in magnitude to a certain cylinder, is a very different thing from undertaking to shew, how to put these three cones into one mass, so as to form a cylinder. *Philalethes* does not distinguish between shewing, that a number of some magnitudes may altogether be equal in quantity to another, and directing how to unite those magnitudes together, so as to form not only an equal bulk, but the same figure with the other magnitude.

§. 9. Can a man reside many years together in a country, where arbitrary power prevails, without being rendered perpetually obnoxious to that power, all the time he stays there?

Sect. XXVI. §. 1–7. Whatever is brought to illustrate any subject, must be an exact parallel to all the particulars of that subject, which are to be illustrated by it. Now Sir *Isaac Newton* could not produce this instance merely to shew, that the *quantitates ultimæ* cannot be assigned, for that he had already observed to be proved by *Euclide*; there remains therefore nothing for this case to illustrate, but how a ratio may be ascribed to quantities as their last, though there are no last quantities.

§. 8. Mr. *Robins* does not charge the author of the *Analyst* with this opinion; but the error, he animadverts upon, is that author's attributing this opinion to Sir *Isaac Newton*; and he apprehends *Philalethes* to be guilty of the same mistake.

That *Philalethes* was of opinion, that vanishing quantities were the subjects of this last proportion is manifest from the following paragraph of the *Minute Mathematician* pag. 31. *as the increments do not come to this proportion before they vanish, so neither do they vanish before they come to this proportion: but at one and the same instant of time they come to this proportion and vanish, they vanish and come to this proportion.* Is not this asserting, that at the instant of their vanishing, they are the subjects of this proportion? Again at pag. 383 of the *Republick of Letters* for *November* last, has he not asserted, that the subject in dispute is in relation to this point, whether vanishing quantities *bear to one another an infinite number of different successive ratios during their vanishing, or one ratio only, at the point, or instant of their evanescence*? Is not this declaring, that in his opinion these quantities at the instant of their evanescence do bear to each other a ratio?

Sect. XXVII. §. 1–14. Since *Philalethes* has a better opinion of the abilities for controversy of the author of the *Analyst* than of his own, why does he officiously undertake this defense of that author, in which he is not concerned.

The objection here talked of is grounded upon that author's interpreting *evanescant jam augmenta illa* by these words, *let the increments be nothing*; if *Philalethes* does not approve of that interpretation, he is no way concerned in this part of Mr. *Robins*'s *Discourse.* And though *Philalethes* does not apprehend the author of the *Analyst to be so weak, as not to know, that a gradual diminution requires a finite time*; yet surely *Philalethes* is not himself so weak as not to know, that quantities may be supposed to vanish at once, as well as gradually: and it is very manifest, that the aforementioned interpretation ascribes to Sir *Isaac Newton* the supposition of an instantaneous evanescence.

§. 15–17. Has not Mr. *Robins* more right to assert, that in such of these passages quoted from Sir *Isaac Newton*, where the words prime or ultimate might without impropriety have been added, their omission is an ellipsis, than *Philalethes* can have to assert, that so very concise a writer as Sir *Isaac Newton* should so frequently add those words for emphasis only; not to mention now the reasons Mr. *Robins* has given, that these words must be expressed or implied?

§. 18–26. After *Philalethes* has spent no less than five pages to controvert Mr. *Robins*'s interpretation of the word *evanescant*, he expresly acknowledges it to be true in these words at the 26th paragraph; *When Sir* Isaac Newton *uses the term* evanescant *simply, he intends his reader should thereby understand, that the quantity was diminished before it came to vanish.* And the error of the Author of the *Analyst* consists in not knowing, that the consequence inferred by Sir *Isaac Newton* is from the consideration of this gradual diminution of the quantities. Whereas he supposes, that as soon as Sir *Isaac Newton* has drawn an inference from the supposition of quantities being increased by an augment of some magnitude or other; he then immediately makes a contrary supposition, that those quantities are not increased, and yet retains consequences drawn from his former supposition.

§. 27, 28. In relation to the second error charged upon the author of the *Analyst* Mr. *Robins* will be at no loss to prove, that that author has accused Sir *Isaac Newton* of meaning by the ultimate ratio of vanishing quantities a ratio, that those quantities must sometime or other exist under. Does he not charge Sir *Isaac Newton* with talking of a ratio between nothings? and could that charge have been made upon any other ground, than imagining, that Sir *Isaac Newton* supposed these quantities to be the subjects of a ratio, after they are vanished and become nothing? Mr. *Robins* has not charged the Author of the *Analyst* with holding this opinion, but with ascribing it to Sir *Isaac Newton*.

§. 29. Is it so very difficult to conceive, how any person may talk inconsistently? Let *Philalethes* reconcile the actual arrival to these quantities to the ratio supposed, and at the same instant vanishing away. Is not this saying, that the two quantities become nothing, and bear proportion at the same instant of time?

Sect. XXVIII, XXIX. Upon this definition *Philalethes* should have explained himself more at large. He has thought himself unjustly accused by Mr. *Robins* of supposing a nascent increment to be some intermediate state of that increment between its finite magnitude, and its being absolutely nothing. To have proved this assertion groundless he ought to have shewn, that this definition does not attempt at describing such an intermediate state.

Philalethes has charged upon Mr. *Robins*, as a great absurdity, that he should refer to the demonstration of Sir *Isaac Newton*'s first *Lemma* for understanding the true force of some expressions in the proposition; and here *Philalethes* has recourse to the second book of the *Principia* to find the meaning of expressions used throughout the whole first book.

Sect. XXX. Here *Philalethes* pretends to justify himself by denying, that he supposed quantities to subsist, to which he notwithstanding ascribes the property of bearing proportion to one another; but can that, which is not, have any property?

Sect. XXXI. §. 1–3. What reflexion is it upon Sir *Isaac Newton* to suppose, that he made use of the methods, he had learnt from others, before he invented better of his own: Or that in an analysis of a problem for dispatch he still continued to make use of such methods, when he conceived it would create no error in the conclusion? Has not Sir *Isaac Newton* said this of himself, and has Mr. *Robins* said any thing more?

§. 4. What Mr. *Robins* has here said of Sir *Isaac Newton*, he still believes to be absolutely true; for he has never so much as insinuated, that either of the methods here spoke of has the least connexion with indivisibles; the chief objection he has to the writings of *Philalethes* is from their being there confounded together.

§. 5. Mr. *Robins*'s meaning in the expression here quoted is more fully explained in the account of his book at *pag.* 263. of the *Republick of Letters* for *October* last, where at the same time it is shewn at large, that no objection can be raised against Sir *Isaac Newton*'s doctrine of prime and ultimate ratio's delivered in the beginning of his *Principia* from any interpretation, that can be affixed to his definition of *momenta*. Is this allowing the doctrine of prime and ultimate ratio's to be in any way tainted with indivisibles?

§. 6, 7. If *Philalethes* be himself mistaken, he is not qualified to determine, who are the best judges in this subject: without doubt he takes those to be such, as are of his own opinion. The design of Mr. *Robins*'s writing being to defend Sir *Isaac Newton* against the author of the *Analyst*, who had accused his doctrine of prime and ultimate ratio's with involving the absurdities of indivisibles notwithstanding all his endeavours to avoid them, that *the doctrine of infinitesimals intruded upon him whether he would or no*; and it appearing by the manner of *Philalethes*'s defence, that instead of vindicating him from that charge, he had in effect admitted it; it was thought convenient, in the account afterwards published of Mr. *Robins*'s book, to make the distinction necessary for justifying Sir *Isaac Newton*, that whenever he had made use of the doctrine of indivisibles, that great man was apprized of his doing so, and was very far from confounding this doctrine with his own, as the author of the *Analyst* had charged him.

What does *Philalethes* mean by this reproach, *that Mr.* Robins *had departed from that candid and ingenuous behaviour, he had hitherto used with respect to our common master*? Mr. *Robins* allows no one the authority of a master over him, though he has received instructions from the writings of many, and from none more than from those of Sir *Isaac Newton*. He presumes he has a full right to declare his thoughts without reserve upon any author, he has read and studied, whenever he shall have a mind so to do. To be awed into silence from the consideration, he may have, of any one's merit, how distinguished soever, or to yield a blind submission to any one's dictates, and give up his opinion to authority, is a low degree of bigottry, he trusts, he shall never submit to. The great value, he sets upon Sir *Isaac Newton*'s works, has engaged him in the present defence, and he has never yet spoken inconsistently with himself upon any particulars. If he has upon some points been at last more explicite, than he thought at first necessary, yet he has never insinuated any objections against what he at any time had fully approved of.

§. 8, 9. Here I shall only ask *Philalethes*, what induced him to assert so direct a falshood, that the term infinitely little does frequently occur in the first section of the *Principia*?

For in no one edition of that book is it to be found in that section more than once; and that in relation to a subject foreign to the present controversy, and it is upon the same occasion, that the terms infinitely greater and infinitely less are also used. In the Treatise of Quadratures (which Mr. *Robins* does truly admire) this term is to be met with at the end of the introduction: but with what design Mr. *Robins* has explained in the discourse, *Philalethes* is animadverting on, *viz.* in the *Republick of Letters* for *April*, p. 330.

§. 10. Here *Philalethes* is too hasty; for such expressions are to be found in Dr. *Wallis*. Sir *Isaac Newton*'s doctrine of prime and ultimate ratio's does not depend merely upon these words. In Dr. *Wallis* these expressions mean the same as in Sir *Isaac Newton*'s *Analysis*, that is, no more than that the difference between two quantities vanishes and comes to nothing. It would be tedious to refer to all the places of Dr. *Wallis*, where such expressions as these occur. It is sufficient to name the two following. In the *Arithmetic of Infinites propp.* 20. 40. are these words, *excessus ille, si in infinitum procedatur, prorsus evaniturus est.* Nay *Huygens* in his illustration of *Fermat*'s method of drawing tangents, though he proceeds undoubtedly upon the principles of indivisibles, yet has made use of the word *evanescens* in the following passage, *Nam termini* &c. *quantitates infinite parvas, sive omnino evanescentes continebunt.* It is certainly very easy to see what Sir *Isaac Newton* meant by the words *infinite parvis* in the *Analysis*; because he tells us expresly, that he used them in imitation of those, who used indivisibles. *Nec vereor loqui de unitate in punctis, sive lineis infinite parvis, siquidem proportiones ibi jam contemplantur Geometræ, dum utuntur methodis indivisibilium.*

§. 11–13. Mr. *Robins*, in order to free *Philalethes* from his mistakes, thought it sufficient in the month of *December* to touch only upon some of the principal points in dispute between *Philalethes* and himself; but an insinuation being thence made in the month of *January*, as if Mr. *Robins* might himself be convinced, wherever he was silent, he found it necessary afterwards to explain himself more at large.

§. 14. All that is asserted of these places is simply matter of fact. Does *Philalethes* expect, that the world should think, he understands even the method of indivisibles, if he will deny the expression *particulæ finitæ non sunt momenta* to be a phrase suitable to the sense of that doctrine?

§. 15, 16. That Sir *Isaac Newton* may reasonably be supposed sometimes to have used indivisibles, is evident from the passage just cited out of his *Analysis*, where he expresly says, he imitated the writers, who followed that doctrine. And whoever has read Sir *Isaac Newton*'s *Lectiones Opticæ*, and will deny, that he has at any time made use of indivisibles, must be very much a stranger to that doctrine, and to the style of those writers, who followed it; but I shall set down two passages, where Sir *Isaac Newton* owns, that he used the phrases of indivisibles in the sense then generally understood. In page 98. *Concipias itaque arcum* b c *in æquales & indefinite multas partes dividi, & ejusmodi tot sumi, quæ minus quam una parte* (*hoc est, indefinite parum*) *differunt ab arcu* c d*, atque adeo ipsi pro more consueto censeantur æquales*, &c. Again in page 127, *Age* N Z *occurentum* C I *in* g*, &, ut mos est, concipe infinite parvum arcum* B N *aequalem esse*, &c. Here the words *pro more consueto*, and *ut mos est* plainly shew, that by the phrases *indefinite multas, indefinite parum*, and *infinite parvum* he meant the same as other writers had done.

§. 17. In the preceding paragraph it is affirmed, that Sir *Isaac Newton* was so far from making use of indivisibles, that in all his works there is no passage, no expression to be found, that should reasonably make him so much as suspected of it; yet here it is maintained, that Sir

Isaac Newton retains in his method the supposition of quantities less than any finite quantity whatever. This certainly is fixing upon him the doctrine of indivisibles, and the absurdity of it, which consists in the pretending to form a conception of infinitely small quantities, of which we cannot possibly have any idea.

§. 18–21. If there be any difficulty in the expression of this *Lemma*, can any thing be more reasonable, than to make Sir *Isaac Newton* his own interpreter, by endeavouring to find out his meaning from other places, where he has spoken of the same subject.

§. 22, 23. *Cavalerius* supposing surfaces to be formed of lines, and solids of surfaces; because a real line is indivisible in breadth, and a surface in thickness; he therefore called his method that of indivisibles; but he was never understood to mean that these indivisibles were of any fixed, determinate magnitude. He never supposed any thing farther, than that by the observation of what he calls *idem transitus*, his lines and planes should be understood to preserve the same distances in the figures compared together.

§. 24. Does *Philalethes* here mean, that a quantity can become less than any finite quantity whatever, before it vanishes into nothing? If not, then the point is given up to Mr. *Robins*, who only contends, that vanishing quantities can never by their diminution be brought at last into any state or condition, wherein to bear the proportion called their ultimate: if otherwise, since *Philalethes* supposes in § 23, that it is nonsense, that it implies a contradiction to imagine a quantity actually existing fixed, determinate, invariable, indivisible, less than any finite quantity whatsoever; because this imports as much as the conception of a quantity less than any quantity, that can be conceived: how can a quantity supposed to be less than any finite quantity whatsoever be rendered more the object of the conception by being understood to be brought into this condition by a constant diminution from a variable divisible quantity? How will the variation, or divisibility, though we continue to ascribe to it these qualities, even after it is become less than any finite quantity whatsoever, aid me in conceiving a quantity less than any quantity, I can conceive?

§. 25, 26. The consideration of quantities infinitely great is no part of the doctrine of indivisibles, though infinite numbers are spoken of in that doctrine. But such infinite numbers were so far from being supposed fixed, determinate, invariable, and immutable, that *Monsieur Pascal* in his *Letters* published under the name of *Dettonville* industriously chuses to call them indefinite.

§. 27–30. This repetition, *Philalethes* makes of his opinion, contributes nothing to the proof of it. If what *Philalethes* here means by *momenta evanescentia* are not finite quantities, it is incumbent on him to shew, that those words can have any meaning at all, since all quantities are finite.

§. 31–33. It is affirmed in the *Account* of the *Commercium Epistolicum*, that Sir *Isaac Newton* did not always use the letter *o* in the same sense.

§. 34–41. Before *Philalethes* so positively charges Sir *Isaac Newton* with introducing the consideration of quantities not finite into his method of prime and ultimate ratio's, he ought to shew, that we are capable of forming an idea of any other than finite quantities.

Sect. XXXII. §. 1–4. Can any words be contrived to express Mr. *Robins*'s sense more distinctly, than those of the passage here quoted in the second paragraph? Will *Philalethes* own himself so absolutely unacquainted with the doctrine of indivisibles, as not to know, that

Cavalerius meant by his lines the very same, as the latter writers in that doctrine understood by surfaces infinitely narrow?

§. 5, 6. When *Philalethes* asserts, that the sense of the passage is not, as Mr. *Robins* has represented it, he certainly transforms it, and reads it, as if it were, that a point in the sense of *Cavalerius* Sir *Isaac Newton* calls a moment.

§. 7, 8. *Philalethes*, while he pretends to contradict Mr. *Robins*, has only quoted the sentence immediately preceeding, and referred to in that quoted by Mr. *Robins*.

Sect. XXXIII. If Sir *Isaac Newton* had upon his first inventing fluxions laid aside absolutely the use of all infinitely small quantities, the words *as much as possible* would have been superfluous. In fact, by his method of fluxions he could omit the consideration of infinitely small quantities in particular problems concerning curve lines, but he still for a time made some little use of them in demonstrating the operations, by which fluxions are found; though afterwards he was inabled by his method of prime and ultimate ratios to demonstrate those operations in a juster manner.

But farther, though Sir *Isaac Newton* had invented the method of fluxions, before he wrote the *Analysis* in 1666, yet since he did not think fit to explain this doctrine in that treatise, he could not make use of it there for avoiding indivisibles.

Sect. XXXIV. §. 1, 2. Is *Philalethes* all this while so ignorant of Mr. *Robins*'s writings as to imagine, that he ever insinuated, that the doctrine of prime and ultimate ratios was not intirely free from the taint of indivisibles? But Mr. *Robins* is of opinion, that *Philalethes*, by his account of moments, does evidently confound them together.

§. 3–7. If Sir *Isaac Newton* had the whole of his method of prime and ultimate ratios from the beginning, how came he to speak of his first invention of fluxions under the limitation, that he introduced them, to avoid infinitely little quantities only as much as possible, whereas by his method of prime and ultimate ratios they are entirely avoided?

Sect. XXXV. Mr. *Robins* has here made no mistake, unless when Sir *Isaac Newton* calls a finite quantity a moment, he even then means an infinitely small quantity.

Philalethes has no manner of ground to understand by decreasing *in infinitum* becoming infinitely little; for that expression is used by Sir *Christopher Wren*, when he is forming demonstrations upon the model of exhaustions, and is expresly defined by him thus. *Magnitudines in infinitum decrescentes sunt, quarum non datur minima.* See *Wallis Op.* Tom. 1. pag. 534. Let not *Philalethes*, after this express definition given by so distinguished a writer, pretend, that decreasing or diminishing *in infinitum* necessarily implies, what in reallity is impossible to be conceived, that the quantities are to become infinitely small.

Sect. XXXVI. §. 1–4. Sir *Isaac Newton* has introduced into use the term moment throughout the whole second book of the *Principia*, and for no other purpose than for the sake of brevity; for his doctrine of prime and ultimate ratios had been before fully explained, and every proposition of the second book might have been treated on without the use of this term, though perhaps with a somewhat greater compass of words. His doctrine of fluxions is rather mentioned in this place than introduced; for there is no use made of it afterwards in this book.

§. 5–7. What could induce *Philalethes* to ask so absurd a question; *How is it possible for him to know, that the magnitude of moments is either finite or infinitely little*? He seems not all this while to have discovered, that the talking of infinitely little quantities is speaking absolutely without ideas. And before he thinks of getting rid of an idea of such quantities, let him first endeavour at getting one.

§. 8–11. The word *other* does not vary the sense of Sir *Isaac Newton*; for so much of the sentence, as shews the construction of it, runs thus, *it would come to the same thing, if instead of these moments you used fluxions, or any other finite quantities proportional to these fluxions.* The word *other* only implies fluxions to be finite quantities. If I should say, that this proposition, the three angles of every rectilinear triangle are equal to two right, is demonstrated by *Euclide*, and *other* writers of the elements of geometry, should I by this form of speech call the proposition a writer of elements. Surely *Philalethes* reads the sentence backwards, as if it had been *instead of fluxions you may use moments or any other finite quantities proportional to those fluxions.* If *Philalethes* does not understand the use of this passage, which Sir *Isaac Newton* himself thought fit to subjoin to his description of moments, he must ascribe it to his own unskilfulness.

§. 12. Does not *Philalethes* know, that it is customary for writers to insert in *errata* their own oversights, as well as those of the printer? and the only reason, why this was thought worthy of notice, was, that as Mr. *Robins* intended the passage for a translation of Sir *Isaac Newton*'s words, it was hoped by this exactness cavils might have been prevented, though it now appears one may be mistaken.

§. 13–15. The paragraph here mentioned I shall not think fit to be omitted, till *Philalethes* can shew, that if Sir *Isaac Newton* did not mean, what is here said, he has any real meaning at all.

Sect. XXXVI. Cannot the increment and decrement of the same quantity be generated together by the motions proper to produce each. However the passage of Mr. *Robins* is incorrectly expressed on another account; for the moment is an arithmetical mean between the increment and decrement generated in the same or equal portions of time, only when the sides of the rectangle are varied uniformly. But however, will *Philalethes* still persist, that defending this demonstration under this particular circumstance only, is defending it against one, who objects to it, as it stands in Sir *Isaac Newton*, where it is by no means confined to this particular circumstance?

Sect. XXXVIII. §. 1–3. Has not *Philalethes* in the *Republick of Letters* for *November* last pag. 386 asserted, (how justly I shall not now enquire) that Sir *Isaac Newton* has plainly declared by the words subjoined to the *Scholium* of this *Lemma* in the first and second editions of the *Principia*, that the letters may be understood to represent the differences of *Leibnitz*.

§. 4–6. If *Philalethes* is so disgusted at the repetition of the word *these*, he is at liberty to leave the second out.

Sect. XXXIX. *Philalethes* does not understand the passage here quoted from the *Introduction* to the *Quadratures*; for there *Leibnitz*'s name is neither mentioned, nor is any reference had to him, or his followers. The passage is this, *Fluxions are very nearly in the* *proportion of such augments of their fluents, as are generated in very small equal portions of time, and, to speak accurately, are in the first proportion of the* augmenta nascentia. The first part of this sentence is evidently intended only to make the other more easily understood; for in the method of indivisibles, followed by *Leibnitz*, the augments of quantities are not supposed only very small, but infinitely small, and thereby expressive of the real proportion.

Sect. XL. §. 1–3. Mr. *Robins* does not imagine Sir *Isaac Newton* to be so bad a writer, that his first *Case* is not to be understood, till we have read the second, but believes whoever reads this *Case* with care, must be a very unskilful reader to understand it any otherwise, than he contends for. Besides he does apprehend it most natural, at least most usual for unprejudiced readers not to think an Author guilty of absurdity, without good grounds for such a supposition. That Sir *Isaac Newton* should quote the first *Case* in the demonstration of the second, when he had demonstrated only some particulars of that *Case* of no use for the second, is a proceeding, which Mr. *Robins* will never charge upon Sir *Isaac Newton*.

§. 4–13. *Philalethes* will find it much easier to boast of the distinctness of his explanation, than shew the truth of it. How Mr. *Robins* understands Sir *Isaac Newton*'s demonstration, *Philalethes* may know not only from the brief description, he has given, of the principles of that demonstration in his book, but more fully from the paraphrase of it in the account of that book at page 267 of the *Republick of Letters* for *October* last. If there is any error in that paraphrase, *Philalethes* would have done better to have shewn it distinctly, than thus to collect together a few passages in an irregular and distorted manner, as he has here done in the 11th paragraph, in order to make nonsense of them.

Sect. XLII. Mr. *Robins* has endeavoured to defend Sir *Isaac Newton* both against the accusation of the author of the *Analyst*, and the misrepresentation of *Philalethes*. He has shewn, that Sir *Isaac Newton*'s doctrine of prime and ultimate ratio's has no connexion with indivisibles, and that, if he ever allowed himself in the use of indivisibles, he knew that he did so, and did not confound both the methods together, as the author of the *Analyst* accuses him, and *Philalethes* without knowing it has owned.

Sect. XLIII. What does *Philalethes* mean by an errant *indivisibileist*? Mr. *Robins* has declared his opinion, that though Sir *Isaac Newton* did sometimes allow himself the use of indivisibles, he always had a dislike to that method. See *Republick of Letters* for *October* last page 265. Does not *Philalethes* think, that if he had distinctly quoted all the places, from whence he has collected this string of sentences, it would have totally frustrated the design, for which he has in this irregular manner huddled them together.

Sect. XLIV. §. 1–4. This requires no answer; because *Philalethes* has not told us, with what intent he has made this collection out of Mr. *Robins*'s writings.

§. 5, 6. Mr. *Robins* has not said, that Sir *Isaac Newton* never read the ancients; he has only suggested, that he might not have read them, when he first invented the doctrine of fluxions. Mr. *Robins* never imagined, Sir *Isaac Newton* had not read the ancients before he publickly gave his opinion concerning them. But Mr. *Robins* thinks, it would have been no cavil to have supposed, that when Sir *Isaac Newton* altered the character, he had given of any of their writings, he had also changed his mind. That *Philalethes* should think any of their works perplexed, is not surprising: the *Elements* of *Euclide* appear so to those, who understand them not.

Sect. XLV. If Sir *Isaac Newton* did really stand in need of any apology, would it not be better to own it, than to ascribe errors to him, and then undertake their defense.

This is all we think necessary to remark upon these *Considerations* of *Philalethes*, and we trust our unprejudiced readers will not find many of his exceptions to Mr. *Robins* unanswered; and we farther hope, that we have not allowed ourselves any warmth or freedom of expression, which the present intemperate manner of *Philalethes*'s writing will not sufficiently excuse. We shall not here draw the usual inference from this change of behaviour in *Philalethes*, that he is become doubtful of his own cause; for Mr. *Robins* rather desires his readers to compare impartially without any bias the writings of *Philalethes* with his own, and to give the preference, where they find the greatest weight of unmixed and undisguised argument. But to remove a difficulty, which naturally lies in the way, I shall conclude this paper with examining, how it has come to pass, that *Philalethes* and Mr. *Robins* should both carefully have studied Sir *Isaac Newton* with a sincere intention of understanding him, and yet differ so much from one another.

I think it evidently appears from the paper of *Philalethes*, we have been considering, that his reading in the mathematicks has been very much confined. Had he been acquainted with the ancient writers, he could have been at no loss to understand, what Mr. *Robins* meant by saying, that he described the parallelograms of the second *Lemma* after the manner of the ancients by subdividing the base of the curve. He could never have thought, that Mr. *Robins* had there reference to two propositions of *Euclide*, in one of which *Euclide* had no view at all to demonstrations by exhaustions, and the other, though used by *Euclide* in one or two such demonstrations, is very unfit to be applied to the subject, Mr. *Robins* mentions.

Had *Philalethes* been versed in the ancients, and in the latter writers who have imitated them, he could have been at no loss about the true sense of *data quavis differentia* used by Sir *Isaac Newton* in his first *Lemma*. For this expression is borrowed from the writers, that made use of exhaustions.

The first proposition of the tenth book of the *Elements*, which is applied by *Euclide* both in his comparing of circles, of pyramids of equal altitudes, and in one or two propositions more, is thus expressed, Two unequal magnitudes being proposed, if from the greater be taken more than half, and from the residue more than half, and so on, there will be left at length a magnitude less than the lesser of the proposed magnitudes. This is directly, as Mr. *Robins* has represented it, first assigning a difference, according to which the degree of approach is afterwards to be regulated.

Archimedes in his treatise on the *Sphere and Cylinder* proposes to shew, when a circle and another space are given, that it is possible to circumscribe a polygon, so that the excess of the polygon above the circle shall be less than the space given. In his book of *Conoides and Spheroides* it is shewn in Prop. XXI, that any segment of a conoide being given cut off by a plane perpendicular to the axis, or any segment of a spheroide not greater than half the spheroide cut off in like manner, it is possible to inscribe a solid figure, and to circumscribe another consisting of cylinders, so that the circumscribed shall exceed the inscribed by less than any solid magnitude, that shall be given. And this he performs by a continual bisection of the axis, till a cylinder is found less than the space, that should be given, by which cylinder the inscribed and circumscribed figures differ from each other. From this proposition *Philalethes* may know, what Mr. *Robins* means, when he speaks of describing the parallelograms in Sir *Isaac Newton*'s second *Lemma* by continually subdividing the base of the curve.

After the same manner the following excellent writers express themselves.

Fed. Commandinus de Centro Gravitatis solidorum, Prop. xi.—*ita ut circumscripta superet inscriptam magnitudine, quæ solida magnitudine sit minor.* Ibid. Prop. xxviii.—*ita ut recta linea quæ inter centrum gravitatis portionis & figuræ inscriptæ, vel circumscriptæ interjicitur, sit minor qualibet recta linea proposita.*

Lucas Valerius de Centro Gr. solid. Lib. 1. Prop. vi.—*ita ut circumscripta superet inscriptam minori spatio quantacunque magnitudine proposita.*

Joannes della Faille de Centro Gr. Partium Circuli & Ellipsis, Prop. viii.—*superet latus sectoris intervallo, quod minus sit quolibet dato.* Id. Prop. xxvii.—*minus distet a centro gravitatis sectoris, quolibet intervallo dato.* Id. Prop. xlii.—*sit quacunque linea recta data minor.*

Huygens de Quadratura Hyperbolæ, Ellipsis & Circuli, ex dato portionum gravit. centro, Theor. 1.—*quæ portionem excedat spatio, quod minus sit quovis dato.* Id. Theor. 11.—*sit minor spatio quovis dato.*

James Gregory in Vera Circul. & Hyperbol. quadratur. in schol. prop. 6. *Continuando subduplam polygonorum descriptionem inveniri possunt duo polygona complicata, quorum differentia sit minor qualibet exhibita quantitate.*

Simson Sect. Conic. Lib. v. Prop. xlviii. *Portioni Parabolæ, vel portionis dimidio circumscribi potest figura, aliaque in ipsa inscribi ex parallelogrammis æqualem latitudinem habentibus, quarum quæ circumscribitur portionem excedat, quæ vero inscribitur ab eadem deficiat spatio, quod minus sit spatio quovis dato.*

And in all these propositions they always supposed a quantity first named, and then shew how to make the approach expressed in the proposition. I have set down this large number of quotations, because the diversity of phrase, whereby these authors express the same sense, renders it impossible to be mistook. Though I confess this long disquisition in so evident a matter is much more than necessary; for had any of these writers been familiar to *Philalethes*, he could not have had the least doubt of Sir *Isaac Newton*'s meaning. He might indeed have discovered it even from Sir *Isaac Newton* himself, who in the eleventh *Lemma* has thrice interpreted the phrase *pro data quavis differentia* by these words, *pro differentia quavis assignata*: so that by *data* he could not mean assignable unless *assignata* and *assignabilis* were synonymous words.

Again, had *Philalethes* been at all acquainted with the writers of indivisibles, he could not have attempted at that vain distinction between the sense, in which those writers use the phrase infinitely small, and the sense, he imagines Sir *Isaac Newton* had affixed to it; that by infinitely small quantities they meant a quantity fixed, determinate, invariable, but Sir *Isaac Newton* meant thereby a variable quantity; whereas it appears, that several writers in indivisibles on set purpose avoided expressions, that implied any thing fixed, determinate, or invariable. We observed above that *Monsieur Pascal*, though he never imagined, that he did not follow the method of indivisibles, on the contrary he particularly at pag. 10. defends himself for so doing; yet is so cautious, as to avoid the word infinite in expressing their number, but constantly calls it indefinite; and certainly, when their number is considered as indefinite, it cannot be pretended, that their magnitude was supposed fixed, determinate, and invariable.

Again Dr. *Barrow*, who has declared himself very expresly in favour of indivisibles, and defends his using of them, almost constantly applies the term indefinite both to the number, and the magnitude of these particles. In his first Lecture on *Archimedes* he says, *Ponatur circulum esse figuram regularem habentem latera indefinite multa & parva* &c. In his second Lecture Prob. 2. *Supponatur cylindrum esse prisma quoddam super polygonam basem, latera habentem indefinite parva & multa*; and Prob. 9. *Posito igitur* V M *esse infinite* (*vel indefinite*) *parvum.* Again in his 5th Geometrical Lecture Art. 6. *arcus* M N *indefinite parvus ponatur.* In his Differential Method of Tangents (as Sir *Isaac Newton* calls it in the Philos. Trans. No. 342. pag. 197.) at the end of his tenth Lecture he says *curvæ arcum* M N *indefinite parvum statuo.* In Lect. 11. Art. 1. *æquisecetur recta* V D *indefinite punctis* A, B, C; and a little after *ob indefinitam sectionem curvula* G H *pro recta haberi potest.* In Lect. 12. at the beginning, *arcum* M N *indefinite parvum esse*; and at Art. 1. *spatium vero* $\alpha\ \beta\ \delta$ *minime differt ab indefinite multis rectangulis, qualia* $\mu\ \theta$; &c. Again in his 9th Optical Lecture Art. 12. *arcus* N R*,* P S *ex hypothesi sunt indefinite parvi* (*seu minimi*) Lect. 13. Art. 24. *ob sumptam arcuum indefinitam parvitatem.*

Thus it appears, that Dr. *Barrow* was so far from considering these infinitely small quantities as fixed, determinate, invariable, that he has purposely chosen the most loose and indeterminate expressions he could contrive to denominate them by. If *Philalethes* had been in the least acquainted with the writings of this most excellent geometer, surely it would have been impossible for him to have asserted, that in all Sir *Isaac Newton*'s works there is no passage, no expression to be found, that should reasonably make him so much as suspected of using indivisibles. Has he not throughout his *Lectiones Opticæ* upon all occasions used either the same, or even less guarded expressions than these of Dr. *Barrow*?

Sir *Isaac Newton*'s doctrine of prime and ultimate ratio's is not to be defended from the accusation of resembling indivisibles by any minute variations, that may be found in his expressions, from what are used by the writers, who followed that other doctrine; for those writers are by no means exactly uniform with one another, but some have expressed themselves with more and some with less caution. What separates the doctrine of prime and ultimate ratio's from indivisibles is the declaration made in the *Scholium* to the first Section of the *Principia*, that Sir *Isaac Newton* understood by the ultimate sums and ratio's of magnitudes no more than the limits of varying magnitudes and ratio's; and he puts the defence of his method upon this, that the determining of any of these limits is the subject of a problem truly geometrical. To insist, that the variable magnitudes and ratio's do actually attain, and exist under these limits, is the very essense of indivisibles. For in supposing this, we pretend to see directly, as Mr. *Robins* has expressed it, in these last forms or limits the properties, which the variable figures had before; and under this notion these limits must be allowed capable of being compared together by a direct form of demonstration. The impossibility only of this actual coincidence obliged Sir *Isaac Newton* to build his demonstrations concerning these limits upon the negative form by *deductio ad absurdum.*

If a choice of expression only were sufficient to distinguish between the two methods, *Heuraet* and Dr. *Wallis* may both be supposed to have avoided indivisibles, though they make no such pretensions. I here allude to this passage of *Heuraet* at the end of the first vol. of *Cartes*'s *Geometry. Unde cum illud* [nempe quod supra demonstraverat] *verum sit, quot-*

cunque rectangula atque tangentes extiterint; & figura ex parallelogrammis constans, si eorum numerus in infinitum augeatur, definat in superficium A G H I K L F*, ac tangentes similiter in lineam curvam* A B C D E*, liquet superficiem* A G H I K L F *æqualem esse rectangulo sub* Σ *& recta æquali curvæ* A B C D E. Again Dr. *Wallis* in his treatise of the Cycloide &c. (*Oper.* Tom. 1. p. 563.) expresses himself thus,—*rectas o* i*, o* c *angulo contactus subtensas prop diminutione* o c*,* o i *tangentium ita minui, ut illæ ad has rationem tandem subeant data quavis minorem; ideoque evanescentibus o* i*, o* c *tangenti & curvæ interjectis, coincident tum* o c*, tum* o i*, tangentis particulæ, particulis curvæ* o *o.*

Again *Philalethes* is but imperfectly instructed in the precepts of common algebra; else he could not have imagined an incurvated line, which he saw would meet a right line in an infinite number of points, to be one and the same curve, and expressible by a finite equation; whereas the number of intersections of a right line with every algebraick curve is limited, the number of such intersections determining the order of the curve.

Farther, it was only owing to his little exercise in geometrical subjects, that made him unapprised of Mr. *Robins*'s meaning, when he spoke in general of vanishing quantities, as if they might be capable sometimes of bearing the ratio, which he calls their ultimate; for had *Philalethes* been accustomed to contemplate the different figures of curves, and how they stand related to their tangents, he must at once have apprehended what Mr. *Robins* had in view. Nay had *Philalethes* been as well versed in the writings of Sir *Isaac Newton*, as might have been expected in one, who has appeared in his defense, he never could have imagined, that because Mr. *Robins* admits, that some quantities capable of an actual equality might be brought under the *Lemma* so often mentioned, therefore he begin to think himself obliged to allow an actual equality, where *Philalethes* contends for it. But above all it is most astonishing, that *Philalethes* should have taken so little care even to understand the person he is writing against as must be supposed, if we are are to think him indeed sincere in his accusation, that Mr. *Robins* has given no less than four different interpretations of this *Lemma.*

With these specimens of *Philalethes*'s imperfect knowledge in the mathematicks, it would have been more becoming him to have been something less free of his censures, and not so hastily to have charged with gross errors, false reasoning, and self-contradictions, a person, who at least seems to have used better endeavours to be well instructed than himself; nor should he be at all surprized, if he has miss'd in any measure Sir *Isaac Newton*'s meaning, who has wrote in a style, which supposes his reader thoroughly conversant in geometrical subjects.

POSTSCRIPT

By Dr. PEMBERTON.

Since the Gentleman, who in this dispute is pleased to assume the name of *Philalethes Cantabrigiensis*, has made a difficulty in relation to the motive, that induced Sir *Isaac Newton* to alter in the last edition of his *Philosophiæ Naturalis Principia Mathematica* the character, he had given of the ancient demonstrations by exhaustions; I can inform the publick, that the true reason was, because he apprehended his former censure too severe; for I received this very reason from his own mouth, while I took care of that edition. As I had then the very best opportunity of knowing his true mind in every part of that treatise, and as *Philalethes* seems to triumph in the approbation of some, whom he is pleased to complement with the name of the best judges; I on the contrary am fully satisfied, that Mr. *Robins* has expressed Sir *Isaac Newton*'s real meaning. And as the respect due from me to Sir *Isaac Newton*'s memory makes me desirous, that his doctrine may be clearly understood, and freed from objections as speedily as possible, I here desire leave of Mr. *Robins* to give *Philalethes* an opportunity of contracting the present debate, and returning from a style not quite befitting a matter of mere science, by entering with him myself into the examination of his interpretation of the first *Lemma* of Sir *Isaac Newton* concerning prime and ultimate ratios, and into the definition, *Philalethes* has given, of nascent, and evanescent quantities.

Or if any of those judges, who approve of *Philalethes*'s opinion, will appear under his own name in the defense of it, it will probably contribute still farther to abbreviate the controversy; for then it is most likely, we shall both pay that regard to the publick, and our own characters, as to avoid what does not directly relate to the point in question.

Philalethes in the *Minute Mathematician* pag. 88, 89. ascribes to this *Lemma* the four following suppositions.

1. *That the quantities or ratios of quantities, tend to equality.*
2. *That this tendency to equality constantly holds during a given time.*
3. *That they come nearer to equality than to have any assignable difference between them.*
4. *That they come thus near to equality before the expiration of the given time.*

The second of these *Philalethes* has explained more distinctly in another place, That *their tendency to equality does not cease during that time, they do not become perfectly equal during that time,* i. e. before *the end of that time. Nor is their tendency to equality supposed to continue beyond that time. Consequently, they cannot become equal* after *that the end of that time. It follows therefore, that they become equal* at *the end of that time, at the instant of the expiration of that time, and at no other.* See *Republick of Letters* for *January* last pag. 82, 83.

Now if two quantities come nearer to equality than to have any assignable difference between them before the expiration of any given time, they will become actually equal before the expiration of that time; for to have no assignable difference, and to be actually equal are synonymous expressions. By this Interpretation therefore Sir *Isaac Newton* is made first to suppose, that the quantities imploy the whole of some given time in their approach, and yet before the end of that time to become quite equal.

The definition of nascent and evanescent increments is contained in the following words.

A nascent increment is an increment just beginning to exist from nothing, or just beginning to be generated, but not yet arrived at any assignable magnitude how small soever. An evanescent increment is the same thing as a nascent increment, but only considered in a different manner, as by a continual diminution becoming less than any assignable quantity, and at last vanishing into nothing, or ceasing to exist. Minute Mathematician pag. 19.

Now Sir *Isaac Newton* in the *Introduction* to his treatise on the *Quadrature of Curves*, after having investigated the fluxion of powers by the doctrine of prime and ultimate ratios, has these words, *Similibus argumentis per methodum rationum primarum et ultimarum colligi possunt fluxiones linearum seu rectarum seu curvarum in casibus quibuscunque, ut et fluxiones superficierum, angulorum, et aliarum quantitatum. In finitis autem quantitatibus Analysin sic instituere, et finitarum nascentium vel evanescentium rationes primas vel ultimas investigare, consonum est Geometriæ Veterum: et volui ostendere quod in Methodo Fluxionum non opus sit figuras infinite parvas in Geometriam introducere.*

Here Sir *Isaac Newton* expresly calls the *quantitates nascentes* and *evanescentes*, whose prime and ultimate ratios he investigates, by the appellation of finite. Now I desire *Philalethes* to reconcile this passage with his notion of a *nascent quantity being a quantity not yet arrived at any assignable magnitude how small soever.* And I must farther ask *Philalethes*, whether he has not here attempted to define a non-entity.