

REMARKS ON THE CONSIDERATIONS
RELATING TO FLUXIONS &c. THAT WERE
PUBLISHED BY PHILALETES CANTABRIGIENSIS
IN THE REPUBLICK OF LETTERS
FOR THE LAST MONTH

By

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NOTE ON THE TEXT

This article appeared in *The Present State of the Republic of Letters* for August 1736.

The following spellings, differing from modern British English, are employed in the original 1736 text: expresly, falsly, surprizing, consession, rendred.

David R. Wilkins
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REMARKS *on the Considerations relating to Fluxions, &c. that were published by Philalethes Cantabrigiensis in the Republick of Letters for the last month.*

[*The Present State of the Republick of Letters*, August 1736, pp. 87–110.]

IN these *Remarks*, for brevity, the words of *Philalethes* are not transcribed, but the *Sections* and *Paragraphs*, wherein they are contained are particularly quoted.

The first four *Sections* contribute nothing towards determining the points in question.

Sect. V. §. 1–4. By the idea of fluxions Mr. *Robins* does not mean the doctrine of fluxions, but only one part of that doctrine; for it is expressly said in the *Republick of Letters* for *October* last, pag. 253. that the doctrine of fluxions consists of two parts, the form of conception there described (that is the idea of fluxions) and the method of applying it to the solution of mathematical problems.

Again, Mr. *Robins* does not conclude, that the method of fluxions is absolutely distinct from that of first and last ratios, only because Sir *Isaac Newton* had formed his idea of fluxions before he had invented the other method, but also because that other method is no otherwise made use of in this than for demonstrating the proportions between different fluxions.

Sect. V. §. 5–8. Here it is attempted to be proved, that one of these methods could not possibly be invented before the other; because Sir *Isaac Newton* has in his writings made use of that of prime and ultimate ratios in demonstrating propositions in the other doctrine; whereas to make this a conclusive argument, it was necessary to shew, it were impossible to form these demonstrations by any other means; but it appears in fact Sir *Isaac Newton* did at first content himself with such demonstrations, as the method of indivisibles did afford; and Mr. *Robins* has shewn, that their propositions may be perfectly demonstrated another way by exhaustions.

§. 9. Because *Philalethes* sees not the use of this distinction, is that a proof it is of no importance? How could *Philalethes* imagine the letter S to be inserted through inadvertency, when that is to suppose the title page printed before the book was writ; for these two methods are as much distinguished in the book itself, as in any thing Mr. *Robins* has since published.

§. 11, 12. Has not Sir *Isaac Newton* sufficiently distinguished them in delivering his method of prime and ultimate ratios by itself at the beginning of his *Principia*? Has he not sufficiently distinguished between them, when at the end of his *Introduction* to the *Quadratures*, he speaks of determining the fluxions of quantities by the method of prime and ultimate ratios?

§. 13. Sir *Isaac Newton* does not intermix his simple and plain description of fluxions with the terms used in the doctrine of prime and ultimate ratios; for his description of fluxions is contained in the two first paragraphs of his *Introduction* to the *Quadratures*, in which no terms of the other doctrine occur: the following part of the *Introduction* is not to instruct us in the idea, he designs to be annexed to the word fluxion, but only contains directions how by the method of prime and ultimate ratios to find the proportions between different fluxions.

Therefore it still appears, that the idea of fluxions has no relation to the doctrine of prime and ultimate ratios, and that the doctrines are absolutely distinct from each other. For is not the fifth proposition of the first book of *Euclide* absolutely distinct from the fourth, though the fourth is the means whereby the fifth is demonstrated. By the argument here made use of the doctrine of fluxions might as well be proved not to be distinct from the ancient method of exhaustions, since Mr. *Robins* has shewn how to apply that method to the determining the proportions of fluxions.

§. 14. Here the distinction made at the beginning of Mr. *Robins*'s last Discourse is not attended to; where it is observed, that the same method of calculation is applicable to both methods. And if these two methods being equally subservient to the same method of computation makes them the same, then they are also the same with the differential method of *Leibnitz*.

Sect. VI. §. 1–4. In reality the words here quoted from Sir *Isaac Newton* are the most unguarded, that great man has used upon these subjects, and upon these *Philalethes* has unhappily built his whole explanation of prime and ultimate ratios. If these expressions are unintelligible or incorrect, what justification will it be to *Philalethes* not to have discovered their imperfection?

§. 5, 6. Why will *Philalethes* so positively assert, that Sir *Isaac Newton*, when he wrote his *Analysis*, did not there proceed upon the principles of indivisibles, since they are apparently used in that treatise, and it is affirmed in the account of the *Commercium Epistolicum*, that he was accustomed to make use of indivisibles at that time, and the terms relating to them, in the same sense with *Cavalieri* their original author. But it seems this account was writ after Sir *Isaac Newton* had corrected that faulty doctrine; but is that a reason, why in giving an account of what he had done before, he should either disguise or conceal the truth. It is also, it seems, in this account declared, that Sir *Isaac Newton* continued to use the letter *o* for an infinitely small quantity, even to the time, when that account was writ; but Mr. *Robins* has never denied, that Sir *Isaac Newton* might sometimes have allowed himself in the use of indivisibles in the investigation of problems, even to his dying day. Mr. *Robins* has shewn, where he used them in his *Principia*. Here I must take leave to inform *Philalethes*, that he does not understand, what use Sir *Isaac Newton* made of the letter *o*; for he asserts, that it denotes the only infinitely small quantity used in his calculus; but a mathematician ought to have known, that of one infinitely small quantity only no use at all can be made; it is only here meant, that Sir *Is. Newton* used no other symbol to express an infinitely small quantity, nay this means only no other simple symbol; for in the passage just quoted by *Philalethes* at §. 5. it is said *prick'd letters never signify moments, unless when they are multiplied by the moment o either expressed or understood to make them infinitely little, and then the rectangles are put for moments*. Now according to *Philalethes* all moments are infinitely little quantities; therefore when Sir *Isaac Newton* used any infinitely little quantities at all, he really used as many different infinitely little quantities as he considered moments, though he did not mark them by different simple characters, as others had been accustomed to do. Besides, Mr. *Robins* has shewn, that Sir *Isaac Newton* did not always intend an infinitely little quantity by the letter *o*. See *Republick* for *April* last, pag. 324. and this Discourse of *Philalethes*, §. 10. of the present section.

§. 7, 8. Surely *Philalethes* has more respect for Sir *Isaac Newton*'s memory than to insinuate, that, when he was defending his method, he was not careful to give a true and

unexceptionable description of it.

§. 9–12. Here in relation to the words *volui ostendere, &c.* I will only ask *Philalethes*, whether Sir *Isaac Newton* does not mean, that he intended in the *Introduction* to the *Quadratures*, which is closed by these words, to shew, that it was not necessary infinitely small quantities should be introduced into geometry; and whether he has executed that design he proposed? If he has, the doctrine of prime and ultimate ratios made use of throughout that Introduction does not imply the consideration of any infinitely small quantities.

§. 13, 14. Because Sir *Isaac Newton* thought fit to own, that it is possible to avoid error in the use of the principles of indivisibles with proper circumspection; does it therefore follow, that he himself made use of infinitely small quantities at the very time he undertook to shew there was no necessity for so doing.

Sect. VII. §. 1–4. Any given difference does not mean merely any assignable difference, but any assignable difference that shall be given. Sir *Isaac Newton* at the end of his *Analysis* uses the like phrase exactly in the sense Mr. *Robins* contends for; for he there observes, that by adding continually fresh terms to the series there discoursed of, *ultimus terminus* (*per 1. 10. Elem.*) *tandem evadet minor quavis data quantitate; & prorsus evanescet, si opus infinite continuatur.* Now it is impossible to continue the series till you find a term less than any assignable. And Sir *Isaac Newton* himself expressly distinguishes the diminution he first speaks of from the vanishing of the term, (whereby alone it can become less than any assignable,) since he adds, that for this purpose the series must be infinitely continued.

§. 6, 7. In the long string of phrases here collected, they are so far from being all designed to express the sense of *data quavis differentia*, that they have not all the same meaning one with another; *assignable difference*, or *difference that can be assigned*, means *differentia quæ dari potest*; in a translation of the fifth phrase not one of these words ought to be used.

§. 9. This is a wrong representation of Mr. *Robins*'s meaning, he does not expound the words in question by *any difference how minute soever, that can be assigned*, but has represented the sense of the *Lemma* in another turn of phrase, wherein the words *data quavis differentia* [*any difference that is or shall be given*] would have been improper. This author does not here distinguish between the expounding the sense of a single phrase and representing the meaning of the whole sentence, in which that phrase is used.

§. 10. Any given difference is not in Mr. *Robins*'s opinion consonant to any difference that can be assigned, though it be not difficult to make use of either of these phrases in expressing the same thing, provided the rest of the expression be rightly accommodated to it.

Sect. VIII. §. 1, 2. Mr. *Robins* has never differed in the sense, he has ascribed to this *Lemma*, though he thought himself to apprehend its meaning so truly, as to be in no fear of expressing that meaning by different forms of words.

§. 3–9. To what purpose are these four suppositions here again repeated? Has not *Philalethes* yet discovered, that this interpretation of the *Lemma* makes it an erroneous proposition? For two quantities may constantly tend to equality during some finite space of time, and before the end of that time come nearer together than to have any difference, which shall be given; and yet at the end of that time have still a real difference. For however small the difference be, which is given; from the quantities attaining to that difference before the end of the time named, it will only follow, that at the end of this time they will have a less difference.

But *Philalethes* perhaps will still insist upon the words *any given difference* being synonymous to *any assignable difference*. And then two quantities are supposed constantly to tend to equality during some finite space of time, and yet before the end of that time to come nearer together than to have any assignable difference. But thus the quantities will become equal before the end of that time, the whole of which is supposed to be taken up in their approach. [See *Republick of Letters* for *January* last at the bottom of pag. 82, and beginning of pag. 83.] As soon as quantities come to have no assignable difference, they are actually equal. By this interpretation therefore the terms of the *Lemma* are rendered inconsistent with themselves.

§. 10–12. Does not *Philalethes* know, that the words here quoted from Mr. *Robins*'s Book were intended to express only a part of Sir *Isaac Newton*'s method, which relates to quantities approaching each other, and that the other part relating to ratios is expressed soon after in another paragraph? and has not Mr. *Robins* in his last paper united both these together? Why then does *Philalethes* represent Mr. *Robins*'s description of Sir *Isaac Newton*'s method thus imperfect?

§. 13. None of the suppositions really contained in this *Lemma* are omitted by Mr. *Robins* in the passage here cited. For suppose the quantities are approaching to all eternity, they certainly then approach during any finite space of time; and since the varying quantity or ratio is supposed capable of being made in its approach to the other to differ from it by less than any quantity, how minute soever, that can be assigned; certainly some finite space of time may be always assigned, before the end of which the quantities of ratios may come nearer than by any difference whatever, that shall be proposed. The conclusion of the *Lemma*, that the quantities become *ultimo æquales*, is not omitted; but Mr. *Robins* conceives truly interpreted; and *Philalethes* has no right to assert the contrary, till he has proved, that Sir *Isaac Newton* designed no quantities or ratios to be comprehended within the Sense of this *Lemma*, which do not become actually equal. This *Philalethes* has not yet proved, and Mr. *Robins* believes, never will. That the quantities or ratios are to become equal at the end of the finite time mentioned in this *Lemma*, Mr. *Robins* apprehends is not the sense of the *Lemma*.

§. 14. This second interpretation was not given to supply the defects of the first, but to shew the truth of the first by expressing the same sense in words nearer to those of Sir *Isaac Newton*.

§. 15. Here I conceive no supposition of Sir *Isaac Newton* to be omitted, but that Mr. *Robins* has spoken of the finite time here mentioned in the sense of Sir *Isaac Newton*, although by a different rangement of the words it does not stand in the same place of both sentences. Mr. *Robins* has indeed here omitted, and always intends to omit the second supposition ascribed by *Philalethes* to Sir *Isaac Newton*, that the approach of the quantities or ratios mentioned is always to be considered within the limits of some finite space of time; because, he thinks, he has proved Sir *Isaac Newton* to mean otherwise.

§. 16. Mr. *Robins* does not advise to read the demonstration of this *Lemma* before the *Lemma* itself, in order to understand the meaning of the expressions used in it; that would have been to advise to seek after the meaning of the words, before the words themselves were read.

§. 17, 18. I am not of opinion, that the answer given was either full or conclusive; nor has Mr. *Robins* to evade it ever altered the sense of the remark here quoted.

§. 19–22. Here Mr. *Robins* did not intend any third interpretation of this *Lemma*, and

in order to represent what is quoted in §. 20, as such, the words in §. 21 are falsely cited. For the words *the latter sense* are turned into the word *this*. Thus Mr. *Robins* is made to call the preceeding words an interpretation of the *Lemma*, whereas *the latter sense* he speaks of, is the interpretation, he assigns to the words, *given difference*, in opposition to the sense they must have according to *Philalethes*'s interpretation of that *Lemma*. The words cited in the 20th §. are not an interpretation of the *Lemma*, but an account how certain quantities may be brought under it, which *Philalethes* denies to be subjects of it.

§. 23. This is begging the question, and proceeds upon the supposition, that *Philalethes* understands the true meaning of the words *ultimo æquales*, which Mr. *Robins* denies.

§. 24–26. Mr. *Robins* does not refer to the demonstration of this *Lemma* for the sense of the words *any given difference*, but expressly fixes the sense of these words from their genuine use in geometry. Mr. *Robins* refers to the demonstration of this *Lemma* only for discovering the sense of the phrase *fiunt ultimo æquales*.

§. 27, 28. The supposition, that D is the last difference, is equally erroneous, whether the quantities are still approaching, when they have this difference, which is supposed their last, or whether they have ceased to approach, but are already come nearer than by that difference.

§. 29. If *Philalethes* cannot understand Mr. *Robins*'s meaning, here called the fourth interpretation of the *Lemma* in question, it is to be hoped his unprejudiced readers can. Suppose Mr. *Robins* means, that some of these quantities may perpetually and to all eternity have some difference, and yet no difference, that can be called their last; to what purpose is any question here made about the meaning of the words *fiunt ultimo æquales*; for Mr. *Robins* has sufficiently declared, what he thinks Sir *Isaac Newton* meant by them, and believes he has proved, that consistent with truth Sir *Isaac Newton* could not mean otherwise. And Mr. *Robins* supposes the sense, he has ascribed to them, not to be the plain and ordinary meaning of those words, but a new sense Sir *Isaac Newton* thought fit to put upon them. Nay farther, it is not difficult to assign a very probable reason, which led Sir *Isaac Newton* to the use of this expression; for before him it had not been unusual for geometers to speak of the last sums of infinite progressions, which is an expression somewhat similar to this. Surely here no one will pretend, that an infinite number of terms can, in strict propriety of speech, and without a figure, be said to be capable of being actually summed up and added together.

§. 30. This paragraph depends entirely upon the use made of the finite time mentioned in this *Lemma*, and if Mr. *Robins* agreed in opinion with *Philalethes* relating hereto, the whole question would be given up, and this paragraph unnecessary; but, while he does not agree to it, this paragraph contributes nothing to the deciding the point in debate.

§. 31. In answer to the demand what is to be understood by the word *perpetually*, suppose it were answered, that it means all the time, that the approach of the quantities or ratios in question is under consideration.

§. 33. If it appears so surprizing to *Philalethes*, that one person should in his opinion give four disagreeing interpretations to the same thing, why may it not appear as strange to Mr. *Robins*, that another person should apprehend in four different senses, what he thinks very evidently unite in the same?

Sect. IX. §. 1, 2. When in any phrases common words are used in a new sense, those words may properly be called new terms; for though the sound is the same, the sense is different.

§. 3, 4. This is the only purpose, for which Mr. *Robins* refers to the demonstration of this *Lemma* for the true understanding of it. And Mr. *Robins* thinks, that the method taken here by Sir *Isaac Newton* is consistent with his being a good writer, and has as much right to hold that opinion, as *Philalethes* has to be positive of the contrary. For *Euclide*, whom Mr. *Robins* esteems as one of the correctest writers, that ever was, appears to have taken the like method with respect to the phrase of compound proportion. For the definition found in the present copies of the *Elements* is neither alluded to by *Euclide*, when he first makes use of that term, nor is it well adapted to explain the sense, in which it is used. But the sense of the phrase appears very evidently from his first use of it.

§. 5. How will *Philalethes* prove, that Mr. *Robins* was ever of any other opinion, than that Sir *Isaac Newton*'s demonstration is applicable to such quantities, as at last become actually equal, as well as to quantities, which only approach without limit to the ratio of equality?

§. 6–8. Mr. *Robins* has indeed asserted, that some of these subjects do become actually equal. But suppose it were agreed, that, when quantities are capable of an actual equality, Sir *Isaac Newton*'s demonstration will prove it, what concession is that to *Philalethes*? For whatever quantities *Philalethes* has named as becoming actually equal, which Mr. *Robins* has once denied to do so, he still continues to deny as strongly as ever. Suppose Mr. *Robins* should allow, that *Philalethes*'s interpretation of the *Lemma* would agree with some of the quantities comprehended under it, is that a concession, that *Philalethes* has expressed Sir *Isaac Newton*'s true meaning, while Mr. *Robins* is of opinion, that his interpretation will exclude other quantities necessary to be brought under it? Surely that is declaring, that the interpretation of *Philalethes* is erroneous; because the true interpretation ought equally to comprehend both.

§. 9. Mr. *Robins* perceived, that *Philalethes* mistook his meaning in the first of the passages here cited; and therefore added the words, *necessarily implied*, for his information. What induced *Philalethes* to produce this latter quotation as if asserted of all quantities relating to this *Lemma*, when it only concerns a particular case, the parallelograms inscribed and circumscribed to a curve in the second *Lemma*.

Sect. X. S. 1. Where has Mr. *Robins* admitted, that any quantities whatever will become actually equal at the end of the finite time referred to in the *Lemma*? If he has not, how has he allowed of *Philalethes*'s interpretation? What does *Philalethes* then mean by fancying, that Mr. *Robins* has at any time allowed Sir *Isaac Newton*'s meaning to be the same with that of *Philalethes*? Mr. *Robins* says, that the *Lemma* in Sir *Isaac Newton*'s sense of it did not necessarily imply, that the quantities compared in it should become actually equal, because in fact many of them cannot; nay that Sir *Isaac Newton* neither demonstrated the actual equality of all quantities capable of being brought under this *Lemma*, nor that he intended to do so. See *Repub. of Lett.* for *April* last, p. 309.

§. 3, 4. Since *Philalethes* does not perceive, how the demonstration of this *Lemma* can be applied to the case mentioned of the hyperbola, I shall now shew it at large. Here this hypothesis assumed is a property of the hyperbola commonly known, That the curve continually approaches its asymptote, as it is farther extended, so that by removing any ordinate farther and farther from the vertex. it will approach nearer and nearer in magnitude to the same continued to the asymptote, without limit. Now to prove the ultimate equality of these two lines, let us say thus. If you deny it, let them be ultimately unequal, and let their

last difference be D; therefore they cannot approach to equality nearer than by the given difference D, contrary to the hypothesis. Thus is the demonstration of this *Lemma* without changing a single word applied to the present case.

§. 5–9. The reasons here mentioned are not unanswerable; one of them is this, *The demonstration of this Lemma cannot possibly be applied to this case; since these two variable quantities can have no last magnitude, and consequently cannot be supposed to have any last difference.* This argument of *Philalethes* depends upon this mistake, that the difference proposed as the last is a difference, which the quantities are supposed to have after the time of their approach is over. But there is no ground for such a supposition from the words of Sir *Isaac Newton*'s demonstration. See the Remark on §. 28. of Sect. VIII.

The argument produced in *January* by interpreting the time mentioned in this *Lemma* to be one definite portion of time, and supposing the quantities to become equal at the end of that time, renders the *Lemma*, as we have above observed, a false proposition.

§. 10. It never was any matter of doubt with Mr. *Robins*, whether the limitation of the finite time mentioned in this *Lemma* to some one definite portion of time be necessary to the purpose of interpreting Sir *Isaac Newton*'s meaning; but Mr. *Robins* declined in this place discussing that point, because he could confute the objection under consideration without it: for he shewed, how to put this case of the hyperbola so as to comprehend the motion proposed in it within a definite space of time. What does *Philalethes* mean in this next paragraph by saying, Mr. *Robins* has shewn this pretended method to be absurd, fallacious and inconclusive? Mr. *Robins* has only shewn, that the conclusion, which would be drawn after the addition of this circumstance from *Philalethes*'s interpretation of this *Lemma*, is absurd, fallacious and inconclusive.

Sect. XI. How was it possible for *Philalethes* to assert, that Mr. *Robins* has not even offered to shew, that any quantities or ratios incapable of an actual equality are compared in this *Lemma*, when it is expressly endeavoured to be proved at pag. 315 of the *Republick of Letters* for *April* last, that this is absolutely the case in all vanishing quantities, and that by the most natural interpretation of Sir *Isaac Newton*'s second and third *Lemmas*, those *Lemmas* apply quantities to this first, which are incapable of an actual equality.

But does *Philalethes* really know, what those quantities are which Mr. *Robins* acknowledges to be the subject of this *Lemma*, and capable of an actual equality? When *Philalethes* shall name them, Mr. *Robins* will shew, that this *Lemma* is so far from being confined to that case, that the equality of all those quantities may be proved by another method much more natural than by referring them to this *Lemma*.

The demonstration of this *Lemma* is by the indirect form called *deductio ad absurdum*. Now there are two kinds of quantities, which may be brought under this *Lemma*: one sort are capable of having their ultimate equality proved by the direct method of demonstration; and these quantities do become actually equal: to the other subjects of this *Lemma* the direct form of demonstration cannot be applied; these are the subjects, for the sake of which this *Lemma* was principally writ; and these, in the opinion of Mr. *Robins*, never can arrive at an actual equality.

Sect. XII. §. 1–3. Here is a very positive charge of falsly translating and interpreting and unfairly quoting certain words of Sir *Isaac Newton*. It seems the words *pro æqualibus habeantur* should have been rendered *let them be esteemed equal*. But why is *habeantur* the Imperative mood? *Philalethes* was deceived by the preceding words, which direct certain

constructions; but here, where a consequence is concluded from those constructions, the Potential mood is required, in which, to use the learned *Linacer's* words, *indicatio est potentiæ, debetive, aliquando voluntatis*. How then is, *are to be esteemed equal*, a false translation? Is not that *pro æqualibus haberi debent*? Just as *non expectes, ut statim gratias agat, qui sanatur invitatus*, is expounded by this great Grammarian *non debes expectare*. See fol. 10. of his book *De Emendata structura Latini Sermonis*, Ed. R. Steph. 1527.

I have quoted *Linacer*, because he was the first who gave the name of potential to this mood, when it bears any of the three significations here mentioned: but this form of the verb having all these senses is a point agreed among Grammarians. See *Alvarez* and *Vossius*.

The charge of quoting unfairly is wholly grounded upon certain words not being produced, which are nothing to the purpose: for it was the form of the expression only, which Mr. *Robins* had here in view. Does *Philalethes* take it for granted, that Mr. *Robins* had so much as any shadow of suspicion, that these subsequent words were anywise inconsistent with the sense he has put upon the other? Surely *Philalethes* does not yet comprehend distinctly the point in dispute between Mr. *Robins* and himself: else he would scarce have been guilty of so manifest a begging the question, as is contained in these words, *When they come to vanish, they will arrive at the ratio of an absolute equality*. Knows he not, that Mr. *Robins* allows the ratio called the ultimate of the vanishing quantities DF , df to be that of equality; though he will challenge *Philalethes* to shew, that it is consonant either to the geometry of the ancients, or to truth, that these lines can be ever equal. If PS is greater than ps , DF must always be greater than df .

§. 4. Mr. *Robins* does not see, that of the two expressions, *ultimo in ratione æqualitatis* and *ultimo æquales*, the one will admit of a laxer interpretation any more than the other, for Mr. *Robins* has ever contended, that they must both have such a lax sense, when applied to quantities, that cannot actually exist under the condition of equality. What is here asserted concerning Mr. *Robins* ought to have been postponed, till the assertion be attempted to be proved.

§. 5. The second *Lemma* was not produced in the account of Mr. *Robins's* Discourse as an example of the whole of the assertion just quoted from him, but only as an instance of one part of it; and therefore, although *Philalethes* had as manifestly shewn, as he here groundlessly presumes, that *Lemma* to be an example quite contrary to Mr. *Robins's* purpose, it would not be an entire confutation of Mr. *Robins's* assertion; nay it would be no confutation at all; for if Sir *Isaac Newton* has applied equality to vanishing quantities, Mr. *Robins* has still shewn the truth of his assertion.

§. 6. Surely *Philalethes* wrote this paragraph, before he had read Mr. *Robins's* Dissertation through; because what he charges him with taking no notice of, he has discoursed of very largely at pag. 316. Mr. *Robins* thinks he had at first sufficiently considered, what was said by *Philalethes* in *November* last. *Philalethes* is desired to explain, what he means here by saying Mr. *Robins* has made too free with Sir *Isaac Newton's* name; because, I believe, Mr. *Robins* has such a veneration for Sir *Isaac Newton's* memory, that he would be greatly grieved, that the publick should suspect him of treating him with the least disrespect.

Sect. XIII. S. 1–11. *Philalethes* might easily have extricated himself out of all his confusion by attending to one expression of Mr. *Robins's* original book, page 64. where it is observed, that by the form of a curve its tangent may meet the curve again in some other point: and at such point the quantities, by whose ultimate ratio the situation of the tan-

gent is determined, will bear that ratio, which is called their ultimate. And since quantities, whose ultimate ratio is sought, may often have such a relation to each other, as accidentally in particular magnitudes of them to bear the same ratio with that, which is called their ultimate; for that reason Mr. *Robins*, where he speaks of vanishing quantities in general, has often taken care to turn his expression so, as not to exclude this accident; but whenever he speaks of vanishing quantities not subject to this casualty, he then always speaks positively, that those quantities never can be the subject of that ratio, which is called their ultimate. This is the case of the *Quadrilaterals* here mentioned in the third paragraph. However this restriction is of so little consequence, that sometimes perhaps it may not be so expressly attended to. Mr. *Robins*'s present opinion concerning vanishing quantities is the same, it always was, and in short is this; That there are no quantities at the instant of evanescence actually subsisting to be the subjects of the ultimate proportion of the vanishing quantities; and this was Sir *Isaac Newton*'s opinion, as appears from this expression of his, there are *rationes primæ quantitatum nascentium*, but not *quantitates primæ nascentes*. Farther the definition of *Philalethes* of a nascent increment being *an increment just beginning to exist from nothing, or just beginning to be generated, but not yet arrived at any assignable magnitude, how small soweever*, conveys to Mr. *Robins* no idea, nor does he believe to any one else, however any person through an indistinctness of conception may deceive himself. For surely this is a description of infinitely little quantities, of which Sir *Isaac Newton* says we have no idea.

§. 11. Mr. *Robins* had no occasion to pass any censure upon the passage here referred to. He never imagined, that *Philalethes* had not learnt from Sir *Isaac Newton*, how to assign the ultimate proportion of vanishing quantities truly; his only objection to *Philalethes* is his giving a wrong and unintelligible notion of these quantities. This is an endeavour to divert the question from the real point in dispute.

Sect. XIV. §. 1, 2. Mr. *Robins* approves of the word *endlessly*, and is so far from acknowledging any mistake it led him into, that on the contrary *Philalethes* has himself erred by imagining the words *in infinitum* have any other signification.

§. 3–6. *Philalethes* has given two senses to this word *endlessly*, but neither of them will lead into error, but are both applicable to the present case. Nay the first sense necessarily implies the second; for if the number of these parallelograms is to be increased still more and more without limit, it is certainly impossible for the imagination in any finite time to pursue to the end, what has no limit.

§. 7. Before *Philalethes* charges this interpretation as *petitio principii*, he ought to shew, that the words are capable of any other sense, which I apprehend, it is impossible for him to do.

§. 8. Mr. *Robins*, I believe, is not ashamed of his skill in selecting those expressions, which he thinks contribute most readily to prove his opinion. Mr. *Robins* apprehends *Philalethes* in this paragraph to be chargeable with *petitio principii*; for that Sir *Isaac Newton* by diminishing a quantity *in infinitum* is always to be understood, that he requires you to pursue that diminution in the imagination till the quantity is actually vanished, Mr. *Robins* has as much liberty to deny, as *Philalethes* to assert. And that Sir *Isaac Newton* did not intend this in the proposition in question, he is well persuaded, because it is impossible.

§. 9. There is certainly no other harm in using improper expressions, when the writer is careful to explain his meaning, than in being guilty of a needless obscurity.

§. 10. The fear Mr. *Robins* has of borrowing any terms from the language of indivisibles

arises from this, that they are absolutely unnecessary, and he believes, that Sir *Isaac Newton*'s use of such terms has been the whole occasion of the misunderstanding his doctrine; for had no such terms been used, whoever was capable of reading him, he thinks, must have come nearer to his true sense.

Sect. XV. S. 1, 2. Mr. *Robins* thinks himself directed by the words of Sir *Isaac Newton* to make the subdivision in the manner here proposed. *Philalethes* in not understanding this place, confirms Mr. *Robins* in his opinion, how needful a knowledge in the ancients is to qualify a person for understanding either Sir *Isaac Newton* or himself. But however *Philalethes* has endeavoured to shew his knowledge of the ancients by quoting two propositions from *Euclide*'s *Elements*; one of which teaches how to divide a line in the same proportion as some other line is divided, the other shews that by taking from any quantity more than half, and from the remainder more than half continually, the residue may be reduced within any degree of smallness. How much these propositions are to the purpose of inscribing and circumscribing parallelograms to a curvilinear space, let *Philalethes* shew. When *Philalethes* has gone farther in the ancients than the *Elements of Euclide*, he will be better able to comprehend Mr. *Robins*'s meaning, and judge upon the point in question.

§. 4. The motion here proposed does not divide the base of the curve, as is required; at the end of half an hour the base will indeed be divided into two equal parts; at the end of three quarters of an hour one of those parts will again be divided into two halves; at the end of $\frac{7}{8}$ of an hour one of these will again be bisected; but is this dividing the whole line continually into equal parts?

§. 5. Mr. *Robins* would perform nothing in relation to this third, nor yet the second *Lemma* from the propositions above cited from *Euclide*, but such subdivisions as are required in both, he has performed in pag. 60 of his first *Discourse*; for the base AD of the curvilinear space ADB is divided into equal parts, and the base EH of the figure EFGH is divided unequally.

§. 6. After what has been written on the fourth paragraph, I believe *Philalethes* will not desire the question here to be answered.

§. 7. I believe *Philalethes* will not repeat again this heap of assertions without proof, till he has found out some other motion, than he has yet been able to contrive, whereby to support them; for, if there be no other method of inscribing and circumscribing the parallelograms, as Sir *Isaac Newton* requires, but by subdividing continually the base of the curve; it is a wrong assertion, that Sir *Isaac Newton* says not one word of continual subdivision or subdivision into parts of the base of the figure.

Sect. XVI. §. 1, 2. Besides what has already been said on this pretended division of the base by motion, it is farther necessary for *Philalethes*, in order to perform by continued motion what Sir *Isaac Newton* requires, not only to divide the base of the curve, but form the parallelograms also by such continued motion. And what motion will he contrive for this purpose which shall not be perplexed or even confused?

Here let *Philalethes* be ask'd, where is to be found *Euclide*'s method of dividing the base of a curve either to measure that curve, or to compare it with another curve?

§. 3. I answer, that the hour itself can certainly be pursued by the imagination to the end of it, but not the innumerable subdivisions, which *Philalethes* pretends to have made by his motion. Perhaps I may be easiest understood by comparing the present point with the old argument against motion from *Achilles* and the *Tortoise*. It is impossible to pursue in

the imagination their motion by the means proposed in that argument to the point of their meeting; because the motion of each is described by the terms of an infinite progression; but if we seek after the place of their meeting by the method proposed in that argument, we must have recourse to the doctrine of prime and ultimate ratios, and find the nearest limit of each of those progressions.

§. 4. Does *Philalethes* here mean, that it is not necessary in every geometrical demonstration to form in the imagination a distinct conception of the subjects under consideration? to do otherwise is not only contrary to the practice of the ancients, but to that of every other just reasoner.

THUS we have gone through the *Considerations* of *Philalethes* published last month, paragraph by paragraph; and design to examine the rest after the same manner, as soon as they shall be published. But to prevent the dispute running to an unmeasurable length, we shall afterwards reduce our *Remarks* into as narrow a compass as possible; and for that end intend to confine ourself to the discussion of one single point, by which we apprehend the merits of the question will in a manner be wholly decided. This is the examining whose interpretation of Sir *Isaac Newton*'s first *Lemma* is agreeable to the nature of vanishing quantities, as soon as *Philalethes* shall have rendered his interpretation a true and consistent proposition. But after this point is settled, we shall be ready to explain ourselves upon any other, *Philalethes* shall desire.