THE REMAINDER OF THE PAPER BEGUN IN OUR LAST, ENTITULED CONSIDERATIONS UPON SOME PASSAGES OF A DISSERTATION CONCERNING THE DOCTRINE OF FLUXIONS, PUBLISHED BY MR. ROBINS IN THE REPUBLICK OF LETTERS FOR APRIL LAST.

By

James Jurin

(The Present State of the Republick of Letters, August 1736, pp. 111–179)

Edited by David R. Wilkins

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NOTE ON THE TEXT

This text is transcribed from *The present state of the republick of letters* for August 1736.

The following spellings, differing from modern British English, are employed in the original 1735 text: streight, waved [waived], instable, throughly, candor, meer.

The following erratum, noted at the conclusion, has been corrected:—

‘*analagous*’ has been corrected to ‘*analogous*’.

The ‘ERRATA’ also instruct us to delete the whole of section XXXVI. from the words ‘We are told, Sir *Isaac Newton* “did not scruple there...”, to the end of the section. This text has however been retained in the present edition.

David R. Wilkins
Dublin, June 2002

The REMAINDER *of the Paper begun in our last, entituled,* Considerations upon some passages of a *Dissertation* concerning the Doctrine of *Fluxions*, published by Mr. *Robins* in the *Republick of Letters* for *April* last. By *Philalethes Cantabrigiensis.*

[*The Present State of the Republick of Letters*, August 1736, pp. 111–179.]

XVII. HOwever, since Mr. *Robins* is pleased to talk so much about *straining our imagination for some involved and perplexed kind of motion*, let us see, if we cannot find some plain and easy way, of representing to the imagination, that actual equality, at which the inscribed and circumscribed figures will arrive with each other, and with the curvilineal figure, at the expiration of the finite time.

For this purpose, perhaps the best method we can take, will be to imitate that judicious expedient, which Sir *Isaac Newton* has made use of in the seventh *Lemma*, to represent to the imagination the last proportion of decreasing quantities.

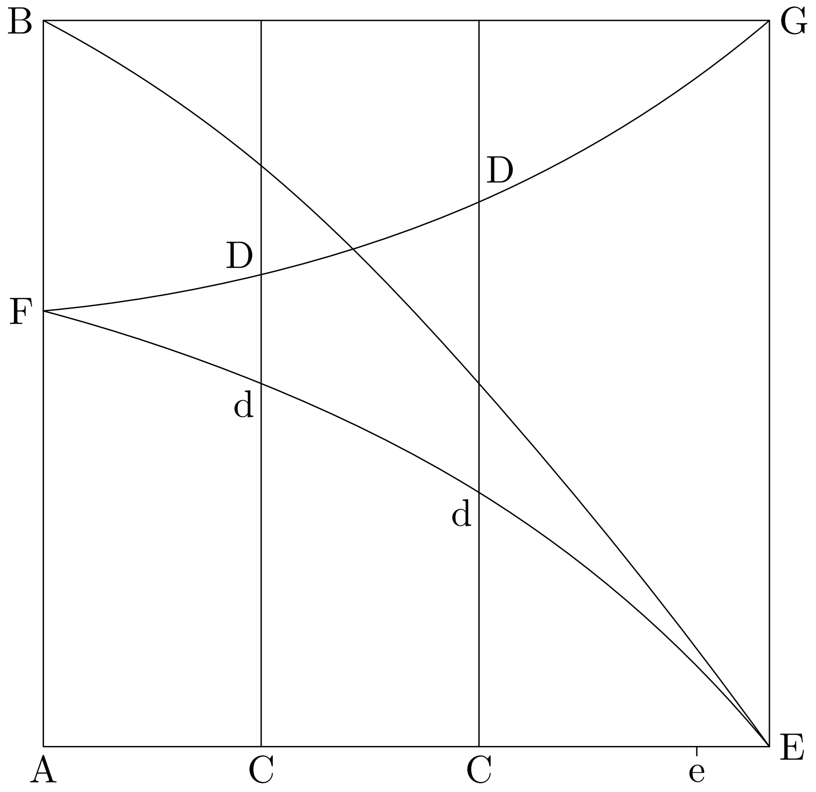
As those quantities, by a constant decrease, are diminished *ad infinitum*, and at last vanish, they arrive at their last ratio at the instant of their evanescence. And as, at that instant, they slip away and withdraw themselves from our conception; for this reason, Sir *Isaac Newton*, to help our imagination, teaches us to contemplate the variable ratio of those quantities, and particularly their last ratio, not in themselves, but in other quantities proportional to them, which do not vanish, but continue and subsist at the instant that they arrive at this last proportion; as we have more particularly observed in the *Republick of Letters* for *November* last.*

So here, as the inscribed and circumscribed figures, at the instant of their coincidence with the curvilineal figure, do, with respect to their rectilineal form, likewise slip away and withdraw themselves from our conception; it will be of use to consider their variable ratio, and particularly their last ratio, not in themselves, but in other quantities, which undergo no alteration of their form, when they arrive at this last proportion.

Let us therefore suppose the curvilineal figure A B E to be equal to the rectangle contained under the base E A and some constant line, as A F. And at the instant that the point, which we have † supposed to describe the base E A in a finite time, arrives at any point as C, let the rectangle under the same base E A and some variable line, as C d, parallel to A B, be equal to the sum of the inscribed parallelograms, at that instant conceived as standing upon C A the remaining undescribed part of the base, and upon as many other parts of the base E A, as can be taken equal to C A and adjoining to it, *i. e.* let the rectangle under E A and C d be equal to the inscribed figure: And let E $d\,d$ be the curve, which the point d continually touches or describes.

* Pag. 378, 379.

† Republick of Letters for *November*, p. 376.



In like manner, let the rectangle under the same base E A and some other variable line, as C D, also parallel to A B, be equal to the sum of the circumscribed parallelograms, at the same instant conceived to be standing upon the same parts of the base as above, and upon E *e* the smaller remainder of the base, if any such there be, adjoining to the point E, after the manner described in the *Republick of Letters* for *January* last;* *i. e.* let the rectangle under E A and C D be equal to the circumscribed figure: And let G D D be the curve, which the point D continually touches or describes.

Then, as the curvilineal, the inscribed, and circumscribed figures are respectively equal to these three rectangles $EA \times AF$, $EA \times Cd$, $EA \times CD$, having all the same common base E A; it is manifest that those figures will be respectively proportional to the lines A F, C *d* and C D; and may consequently be always represented by these lines.

Now, since the two lines C *d*, C D, do

1. Constantly tend to equality with each other, and with the constant line A F,
2. During a finite time,
3. And approach within less than any given difference,
4. Before the end of that finite time; agreeably to the four suppositions of the first *Lemma*;

These three lines will, by that *Lemma*, be equal to one another at the end of the finite time.

This we apprehend not only to be undeniably true, but *now* to be acknowledged even by Mr. *Robins*; since that Gentleman is at last brought to confess, that † whenever the "quanti-

* Page 90.

† Republick of Letters for *April*, p. 309.

ties compared in this *lemma* are capable of an actual equality, they must really become so;" and also that " * There is no ultimate sum of these parallelograms, nor no ultimate figure compounded of them, distinct from the very curve itself." "This," he says, "Sir *Isaac Newton*, in the corollaries annexed to the third *lemma* expressly declares." Here it is manifest, that, by the *ultimate sum of these parallelograms*, can only be understood the *summa ultima parallelogrammorum evanescentium*, which is said *coincidere omni ex parti cum figura curvilinea*, in the first of those corollaries; and that by the *ultimate figure compounded of them*, nothing else can be meant, than the *figureæ ultimæ*, of which it is said, *non sunt rectilineæ, sed rectilinearum limites curvilinei*, in the fourth corollary; both which corollaries had been expressly † quoted and insisted on by me against Mr. *Robins*.

With this allowance therefore of Mr. *Robins*, we may safely assert, that, at the end of the finite time, the variable lines C d, C D, since they *are capable of an actual equality, must really become* equal to each other, and to the constant line A F.

And, if there were occasion, the same thing might also be proved after another manner. For, since the relation between C d, the ordinate to the curve E $d\,d$, and the absciss E C, may always be expressed by an equation to the curve E $d\,d$; and since the relation between C D, the ordinate to the curve G D D, and the same absciss E C, may always be expressed by an equation to the curve G D D: by means of those two equations it may be made to appear, that, when the common absciss becomes equal to the whole base E A, these ordinates C d, C D, will be equal to each other, and to the constant line A F.

This being premised, and the proportion between the curvilineal, the inscribed and circumscribed figures, being always represented to the imagination by means of the lines A F, C d, and C D; it is easy to conceive, and as it were to see, that the circumscribed figure, from being at first equal to the rectangle A B G E, and represented by the line G E, does by a constant diminution, while represented by the line C D, tend to an equality with the inscribed and curvilineal figures, represented by the lines C d, A F, and does at last actually arrive at that equality; as likewise that the inscribed figure, beginning from nothing, does by a constant increase, while represented by the line C d, tend to an equality with the circumscribed and curvilineal figures represented by the lines C D A F, and does at last actually arrive at that equality, when the describing point arrives at A the end of the base.

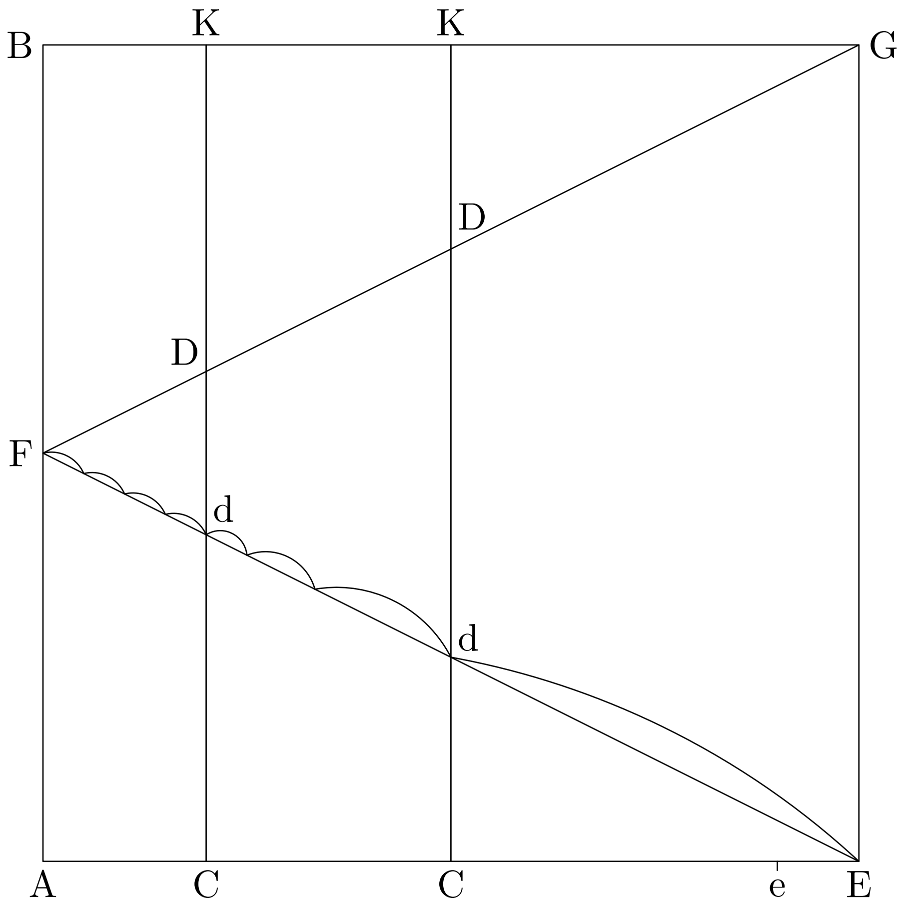
It may not be improper to illustrate what we have been saying, by an easy example; and for my own and my reader's ease, I shall chuse the simplest example I can think of.

Instead of a curvilineal figure, let us take a rectilineal one, *viz.* the rectangular triangle A B E, whose base A E is equal to the altitude A B. Let us imagine two figures, one inscribed, and the other circumscribed about this triangle, in the same manner as about the curvilineal figure in Sir *Isaac Newton*'s second *lemma*: And let these three figures be respectively represented, as before, by the constant line $\mathrm{A\,F} = \frac{1}{2}\mathrm{A\,B}$, and the variable lines, or ordinates, C d, C D.

Then, if E A, or A B be called a, and the part of the base, as EC, which at any instant of time is already described by the motion of the point C, be called x; it will easily be found, that, when CA the remainder of the base, is any aliquot part of the whole base, the ordinate C d, representing the inscribed figure, is equal to $\frac{1}{2}\,\mathrm{E\,C}$, or $\frac{1}{2}x$. Consequently, all

* Ibid. p. 311.

† Rep. of Let. for *Jan.* p. 84, 85.



these ordinates will be terminated by the right line E F, drawn from E to the point F bisecting the altitude A B.

Likewise, it will be found, that the ordinate C D, representing the circumscribed figure, will be equal to $a - \frac{1}{2}x$. And if, instead of C D, we take K d, which is always equal to it, these ordinates K d to the base B G, will be all terminated by the same right line E F already described.

But when C A, the remainder of the base E A, is not an aliquot part of the whole base; if we divide the base into as many parts as may be, severally equal to CA and adjoining to it; there will be left, adjoining to the point E, a portion as E e, less than any of the rest, which let us call r. And in this case it will be found, that the ordinate C d = $\frac{\overline{x+r} \times \overline{a-r}}{2a}$; and all these ordinates will be bounded by the continued curve E d d F, rising a little above the right line E F, and touching it in every point, where an ordinate can be drawn to that curve from the end of any aliquot part of the base, lying between A and the middle of the base.

Also, it will be found, that the ordinate C D, or K $d = a - \frac{\overline{x+r} \times \overline{a-r}}{2a}$; and all these ordinates K d will be bounded by the same curve E d d F.

Therefore, when $x = a$, and r vanishes, C $d = \frac{1}{2}a$, and K $d = \frac{1}{2}a$, i. e. C d and K d do then become equal to each other, and to the line A F. Consequently the inscribed and circumscribed figures do then become equal to each other, and to the triangle A B E.

XVIII. Mr. *Robins* is pleased to ask, * "What idea can we form of the inscribed or circumscribed figure, to which we are at last actually to arrive, which with any propriety of speech is to be styled equal to the curve?"

I answer, The idea of the figure, at which we conceive the inscribed or circumscribed figures at last actually to arrive, is no other than that of the curvilineal figure itself.

Mr. *Robins* proceeds to tell us †, "Sir *Isac Newton* in the corollaries annexed to the third *Lemma* expresly declares, there is no ultimate sum of these parallelograms, nor no ultimate figure compounded of them, distinct from the very curve itself. *Philalethes* himself acknowledges this." And in the next line this is called a *concession* of *Philalethes. Concession*! To whom? Surely not to Mr. *Robins.* This Gentleman in the *Repub. of Letters* for *December, expresly declares,* ‡ *The supposed last form of the inscribed figure must essentially differ from the curve itself.* And again. § *The curve is not the last form of the inscribed figure.* ¶ *The coincidence does involve an impossibility.*

Would not therefore the passage before us shew a greater regard to truth, and be much more properly expressed, if it were to run in the following manner?

Sir *Isaac Newton* in the corollaries annexed to the third *Lemma* expresly declares, that ‖ the last sum of the evanescent parallelograms does perfectly coincide, *coincidit omni ex parte*, with the curvilineal figure; and ** that these last figures are not rectilineal, but the curvilineal limits of rectilineal figures; and accordingly in the first proposition of the second section of the *Principia*, he tells us, the last perimeter of the triangles will be a curve line, by the fourth corollary of the third *Lemma. Philalethes* has always contended for this, has clearly proved and invincibly demonstrated this, has inevitably forced and compelled me *B. R.* under the utmost unwillingness and reluctance, to confess and acknowledge this to be true.

Castigatque auditque dolos, subigitque fateri.

XIX. It seems, there is *a* †† *general complaint of the difficulties attending the study* of Sir *Isaac Newton*'s doctrine, from the *brevity* and *conciseness* with which it is delivered; *insomuch that it often requires careful attention to discover the exact meaning, and full force of the expressions.*

A very sad complaint truly! Sir *Isaac Newton*'s doctrine of fluxions is not so easy to read as a Gazette, or a Play! If I will discover not only the meaning but *the exact meaning*; not barely the force but *the full force of* his *expressions*; it requires, nay, *it often requires*— What? Twenty years study by candle-light? No. But it requires attention. Nay more; it requires *careful attention.* I must confess, I was always of this opinion, and therefore have submitted to this grievance of *careful attention*, and would advise other People to do likewise.

* Rep. of Lett. for *Apr.* p. 310.
† Pag. 311.
‡ Pag. 446.
§ Ibid.
¶ Ibid. p. 447.
‖ Coroll. 1.
** Corol. 4.
†† Rep. of Lett. for April, p. 311.

XX. So much for the difficulty Sir *Isaac Newton* puts upon those who study his Doctrine: But it seems he is to blame likewise for the ease, which he gives some persons in understanding him. "More * than once, to convey his intention the *easier* to those, who had been accustomed to the language of indivisibles, he has introduced some expressions analogous to the phraseology of that doctrine, when the brevity, he had prescribed to himself, occasion'd his not giving express notice of it."

Now, for my life, I cannot see that Sir *Isaac* was at all to blame. In order to be understood, he used a language familiar to those he wrote for. Mathematicians, when he first wrote, were universally accustomed to the method of indivisibles, and the terms and expressions therein used; not only those Mathematicians who approved that doctrine, but those also, who upon examination rejected it, as inaccurate and erroneous. If therefore Sir *Isaac Newton*, while *he addresses himself to those, who had been accustomed to indivisibles, accommodates himself to their language*; if, *to convey his intention the easier to them*, he *has introduced some expressions analogous to the phraseology of that doctrine*, or even sometimes used the very terms and expressions of that doctrine, I profess, I can see no harm in it. For, though he did not always *give express notice of it*, yet he took care to caution his reader more than once, in what sense he used those terms or expressions; and thereby he was always understood, and never misled any body that used *careful attention.* Though he never liked the doctrine of indivisibles; yet he had skill enough to avoid the errors of that doctrine, without the ridiculous affectation of running perpetually out of his way, to avoid every term or expression, that the Writers upon that subject had happened to make use of. He thought unintelligibleism, as great a fault as indivisibleism.

XXI. Mr. *Robins* † will not allow of this. He gives us two expressions of Sir *Isaac Newton*, where he *accommodates himself to the language* of indivisibles, not in order *to convey his intention the easier to those, who had been accustomed to that method*; (that were pardonable) but in one of them to speak, as if there were a time when a thing should happen, when his *words mean in reality no more, than that* the thing *will never* happen; and in the other to convey no intention at all, *no kind of meaning.*

— — — Dat inania verba,
Dat sine mente sonum. Fumum, mirabile dictu,
Faucibus ingentem vomit, & caligine sese
Involvit cæcâ, quales volitare figuras,
Morte obitâ, fama est solâ sub nocte per umbram.

One of these expressions, *diminuuntur in infinitum*, is taken from the conclusion of his general *scholium* to the first section of the *Principia*: The other, *summa ultima parallelogrammorum evanescentium*, is used in the first corollary to his third *lemma.*

Now, for my part, I am so unhappy as to differ from Mr. *Robins*, both in thinking that these two expressions are taken from the doctrine of indivisibles, or are in any way *analogous to the phraseology of that doctrine*; and likewise in supposing that they have *no kind of meaning.* Their meaning seems to me to be perfectly clear.

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* Ibid.

† Rep. of Lett. for April, p. 312.

The former expression, *diminuuntur in infinitum* often occurs in Sir *Isaac Newton*'s writings, particularly in the first section of his *Principia*, and signifies a * diminution *sine limite*, till the quantities do *evanescere*, vanish into nothing. In the *Analysis* written in or before the year 1669, we find the equivalent expressions, † *continuo diminuatur donec tandem evanescat*, and *diminuetur donec evanescat*, and *continuo decrescit, donec tandem penitus evanescat*; as also ‡ *in infinitum diminui & evanescere, sive esse nihil*, and sometimes simply *evanescere*, which last word is frequently used either by itself, or after the words *coeuntibus punctis, redeat in locum priorem*, or some such expression, to determine its meaning, both in the first section of the *Principia*, and in the introduction to the *Quadratures*; to say nothing of the like expressions used in the account of the *Commercium Epistolicum*. So that, if this expression have no meaning, or no clear meaning, the greater part of Sir *Isaac Newton*'s writings must be unintelligible.

The second expression, *summa ultima parallelogrammorum evanescentium*, or *summa ultima*, as Mr. *Robins* prudently chuses to quote it, he tells us, "in strict propriety of speech, has no kind of meaning, for it is really infinite."

To which I must beg leave to reply, that his assertion is not true; as also that the proof he gives of his assertion is manifestly false; and farther, that, if what Mr. *Robins* gives for a proof, were ever so true, yet it would by no means prove his assertion.

The sum of the parallelograms, *i. e.* the inscribed or circumscribed figures, during the approach of those figures to one another and to the curvilineal figure, is an aggregate of a finite number of parallelograms finite in magnitude: And as these parallelograms diminish in breadth, and augment in number, *sine limite*, their sum constantly approaches to the curvilineal figure, and at last, at the end of the approach, degenerates into that figure, which is therefore called the *figura ultima*, or *summa ultima*, the figure, or sum, to which they at last arrive. This is plainly Sir *Isaac Newton*'s meaning.

Mr. *Robins*, in order to prove that the *summa ultima has no kind of meaning*, tells us *it is really infinite*. But the circumscribed figure is finite at first, and constantly diminishes; and therefore can never grow *really infinite*. And the inscribed figure, though it constantly increases, yet it never passes the magnitude of the curvilineal figure, which is finite: Consequently, neither can the inscribed figure become *really infinite*. Mr. *Robins*'s proof is therefore manifestly false.

And if this proof were true, if the *summa ultima* were *really infinite*; yet this would by no means prove Mr. *Robins*'s assertion, that *the summa ultima in strict propriety of speech, has no kind of meaning*; unless the words *really infinite, in strict propriety of speech*, have *no kind of meaning*. But this, surely, is what no body can surmise, that Mr. *Robins* himself, who has laboured so much *to preserve propriety of expression*, should have used an expression, which is *in strict propriety of speech, has no kind of meaning*.

So far is that Gentleman from the fault, of having *no kind of meaning* in what he says, that he too often falls into the other extreme of having too much meaning, or rather too many meanings. For where Sir *Isaac Newton* has only one meaning, Mr. *Robins* has sometimes two or three; and what is still a greater misfortune, of those two or three meanings there is none to

* Cogita diminuendas sine limite. Scholium Sect. 1. Princip. sub finem.

† Demonstr. resol. æquat. ass.

‡ Præp. pro Reg. 1. dem.

be found that will answer his purpose. For instance, in the case before us, the *ultima summa* is said to be infinite, and to give more force to the expression, it is said to be *really infinite*. But what is *the genuine meaning* of these words *really infinite*? Is it, that the *summa ultima* is infinite in magnitude? That it contains an infinite area? If so, we have already shown this to be manifestly false.

Or does he mean, that the *summa ultima*, tho' of a finite magnitude, contains an infinite number of parallelograms? But to say *the number* of parallelograms *becomes infinitely great, is the express language of indivisibles.* Besides, if we may be allowed to say, the *summa ultima is really infinite*, because the number of parallelograms may exceed any number that shall be proposed; we may, with equal *propriety of speech*, affirm, that a line of an inch long *is really infinite*, because the number of parts into which it can be divided, may likewise exceed any number that shall be proposed.

XXII. Mr. *Robins* goes no thus. "To assert that any collection of these inscribed or circumscribed parallelograms can ever become actually equal to the curve," (curvilineal figure rather in *strict propriety of speech*) "is certainly an impropriety of speech; for equality can properly subsist only between figures distinct from each other."

And here, I could almost sit down and deplore my own misfortune, in having to do with a Writer, who so often, in a single sentence, almost with one dash of his pen, makes no conscience of cutting me out work for several pages. But the task, however uneasy, must be submitted to, or I should be unworthy the name I bear, of *Philalethes*.

I must therefore again observe, that Mr. *Robins*'s assertion in this sentence is utterly untrue; that the proof he gives of that assertion, is manifestly false, even by his own confession; and farther, that, if what he gives us for proof of his assertion, were true, yet it would be so far from proving his assertion, that it would prove the direct contrary of his assertion to be true.

Whether such collection of parallelograms ever is, or *ever can become* actually equal to the curvilineal figure, is not now the question. I have clearly expressed my sentiment upon this point, though in other words than what Mr. *Robins* would here impute to me, in several places, particularly in the *Republick of Letters* for *November* last, p. 377. and in that for *January*, p. 87, 88. which passages remain as yet unanswered. But let what I have there said, be true, or false, yet, as this collection of parallelograms is a finite area, and the curvilineal figure is likewise a finite area, it can *certainly* be no *impropriety of speech*, to assert that one is, much less that one *can ever become actually equal* to the other. Mr. *Robins*'s assertion is therefore untrue; let us see what we can make of his proof.

That proof is, *Equality can properly subsist only between figures distinct from each other.*

To which I must oppose the known axiom of *Euclid*, upon which is grounded the demonstration of many propositions in his *Elements, Que mutuo sibi congruunt, sunt æqualia.*

And Mr. *Robins* himself, in other places, acknowledges the same thing. In his *Discourse upon Fluxions*, p. 44. he tells us, "The primary method of comparing together the magnitudes of rectilinear spaces is by laying them one upon another." And presently after he says, "This method cannot be applied in comparing curvilinear spaces with rectilinear ones; because no part whatever of a curve line can be laid upon a streight line, so as wholly to coincide with it."

Also, in the *Republick of Letters* for *October*, p. 256. this Gentleman tells us, "One (way of comparing spaces) is by shewing them to consist of such, as by imposition on each other will appear to occupy the same space." And in the very same sentence he gives us to understand, it is *indubitable, that those things are equal, which have no difference.*

From all which I must conclude that, in Mr. *Robins*'s own opinion, when his judgment is not perverted by heat of controversy, *equality can properly subsist between figures* not *distinct from each other*; unless any man tell me, that figures, which are *laid upon one another*, which *wholly coincide* with one another, which *by imposition on each other occupy the same space*, are *distinct from each other*; unless any man will be hardy enough to deny, what Mr. *Robins* avers to be *indubitable, that those things are equal, which have no difference.*

But let us for once allow, that Mr. *Robins*'s proof is true, that *equality can properly subsist only between figures distinct from each other*; yet it will be no proof of his assertion, but of the contrary. For since *any collection of these inscribed or circumscribed parallelograms* is distinct from the curvilineal figure, equality may *properly subsist* between them, one may *in strict propriety of speech* affirm, that they may become equal to one another.

The remainder of this paragraph of Mr. *Robins* does not, I think, concern either myself, or my honoured friend and new Ally, the *Author of the Analyst.* It seems to be levelled against such only, as are tainted with the damnable heresy of indivisibles, which is far from being our case.

XXIII. I therefore proceed to the question put to me in page 313, to which I shall return an answer very clear and explicit.

"Does *Philalethes* here suppose the truth of Sir *Isaac Newton*'s demonstrations to depend on this actual equality of the variable quantity and its limit?" I do.

"He confesses our demonstrations to be just, which do not suppose this actual equality."

Yes, * *in the manner Mr.* Robins *defines, and treats of prime and ultimate ratios*, I allow his demonstrations to be just without this actual equality. But Sir *Isaac Newton* does not define and treat of prime and ultimate ratios, in the same manner with Mr. *Robins*; nor are Mr. *Robins*'s demonstrations at all like Sir *Isaac Newton*'s demonstration.

XXIV. "He (*Philalethes*) also says, that *the supposing this actual equality, seems greatly to exceed the method of the ancients in perspicuity, as well as in the conciseness of the demonstrations.*"

Here I must complain, that I am unfairly dealt with. The passage referred to, is to be found in the *Republick of Letters* for *November*, p. 373. where I first observe, that Sir *Isaac Newton takes in the consideration of a finite time*; secondly, that *thereby he greatly assists the imagination*; and I afterwards say, *On both these accounts*, (not, as Mr. *Robins* represents it, *by supposing this actual equality*) *this method seems greatly to exceed the method of the ancients, in perspicuity as well as in the conciseness of the demonstrations.*

But we are told, "That this method should be more perspicuous is impossible, the method of the ancients being perfect in that respect."

Herein I greatly differ from Mr. *Robins.* A telescope composed of an hundred glasses will not be equally diaphanous with one composed of two; though every one of the hundred

* Rep. of Let. for *Jan.* p. 79.

glasses, singly taken, be equally transparent with each of the two. Nor can a demonstration, consisting of an hundred steps, be equally perspicuous with one consisting of three only; though every one of the hundred steps, singly taken, be equally clear with each of the three.

Besides, a direct demonstration is certainly, *cæteris paribus*, more perspicuous than an indirect one, which reduced the reader *ad absurdam.* In the former, the *Reader* may see that every step tends towards the conclusion: In the latter, he is generally as it were hoodwink'd and led quite a way from the conclusion itself, till he comes to some absurdity, which he must either acknowledge, or admit the conclusion to be true; as finding no possibility of avoiding either the one or the other. Now, as Sir *Isaac Newton*'s first and fundamental *lemma*, though demonstrated *ab absurdo*, is yet a very short and easy one, in which the imagination is led towards the conclusion by the consideration of a finite time, even before one begins the demonstration; and almost all his other demonstrations are direct ones, and are likewise short and clear, I may with reason say, that his method greatly exceeds the method of the ancients, where the demonstrations are indirect, tedious and perplexed, in perspicuity as well as conciseness. Indeed, the perspicuity of a demonstration is always in proportion to its conciseness, if the single steps be clear.

XXV. But it is said, "Certainly there are not in their method, what *Philalethes* (though I think without reason) insinuated of this, any demonstrated truths, that must be owned, tho' we do not perfectly see every step, by which the thing is brought about."

The passage of mine here alluded to, is not, as the Reader may imagine, and as Mr. *Robins* seems to design he should imagine, any way connected to, or so much as to be found in the same treatise with the passage that was last under consideration, wherein I spoke of Sir *Isaac Newton*'s method as exceeding the method of the ancients, in perspicuity, as well as in the conciseness of the demonstrations. That was contained in the *Republick of Letters* for *January* last, p. 87. and runs in these words.

"This equality therefore we are obliged to acknowledge, although we should not be able, by stretch of imagination, to pursue these figures, and, as it were, to keep them in sight all the way, till the very point of time that they arrive at this equality. For a demonstrated truth must be owned, though we do not perfectly see every step, by which the thing is brought about."

It is easy to see, that the steps spoken of in this passage, are not the steps of the demonstration, but the steps of the imagination; not the steps by which the thing is proved to be true, (that proof is supposed to be already over and finished) but the steps by which the imagination can conceive, and, as it were, see the thing to be brought about.

Now, I say, there are in this method of the ancients many demonstrated truths, that must be owned, though it be utterly impossible for the imagination to see how, or by what steps, the thing is brought about.

That the aggregate of three equal cones is equal to a cylinder of the same base and height, is a demonstrated truth in the method of the ancients; and though it be very easy to see that the steps of the demonstration are true and certain, yet it will not be easy for the imagination to see how, or by what steps, these three cones can be united to compose the cylinder.

Also, if one of these cones, keeping always the same base, be supposed gradually to increase in height, till its altitude become triple of the altitude of the cylinder; it is then a demonstrated truth, that such cone will be at last equal to the cylinder; and yet the imagination will not be able to perceive either this equality, or how it is brought about.

It is likewise a demonstrated truth, that the 1000 external angles of a rectilineal figure of a thousand sides, whether those side are all equal or any how unequal, will be equal to four right angles; and yet the imagination cannot be brought to see, how the thousand angles compose the four, in any one of these cases; much less in the endless variety of them that might be produced. Nor indeed can the imagination represent to itself a figure of 1000 sides, so as to distinguish it from another of 1001.

The remainder of this paragraph and the next is chiefly taken up, with representing what we have said, about the motion of a point describing a line, as something *intricate, sufficiently intricate, involved and perplexed, subtle, complex, not to say confused*, such as *will bring us upon the very borders of indivisibles, and* not only so, but will *render us perpetually obnoxious to the absurdities of that doctrine*. Now Heaven preserve all honest men from indivisibles, whose *very borders* are so dangerous!

Nulli fas casto sceleratum insistere limen.

One may travel, not only to the borders of *France*, or *Turky*, by through the heart of those regions of slaves; nay one may reside among them for years together, and yet not be rendered *perpetually obnoxious* to arbitrary power.

XXVI. In the four following pages I find nothing relating to me, except only that in p. 316 it is said, Sir *Isaac Newton* "draws a parallel between this case," (the case of vanishing quantities) "and the case of quantities supposed to augment without end." And a little after we are told, "*Philalethes* is very unwilling to allow this intended for an exact parallel." Mr. *Robins* might have said, *Philalethes* has clearly * proved, that this was not intended for an exact parallel.

"But, says Mr. *Robins*, Sir *Isaac Newton* expresly affirms, his intention of introducing it was to render more clear the thing, he had been immediately speaking of, that is, the nature of the ultimate ratios of quantities decreasing without limit."

To this I reply, that although Sir *Isaac's* intention was to *render more clear* what went before, yet that will not prove this to be *an exact parallel* to what went before. Every illustration is not *an exact parallel*. It is sufficient, if the illustration agrees with the thing to be illustrated, in some one particular, which needed to be rendred more clear. So here the quantities *supposed to augment without end*, agree with the quantities *decreasing without limit*, in this one particular, that the *quantitates ultimæ* cannot be assigned in one case, any more than in the other: but surely the case of quantities increasing without limit cannot be an exact parallel to the case of quantities decreasing without limit. Besides, I have † shown, that the ratio of the decreasing quantities may arrive at its limit; but that the ratio of the increasing quantities can never arrive at its limit: so that these two cases are far from being exactly parallel. Indeed, Mr. *Robins* himself, presently after, indirectly confesses this to be true. "It is true, says he, the quantities in one case may be reduced absolutedly to nothing, but in the other can never be extended to an infinite magnitude."

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* Repub. of Letters for *Nov.* p. 382, 383.

† Ibid.

Now, when the decreasing quantities are reduced absolutely to nothing, I have clearly shown; not only in the last quoted place, but more particularly still, and in a manner uncontroverted by Mr. *Robins*, in the *Republick of Letters* for *January* last, p. 76, 77. how the ratio of those quantities will arrive at its limit. And if that be not sufficient, the same thing may be proved from what that Gentleman himself lays down in this very page. For after telling us, that "these ratios can never pass, nor ever reach those limits, before the quantities are diminished to nothing," he now acknowledges that *the quantities may be reduced absolutely to nothing*: consequently the ratio of those quantities will arrive at its limit.

And as it is agreed on both sides, that the ratio of increasing quantities will never arrive at its limit, I may very safely affirm, that the two cases are not exactly parallel.

I must not forget to observe, that what Sir *Isaac* by this illustration was endeavouring to render more clear, was not, as Mr. *Robins expresly affirms, the nature of the ultimate ratio's of quantities decreasing without limit* but only that, when quantities decrease without limit, the *quantitates ultimæ* cannot be assigned.

The rest of what is said in this and the two next pages, about those decreasing quantities, and their not being the *subjects* of the *ratio ultima*, as well as what is so often repeated in other places, that they cannot, or may not, or will not *subsist*, or *exist*, under that ratio, may serve to shew how well Mr. *Robins* can fight with his own shadow; but does not at all concern me, or my learned friend, the Author of the *Analyst*. We hold no such opinion.

XXVII. But in page 319 that Gentleman is again accused by Mr. *Robins* of committing a twofold error; of neither of which I can in my conscience allow him to be guilty, in the manner those errors are represented by Mr. *Robins*; and when rightly represented, if they are really errors, those errors are much more justly to be imputed to me, than to him. Considering the bickerings we have had together, who could ever have expected, that excellent person should have proved so great a Confessor in my cause? What could move Mr. *Robins* to use him thus severely, on account of a controversy, in which he was utterly unconcerned? Whatever his mathematical sins have been, they were committed long before Mr. *Robins*'s *Discourse* was written; and yet he was never once mentioned in that *Discourse*. Besides, those sins have been atoned for by so severe a penance, that, I profess, my bowels yearn for him, every time I think of it. Has Mr. *Robins* less compassion? I cannot think it. I am no stranger to this Gentleman's generosity and good nature, which will not easily permit him to insult over the unfortunate and unsuccessful. What then can be the meaning of his being now, just now, so hardly treated? It cannot be sure, that, with regard to certain points, it is thought easier to attack him, than to reply to me. For, though by this means the trouble of answering my arguments may be saved, yet will not that learned person be able to produce as good, or better arguments in vindication of his opinions?

Are his abilities to be judged of by his making a bad defence, when unfortunately he happened to be engaged in a bad cause, any more than those of Mr. *Robins*, from his making a worse defence in the like case? Give the Author of the *Analyst* the right side of the question, and he may be turned loose against any disputant in *Europe*. Had this been the case, when he first attacked the *British* Mathematicians, it would by no means have been safe for Mr. *Robins*, or me, to have encountered him.

—*Non illi se quisquam impune tulisset*

Obvius armato. Stetimus tela aspera contra,
Contulimusq; manus: Experto credite, quantus
In clypeum adsurgat, quo turbine torqueat hastam.

But as I know not whether he has at this time leisure or inclination to defend himself, I shall, as I at first engaged, use my best endeavours to do it for him.

The first of these errors is said to consist "in imagining, that the operation, which the increments are by Sir *Isaac Newton* supposed to undergo in order to have their ultimate proportion assigned, and which he describes by the verb *evanescant*, is confined to that point of time only, at which the increments are actually gone and abolished."

And here, before either the *Author of the Analyst*, or I *Philalethes*, can plead guilty, or not guilty to this charge, it will be necessary to understand what the error is. In order whereto, I must beg leave to ask one or two questions.

1. What is meant by *the operation, which the increments are supposed to undergo*?

2. What is *that point of time only, at which the increments are actually gone and abolished*?

1. If by this *operation* is meant a gradual diminution of the increments, ending in their total abolition or vanishing into nothing; I cannot find that Sir *Isaac Newton* has ever *described* this *operation by the verb evanescant.* What he *describes by the verb evanescant*, is only the end of that diminution, the vanishing into nothing; not the diminution itself, which precedes, and which ends in that vanishing. Though the diminution, when it is not mentioned, is sometimes implied by the verb *evanescant*; yet it is never described by that verb.

Much less will it be found, that either the *Author of the Analyst*, or myself, have ever confined such a gradual diminution to one *point of time only.* We are not so weak, as not to know, that a gradual diminution requires a finite time.

And if, by this *operation*, is meant only the end of the gradual diminution of the increments, *i. e.* their total abolition, their vanishing into nothing, which Sir *Isaac Newton describes by the verb evanescant*; I shall then readily admit, that the *Author of the Analyst*, or at least, that I *Philalethes* have imagined, that this *operation* is confined to one *point of time only*: But it is to that point of time, at which the increments vanish into nothing; not to a point of time, *at which the increments are actually gone and abolished.* For

2. It is easy to conceive, that the point of time at which the increments vanish, is one, determinate, only point of time: But I can have no conception of one *point of time only, at which the increments are actually gone and abolished.* When once they are gone and abolished, if we take any point of time at pleasure, during any finite time how great soever, we may truly say, at this point of time the increments are actually gone and abolished; but not at this *point of time only.* A little more accuracy in *style and diction* would do none of us any hurt; we all need it at least as much as Sir *Isaac Newton.* But it is easy to see, that through this cloud of expression, though the *Author of the Analyst* is only named, yet *Philalethes* is the person really aimed at. I proceed therefore to the real point, which Mr. *Robins* has here in view.

That by the word *evanescere* Sir *Isaac Newton* understood, that decreasing quantities did at last vanish into nothing, or cease to exist, in an instant or point of time, was so clearly proved in the month of *November* * last, particularly by the words then quoted from him, *intelligendam esse rationem neque antequam evanescant, neque postea, sed quacum evanescunt*, that I never expected to have heard more of this part of the dispute, especially after Mr. *Robins* had once dropt it in his reply to me in *December* following.

But as this point is now revived, together with several others, which had likewise been dropt, as well as many new ones *now* first brought into controversy; I shall once more resume the consideration of it, and shall confirm my sentiment by new authorities taken partly from Sir *Isaac Newton*, and partly from his great interpreter Mr. *Robins*.

I assert then, That quantities, which, by a continual diminution, at last vanish into nothing and cease to exist, are by Sir *Isaac Newton* understood to vanish in an instant or point of time. Before that instant, they are finite quantities, and after that instant they are nothing.

Likewise, quantities, which beginning from nothing, do by a continual increase, arrive at some finite magnitude, are by Sir *Isaac Newton* understood to begin to exist, to arise out of nothing, *nasci*, at one instant or point of time. Before that instant they are nothing, and after that instant they are finite quantities.

This will plainly appear from the following expressions, in which Sir *Isaac* speaks of the proportion of nascent or evanescent quantities, as one certain, determinate proportion, which consequently is confined to one instant of time, at which the quantities begin to exist, or cease to exist.

[a] *Rationem quacum evanescant.* [b] *Ratio quacum nascuntur.* [c] *Ordinatæ sunt ut arearum augmenta nascentia.* [d] *Augmentum evanescens* B b *erit ad augmentum evanescens* E e*, ut* $\mathrm{AB} \times \mathrm{FB}$ *ad* $\mathrm{AE} \times \mathrm{PE}$. [e] *Incrementum primum seu differentia prima cui nascenti proportionalis est ejus fluxio prima.* [f] *Incrementum secundum, seu differentia secunda cui nascenti proportionalis est ejus fluxio secunda.* [g] *Incrementum tertium, seu differentia tertia cui nascenti fluxio tertia proportionalis est.*

In all these expressions the reader will observe, there is no such word as *prima*, or *ultima*, *primò*, or *ultimò*, to be found; though in other expressions those words are sometimes added by way of *emphasis*, which led Mr. *Robins* into the mistake of thinking them always necessary.

Any one therefore of these passages, much more all put together, is sufficient to shew, that, by *vanishing*, Sir *Isaac Newton* does not mean the same as *decreasing*; that he does not call the *quantities vanishing from the time he first ascribes to them a perpetual diminuation*, nor consequently supposes them to bear to one another an infinite number of different proportions, while they are *vanishing*; but that, having *decreased* by a continued motion, and with a variable ratio, they are understood to *vanish*, at the instant when they come to

* Republick of Letters for *November*, p. 384, 385.

[a] Schol. sect. 1. Princip.

[b] Ibid.

[c] Introd. ad Quadr. Curv.

[d] Ibid.

[e] Schol. Quadr. Curv.

[f] Ibid.

[g] Ibid.

nothing or cease to exist; at which instant their variable ratio arrives at and ends in that one determinate ratio, with which they are said to *vanish.*[h]

This distinction between *decreasing* and *vanishing* is farther confirmed by the following passages of Sir *Isaac Newton*, which I have already quoted on another occasion.

[i] *Continuo diminuatur donec tandem evanescat. Diminuetur donec evanescat. Continuo decrescit, donec tandem penitus evanescat. In infinitum diminui & evanescere.* [k] *Minuetur in infinitum & ultimo evanescet.* These plainly shew, that *vanishing* is only the end or conclusion of the constant decrease or diminuation.

To all which I must beg leave to add a few more quotations of another kind, by which Sir *Isaac Newton*'s intention is still more manifestly to be discerned.

[l] *Punctis coeuntibus angulus evanescet.* [m] *Angulo evanescente coincident arcæ.* [n] *Redeat jam* P b *in locum suum priorem* P B *ut augmenta illa evanescant.*

So many instances of all these sorts, but particularly of nascent or evanescent quantities having one only, determinate ratio, are to be found in the *Principia*, that no body can pretend to have read that book with understanding and *careful attention*, who has not observed them.

To these citations from Sir *Isaac Newton* I must add one or two from Mr. *Robins*, which plainly shew this sense of the word *vanishing* to be so common and natural, that this Gentleman himself cannot help falling into it, even while he is strongly contending that the word is to be taken in another meaning.

In one place he says, * the line DF may be supposed to advance towards AE with an uninterrupted motion, till the quadrilaterals quite disappear, or *vanish.* In another he tells us, † the quadrilaterals become vanishing quantities from the time, we first ascribe to them this perpetual diminution; that is, from the time they are quantities going to *vanish.* In a third he speaks of the INSTANT of their ‡ *vanishing.* In a fourth he says, § when the points A and C coalesce, the rectilinear angle under EAF must *vanish.* And presently after, speaking of several quantities, he takes notice that they ¶ *all vanish*, when the point C coincides with A. And in this very *Dissertation*, he gives us to understand that the term vanishing, ‖ absolutely signifies no more than going to *vanish*; where the participle *vanishing* is used in his own sense, and the term *vanish* is used in mine.

Having now clearly settled the meaning of the word vanish, both by the authority of Sir *Isaac Newton*, and that of Mr. *Robins*, I am ready to acknowledge, that when Sir *Isaac Newton*, instead of the expression, *continuo diminuatur, donec tandem evanescat*, uses the

[h] Rep. of Lett. for *Jan.*, p. 76, 77.
[i] Analysis per æquationes num. term. infinitas.
[k] Princip. sect. 1. Lem. 6.
[l] Princip. sect. 1. Lem. 7, and 8.
[m] Ibid. Lem. 9.
[n] Introd. at Quadr. Curv.
* Disc. p. 50.
† Ibid. p. 51.
‡ Republick of Letters *for* October, p. 257.
§ Ibid. p. 258.
¶ Ibid.
‖ P. 320.

term *evanescat* simply, he intends his reader should thereby understand, that the quantity was diminished before it came to *vanish*, though the word *vanish* does not signify diminishing. Just as if I should say, his Majesty arrived at *Hanover* at such an hour, or Prince *Eugene* died such a day, it would be understood that the King was on the way to *Hanover* before he arrived there, and that the Prince lived before he died; and yet arriving does not signify the same as travelling, nor is dying the same as living. But enough of this.

The second error here imputed by Mr. *Robins* to the *Author of the Analyst*, consists "in imagining, that by the ultimate ratio of varying quantities is meant a ratio, that these quantities do at some time or other exist under."

But how a person, who supposes the quantities to vanish into nothing, before they arrive at the ultimate ratio, and consequently makes the ultimate ratio to be a *proportion between nothings*, can be said to imagine, that the *quantities exist under that ratio*, is hard to conceive.

And it will be equally difficult to conceive, that a person, who makes the varying quantities to vanish at the instant that they arrive at the ultimate ratio, can be charged with imagining that the *quantities exist under that ratio.*

XXVIII. In page 320 we are told, *Philalethes* had no necessity, for avoiding the consequence charged upon this doctrine, to have recourse to that definition of a *nascent increment*, which follows. *A nascent increment is an increment just beginning to exist from nothing, or just beginning to be generated, but not yet arrived at any assignable magnitude, how small soever.*"

This is very true; and accordingly *Philalethes* did not give this definition *for avoiding* any *consequence charged upon this doctrine*, but in order to follow Sir *Isaac Newton.* * *Cave intellexeris particulas finitas. Particulæ finitæ non sunt momenta, sed quantitates ipsæ ex momentis genitæ. Intelligenda sunt principia* JAM JAM NASCENTIA *finitarum magnitudinum.*

XXIX. Mr. *Robins* may see by this, that *Philalethes* can read *latin*, and therefore may have *learn'd this doctrine solely from Sir* Isaac Newton*'s works*, as well as Mr. *Robins*, may as well be supposed to have taken the words *nascent* and *evanescent* from Sir *Isaac Newton* himself, as from *those who have writ upon this subject in our language.* But if Mr. *Robins* continues to † *dislike* those words, he is welcome, wherever he meets with them, to put in their room the *latin* ones *nascentes* and *evanescentes.* I know of no difference but in the sound.

XXX. In the next page, Mr. *Robins* seems to please himself with having once caught *Philalethes* in a contradiction. It were pity, methinks, to deny him this small gratification, being, perhaps the only one he is like to meet with in the whole controversy; and yet justice requires I should observe, that this seeming contradiction arises only from supposing *Philalethes* to *suppose* what he never had a thought of supposing, viz. *evanescent quantities subsisting at each point of the curve, which can be the subjects of the proportion between the ordinate, tangent, and subtangent.*

* Lemm. 2. Lib. II. Princip.

† Republick of Letters *for* April, p. 321.

What *Philalethes* constantly supposes is, not that those quantities subsist under this proportion; but the very contrary; that, at one and the same instant of time, they arrive at this proportion and vanish, they vanish and arrive at this proportion, as is manifest from twenty places of his writings.

XXXI. Mr *Robins* [a] *now proceeds to consider, what Sir* Isaac Newton *has called the momenta of quantities.* But, before I enter upon this consideration, I cannot but take some notice of the manner, in which he has been pleased to treat Sir *Isaac Newton*, under pretence of defending him.

Sir *Isaac*, in several places of his works, has strongly declared his dislike of indivisibles, as strongly as Mr. *Robins* himself has done it, and has given his reasons for that dislike. And yet this Gentleman, though he occasionally quotes the very passages, where that [b] dislike, and the [c] reasons for it, are express'd by Sir *Isaac Newton*; and likewise some other passages, in which Sir *Isaac* [d] cautions his reader, that the expressions he uses either for brevity, or the more easy conveying his notions, are not to be understood in the sense of indivisibles; notwithstanding all this, I say, Mr. *Robins* is perpetually [e] accusing him of making use not only of the *language* or what *resembles the language*, but even of the *sense*, the *principles*, the *Method* of indivisibles.

I must however observe, in justice to that Gentleman, that he did not always treat Sir *Isaac Newton* in this manner. It seems to be with unwillingness and reluctance, great necessity enforcing him thereto, that he is brought to use him so unhandsomely.

In the introduction to his *Discourse* upon Fluxions, having taken notice, that the method of indivisibles was *obscure and indistinct*, and *obnoxious to error*, he presently after tells us, "Sir *Isaac Newton* to avoid the imperfection, with which this method of indivisibles was justly charg'd, instituted an Analysis upon OTHER principles;" "has taught an Analysis free from all obscurity and indistinctness;" has moreover invented a synthetick form of reasoning, which is much more concise than the method of demonstrating used in these cases by the ancients, yet is equally distinct and conclusive;" "has very distinctly explained both these subjects, his method of fluxions, and his doctrine of prime and ultimate ratio's, the first in his treatise on the quadrature of curves, and the other in his Mathematical principles of natural philosophy; which methods have all the accuracy of the STRICTEST mathematical demonstration."

I might produce several other compliments of the like nature, bestowed by Mr. *Robins* upon Sir *Isaac Newton*, in the body of that *Discourse*; nor is there any reflection intermixed, to lessen the merit of his inventions, except the following; which, if it be written with that design, is however expressed in a very cautious and tender manner. "In this (the explanation of the term *momentum*) I shall be the more particular, beause Sir *Isaac Newton*'s definition of momenta, that they are the momentaneous increments or decrements of varying quantities,

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[a] P. 323.

[b] Repub. of Let. for *Oct.* p. 265. and Repub. of Let. for *Apr.* p. 292, 295, 311.

[c] Repub. of Let. for *Oct.* p. 265.

[d] Repub. of Let. for *Oct.* p. 258, 266. and Repub. of Let. for *Apr.* p. 317.

[e] Repub. of Let. for *Oct.* p. 258, 263, 264, 265, 266. and Repub. of Let. for *Apr.* p. 311, 312, 324, 327.

may POSSIBLY be thought obscure." All this while no mention is made of Sir *Isaac Newton*'s using indivisibles, or so much as any expression *analogous to the phraseology* of indivisibles.

But the *Minute Mathematician* happening to be published a little before Mr. *Robins*'s *Discourse*, and some of the best judges being of opinion, that *Philalethes* had rightly understood Sir *Isaac Newton*, where Mr. *Robins* had grievously mistaken him, this made it necessary to give an account, or rather publish a vindication of the *Discourse* upon fluxions, in the *Republick of Letters* for *October* last. In which Mr. *Robins* endeavoured to prove, with what success the learned may easily judge, that in some passages he had rightly interpreted Sir *Isaac Newton*, and that *Philalethes* had mistaken him. But in some other passages Mr. *Robins* seems to have found himself so manifestly in the wrong, as uttely to despair of reconciling his interpretation to what Sir *Isaac Newton* has delivered. This obliged him, rather than own himself at all in an error, to depart from that candid and ingenuous behaviour he had hitherto used, with respect to our common Master. Whenever *Philalethes* produced a decisive passage from Sir *Isaac Newton*, and urged it home, that there was no possibility of evading it; he weakly thought the business was done, that Mr. *Robins* would give up the point, silently at least, if not by an open and ingenuous acknowledgement of his mistake. But he knew little of his antagonist. Indivisibles was the word; and Sir *Isaac Newton* himself must be condemned, rather than *Philalethes* should be acquitted. Let him produce the strongest and clearest passages from Sir *Isaac*'s Writings, it was only saying they were written *in the sense of indivisibles*, and the affair was over.

At first, indeed, this charge against Sir *Isaac Newton* was not extended so far by a great deal, as it has since been carried. Observe how modestly this point is touched upon in the Month of *October*. "As [f] he thought fit (in compliance with the bad taste which then prevail'd) to continue the use of some loose and indistinct expressions RESEMBLING those of indivisibles, for which he has himself apologized, he expresly cautions us against misinterpreting him in this manner." Can any thing be more tender of Sir *Isaac Newton*'s reputation? Mr. *Robins* not only mentions Sir *Isaac*'s caution, which is true and well known; but an apology likewise, which it will be hard to find. Nor can we much complain of what is said soon after, [g] "It must be acknowledged, that Sir *Isaac Newton*'s description (of momenta) is CAPABLE of an interpretation too much resembling the language of indivisibles:" For, if his description is only *capable* of such an interpretation, we may understand that it is likewise *capable* of another interpretation. In the next page this charge is carried a little farther, "Sir *Isaac Newton* did sometimes make use of that method of reasoning." But even here this use of indivisibles is confined to the *Analysis* of 1666, to *the treatise of Quadratures, meaning as he at first writ it, from which we have a transcript in Dr.* Wallis, [h] *not as he afterwards corrected it in his own edition of that treatise*; and to *one proposition of his principles of natural philosophy*. The rest of Sir *Isaac*'s works are clear of this imputation, particularly his treatise of *Quadratures*, and the first section of his *Principia*. For Mr. *Robins* is pleased to inform us, that [i] "In his treatise of *Quadratures*, he freed his demonstrations from this defect, under which they first laboured," and soon after we are told, "he at length invented his other method of prime and

[f] Repub. of Let. for *Oct.* p. 258.
[g] Ibid. p. 263.
[h] Oper. Vol. II. p. 392.
[i] Repub. of Let. for *Oct.* p. 265.

ultimate ratios, and thereby entirely got over that difficulty." And Mr. *Robins* concludes with telling us, "When he (Sir *Isaac Newton*) purposely describes these methods," (that is, in the treatise of *Quadratures* and the first section of the *Principia*,) "and explains their principles, he is not only perfectly consistent with himself, but has also delivered his meaning with such perspicuity, the we need not have recourse to any deference for his authority to be fully satisfied of the truth of these doctrines."

And yet the infallible brand of indivisibleism, the term infinitely little, which is but once used in the *Analysis* of 1666, and but twice in the proposition condemned, Prop. 10. L. II. does frequently occur in the first section of the *Principia*, together with the expressions, infinitely great, infinitely greater, infinitely less; and is to be found in the admired treatise of *Quadratures* itself.

If it be said, that these two parts of Sir *Isaac*'s works are excepted from the censure, on account of the two several cautions given by him in the *Scholium* to the first section, *in sequentibus nolim indivisibilia, sed evanescentia divisibilia intelligi*, and *in sequentibus cave intelligas quantitates magnitudine determinatas, sed cogita semper diminuendas sine limite*; which cautions may be supposed to extend not only to the *Principia*, but to the book of *Quadratures*, especially the introduction, which probably was written after the *Principia*; I must then ask, since in each of these cautions the words *in sequentibus* are inferred, how comes it, that the first proposition of the second book of the *Principia*, is excepted out of Mr. *Robins*'s act of grace? Is not that proposition, equally with all the others of the *Principia*, comprehended *in sequentibus*? A candid reader, although these cautions had never been given by Sir *Isaac Newton*, would easily have undertood in what sense the words *infinite parvorum, infinite parva*, were used in that proposition, by the expressions, *minuatur in infinitum, evanescere, nihil erit*, which he soon after makes use of in the examples to that very proposition.

And in the *Analysis* of 1666, where the words, *infinite parvis*, are once used, it is equally easy to see what Sir *Isaac Newton* meant. For in that same *Analysis* we find the expressions above quoted, *in infinitum diminui & evanescere, sive esse nihil; continuo diminuatur donec tandem evanescat; diminuetur donec evanescat; continuo descrescit donec tandem penitus evanescat*: All which expressions are conformable to his constant doctrine, and are peculiarly his own, being never used, that I can learn, by any of the Writers upon indivisibles.

As in the above mentioned vindication of Mr. *Robins*'s *Discourse upon fluxions*, I found several passages in the *Minute Mathematician* obliquely glanced at, I thought proper to clear up and defend those passages in the *Republick of Letters* for *November* following. Where I not only take notice of the *accuracy and caution and perspicuity with which Sir* Isaac Newton *has delivered those parts of his own works, especially, in which the foundation of his doctrine is laid down*; but with an eye to Mr. *Robins*'s charge against him, of sometimes admitting indivisibles, I use the following words, "a fault, which I am bold to say, he was NEVER guilty of, and the bare suspicion of which he has endeavoured to prevent, both here, (in the *Scholium* to the first section of the *Principia*) and in many other parts of his writings, with the utmost caution."

This had so good an effect, that in Mr. *Robins*'s reply, in *December* following, the accusation against Sir *Isaac Newton* was entirely dropp'd, as well as several other parts of the controversy, and to the rest I replied in *January*, and Mr. *Robins* was silent for some months.

But in *April* last, that Gentleman, whether upon receiving some new reinforcements, or from what other cause I know not, not only replied to my paper of *January*; but resumed the consideration of the points he had waved in *December*, and introduced many new ones from my former pieces; and carried his charge against Sir *Isaac Newton* much farther than he had ever done before. I must needs say, he was forced so to do, unless he would acknowledge himself in more than one mistake.

In this piece, not only the charge against the [f] *Analysis* of 1666, and against the [g] account of fluxions published by Dr. *Wallis*, is renewed, and is extended to the [h] account of the *Commercium Epistolicum*; but even the [i] *Scholium* at the end of the first section of the *Principia* is found fault with, as containing *expressions analogous to the phraseology of the doctrine* of indivisibles, as well as the first [k] corollary to the third *Lemma*. And whereas, before, the description of moments, in the second *Lemma* of the second book of the *Principia*, was said only to be *capable of an interpretation too much resembling the language of indivisibles*; now we are plainly told, Sir *Isaac Newton*, [a] "did not scruple there to describe moments according to the sense of that doctrine, as he had done formerly, to be *incrementa vel decrementa momentanea*." And in another place we are informed that Sir *Isaac*, [b] "speaking of these *momenta*, as *incrementa momentanea*, in the sense of indivisibles, says, *particulæ finitæ non sunt momenta*." Where, I must not omit to observe, that this passage, badly written as it is, in *the unintelligible jargon of indivisibles*, yet serves well enough to be quoted against *Philalethes*. It is also affirmed, [c] "That Sir *Isaac Newton* has made use of indivisibles in the very sense of *Cavalerius*, and that the doctrine of moments was originally founded on them", nay, this is said to have been "proved from his own words". We are likewise told, [d] that "in the Lectures he read at *Cambridge* on his admirable discoveries in Opticks, he used indivisibles in his demonstrations".

To all which methinks the following answer were sufficient, though upon occasion a more particular reply may be given.

Sir *Isaac Newton*, in several parts of his works, particularly in the *Scholium* at the end of the first section of the *Principia*, and in the introduction to the *Quadratures*, has with great care distinguished between his own method and the method of indivisibles. He was so far from making use of this latter method, that in all his works, I may venture to assert, there is no passage, no expression to be found, that should reasonably make him so much as suspected of it. For, although he may sometimes use expressions, that, singly taken, as *capable of* such *an interpretation*; yet those expressions are capable likewise of a very different intepretation. And surely, if a writer makes use of a term or expression, in itself capable of two different senses; yet if one of those senses be agreeable to what goes before and after, and the other be directly contrary to it; if one sense be conformable to the constant tenour of his doctrine,

[f] Pag. 323, 326, 328, 330.
[g] Pag. 324.
[h] Pag. 323, 324.
[i] Pag. 311.
[k] Pag. 312.
[a] Republick of Letters *for* April, p. 324.
[b] Ibid. p. 327.
[c] Ibid. p. 326.
[d] Ibid. p. 329.

and the other manifestly contradict it; all the reasonable and impartial part of the world will agree, that the term or expression ought to be taken in the former sense, and not in the latter: Especially, if the author hath cautioned his reader against understanding him in the latter sense; more especially still, if such caution hath been often repeated.

By the term *momenta*, Mr. *Robins* * seems of opinion, that Sir *Isaac Newton* intended finite quantities. I maintain, that a moment, in the sense of Sir *Isaac Newton*, is a momentaneous increment or decrement of a flowing quantity, less than any finite quantity whatsoever, and proportional to the velocity of the flowing quantity.

As this dispute first arose from the demonstration of the second *lemma, Lib.* 2. *Princip.* methinks it were better for Mr. *Robins* and me, if we aim at truth alone, to confine ourself to what is said in that *lemma*, than to amuse ourselves and our readers with hunting over Sir *Isaac*'s other works, and examining every passage where the term *momentum* happens to be made use of: Especially, as in those passages that term is only transiently mentioned, but is here very particularly and distinctly explained, and proper cautions given against any possibility of misunderstanding it.

However, since Mr. *Robins* chuses to consider, how this term was used by Sir *Isaac Newton*, not at the time when he wrote the demonstration in question, but twenty years before that time; what he understood by it in the *Analysis* of 1666, rather than in the *Principia* published in 1687; republished in 1713, and again in 1726, the year before his death, without altering this *lemma*; so that we may look upon it as containing his last and most settled thoughts upon this subject; I am content to follow this Gentleman whither he thinks fit to lead me; but the better to prepare my reader for making a right judgment in this dispute, I must ask leave to make three or four previous observations.

The first is, that, at the beginning of this *Analysis*, Sir *Isaac* has advertised his reader, that the method he there delivers, is *rather briefly explained, than accurately demonstrated*: And just before he first introduces the term *momentum*, he has this expression. *In istis autem quo ego operor modo dicam brevissime.* And accordingly he takes the latter of the *two ways whereby*, as Mr. *Robins* informs us, *good writers explain the use of the terms they introduce: One by expresly defining them; another, when to avoid that formality, they convey the sense of such terms by their manner of using them.*

But the author of the account of the *Commercium Epistolicum*, said by Mr. *Robins* to be Sir *Isaac Newton* himself, acquaints us that, *In the second lemma of the second book of the principles the elements of this calculus are demonstrated synthetically.* For which reason alone, if there were no other, methinks it were better to content ourselves with what is said in this *lemma*, than to have recourse to that *Analysis*.

Secondly, Mr. *Robins* takes it for granted, that, when Sir *Isaac Newton* uses the words *infinitely little*, he always means an indivisible. But I affirm, that, in Sir *Isaac*'s works, that expression never signifies an indivisible, except only when he uses it in condemning indivisibles. For, although the expressions infinitely little and infinitely great are sometimes used by Sir *Isaac Newton*, as well as by the Writers upon indivisibles; yet the sense, in which he uses those expressions, is very different from what they understand them in.

By an infinitely little quantity, I apprehend them to mean a quantity actually existing, fixed, determinate, invariable, indivisible, less than any finite quantity whatsoever. And I

* Repub. of Let. *Apr.* p. 325, 327.

agree with Mr. *Robins* that is nonsense, it implies a contradiction. I can have no conception of a quantity less than any quantity I can conceive. To assert this were to say, that a quantity may be conceived to be less than itself.

But Sir *Isaac Newton* by an infinitely little quantity means a variable, divisible quantity, that, by a constant diminution, is conceived to become less than any finite quantity whatsoever, and at last to vanish into nothing.

And by a quantity infinitely great, I take the Writers upon indivisibles to mean, a quantity actually existing, fixed, determinate, invariable, immutable, greater than any finite quantity whatsoever. Which is likewise a contradictory and impossible idea. I can have no conception of a quantity greater than any quantity I can conceive. To say this were to assert, that a quantity may be conceived to be greater than itself.

But Sir *Isaac Newton*, by a quantity infinitely great, means a variable quantity, that, by a constant increase, may be conceived to become greater than any finite quantity whatsoever.

Thirdly, I must take notice, that Sir *Isaac* sometimes speaks of these *momenta* as nascent quantities, and sometimes as evanescent. In which soever of these lights they are considered, it answers the same end.

But it will be of use to observe, that when the *momenta* are considered as nascent, they may, by flowing from the instant of their origin, during a finite time, arrive at a finite magnitude. During that finite time their proportion constantly varies: And it is only at the first instant of that finite time, that they are in the same proportion with the fluxions. The proportion of the fluxions is the proportion with which the momenta, do *nasci*, begin to exist.

And when the *momenta* are considered as evanescent, we may conceive them to have been once of a finite magnitude, and by decreasing for a finite time, at last to vanish into nothing. During that finite time their proportion constantly varied: And it is only at the last instant of that finite time, that they arrive at the same proportion with the fluxions. That is the proportion with which they vanish, or cease to exist.

In the former of these lights they are considered by Sir *Isaac*, in the *lemma* and demonstration in question. They are spoke of in the second light, in the *Quadratura Curvarum*, and the *Analysis* of 1666. And in the first section of the *Principia*, particularly in the *Scholium* at the end of that section, and likewise in the introduction to the *Quadratures*, they are considered sometimes as nascent, sometimes as evanescent. From not rightly attending to the sense of those passages, where the *momenta* are considered as having once been finite quantities, and being now by a constant diminution arrived at the point of evanescence, I apprehend Mr. *Robins* to have been led into the gross mistake, of taking the *momenta evanescentia* for finite quantities.

Fourthy, I must take leave to put Mr. *Robins* in mind of what he himself has quoted from the *Phil. Trans.* as the words of Sir *Isaac Newton.* "In his calculus there is but one infinitely little quantity represented by a symbol, the symbol *o*.

"Mr. *Newton* used the letter *o* in his *Analysis*, and in his book of *Quadratures*, and in his *Principia Philosophiæ*, and still uses it in the very same sense as at first."

From which it follows, that we need only consult the *Principia*, or the book of *Quadratures*, to see in what sense the letter *o* is there used: By that we shall know the sense it is used in, in the *Analysis.* In which soever of these three treatises, the *momenta* are most fully and clearly described, it is to that undoubtedly we ought to have recourse, in order to see in what sense they are used in all the three. We must therefore have recourse to the *Principia*, and particularly to the second *lemma, lib.* 2. where only there is any description of them given. In the other of these treatises they are transiently mentioned, but no description is given, that I know of.

This being premised, I come to make out my description of a moment, from what Sir *Isaac Newton* says in this *lemma.* That description consists of three parts.

1. A moment is a momentaneous increment or decrement of a flowing quantity; *Has quantitates ut indeterminatas & instabiles, & quasi motu fluxuve perpetuo crescentes vel decrescentes hic considero; & earum incrementa vel decrementa momentanea sub nomine momentorum intelligo*;

2. Is less than any finite quantity; *Cave tamen intellexeris particulas finitas. Particulæ finitæ non sunt momenta, sed quantitates ipsæ ex momentis genitæ. Intelligenda sunt principia jamjam nascentia finitarum magnitudinum.*

3. And is proportional to the velocity of the flowing quantity. *Neq; enim spectatur in hoc lemmate magnitudo momentorum, sed prima nascentium proportio. Eodem recidit si loco momentorum usurpentur vel velocitates incrementorum, vel finitæ quævis quantitates velocitatibus hisce proportionales.* To make this perfectly clear, we may observe, that Sir *Isaac*'s intention is only to take the first proportion of the nascent moments: And when he tells us, it does *eodem recidere*, his meaning is, that the very same proportion will be obtained, whether we take, for the terms of our proportion, the moments, or the velocities, or any finite quantities whatsoever proportional to those velocities, *i. e.* that the moments are proportional to the velocities.

It will perhaps be said, that, as the quantities here considered by Sir *Isaac*, are indeterminate and instable, and increasing by a perpetual flux or motion; so likewise their momentaneous increments, or moments, are to be consider'd as indeterminate and instable, and perpetually flowing or increasing; and consequently will become finite quantities, as Mr. *Robins* supposes them to be. But to this I answer, that, altho' the moments will by flowing become finite; yet, as we have taken notice in our third observation, when they are become finite, they will not have the same proportion as the velocities, the proportion of the velocities being that of the nascent moments only, being only that with which the moments begin to exist. If you take their proportion at any other instant, than that of their origin, it will not *eodem recidere*, as if you take the proportion of the velocities.

So likewise, if the moments, after becoming finite quantities, be supposed again to decrease, and by a constant diminution at last to vanish indeed to nothing; their variable proportion, while they are finite quantities, will be different from that of the velocities; and they will arrive at the proportion of the velocities, at the instant of their vanishing only, or the evanescent moments only will be proportional to the velocities.

Perfectly agreeable to this explication is the passage above quoted * from the account of the *Commercium Epistolicum*: But as that passage has been already sufficiently explained, and has been applied to vanishing quantities in general, there is no need of being particular in the application of it to evanescent moments.

From what has been said upon this subject, it seems very plain, that the *momenta* described in this *lemma* are not finite quantities, nor yet indivisibles: And as these *momenta* were always used by Sir *Isaac Newton* in the same sense as at first, it follows that the *momenta*

* Repub. of Letters for *July*, p. 54.

in the book of *Quadratures* and in the *Analysis* of 1666, are the very same as those of this *lemma*. Therefore from this alone, although we had given no other argument, we may safely conclude against Mr. *Robins* that the *momenta* of that *Analysis*, though called infinitely little, are not indivisibles. It only remains to consider one or two passages, that Mr. *Robins* has produced in favour of his opinion.

XXXII. The first is taken from the account of the *Commercium Epistolicum*, and is thus quoted by Mr. *Robins*. * "He (Sir *Isaac Newton*) says that *he there called the moment of a line a point in the sense of* Cavalerius*, and the moment of an area a line in the same sense*."

Now the passage itself runs thus, † "The moment of a line he called a point in the sense of *Cavallerius*, THO' it be not a geometrical point, but a line infinitely short, and the moment of an area or superficies he called a line in the sense of *Cavallerius*, THO' it be not a geometrical line, but a superficies infinitely narrow."

If my reader should be curious to enquire, for what end or purpose Mr. *Robins* chose rather to truncate this passage, in the manner above set down, than to give it entire; I can easily inform him. He was willing it should be thought, that, in this *Analysis*, Sir *Isaac Newton* meant the very same thing by the moment of a line, as *Cavallerius* meant by a point; and that Sir *Isaac*, by the moment of a surface, meant the very same thing, as *Cavallerius* meant by a line. For he does not scruple to tell us, a page or two after, ‡ "That Sir *Isaac Newton* has made use of indivisibles, in the very sense of *Cavallerius*, and that the doctrine of moments was originally founded on them, we have already proved from his own words."

Where he intends this very passage, taking it for granted, that the Account of the *Commercium Epistolicum* was written by Sir *Isaac Newton*.

But, from the passage taken entire, it plainly appears, that, what Sir *Isaac* meant, by the moment of a line, was not the very same as *Cavallerius* meant by a point, but was a *line infinitely short*; and that what Sir *Isaac* meant by the moment of a surface, was not the very same as *Cavallerius* meant by a line, but was a superficies infinitely narrow: And consequently Sir *Isaac Newton has* not *made use of indivisibles in the very sense of* Cavallerius.

All that is intended by these words, *in the sense of* Cavallerius, is no more than this: what Mr. *Newton* calls the moment of a line, would by *Cavallerius*, or any follower of *Cavallerius*, who wrote after his manner, and spoke his *sense*, be called a point; though it is not a point, but a line. And what Mr. *Newton* calls the moment of a surface, would also by *Cavallerius*, or his followers, be called a line; though it is not a line, but a surface.

Just after the same manner, it is said, § xo and dx, signify the same moment; and in other places moments or differences are spoken of, as equivalent terms, signifying only that what Sir *Isaac Newton* calls moments in his method, answer to what are called differences in the method of Monsieur *Leibnitz*. So also the method of fluxions, and the differential method of Monsieur *Leibnitz*, are said to be one and the same method, in the report made to the *Royal Society* by the Committee appointed to examine the old letters and papers, of which an account is given in the *Commerc. Epistol.* tho', as Sir *Isaac Newton* observes in the *Scholium*

* Rep. of Lett. for *Apr.* p. 323.
† Phil. Trans. N°. 342. p. 178.
‡ P. 326.
§ Phil. Trans. N°. 342. p. 204.

at the end of *Lemm. 2. Libr.* II. *Princip.* Mr. *Leibnitz*'s method differs from that of fluxions, *in verborum & notarum formulis*, and also in the *idea generationis quantitatum*, Mr. *Leibnitz* considering quantities to be composed of differences added together, and Sir *Isaac Newton* conceiving them to grow and increase by a continual flux.

Mr. *Robins*, after the above-mentioned quotation, tells us, "The passage in the book, to which this relates, is as follows. *Nec vereor loqui de unitate in punctis, sive lineis infinite parvis, siquidem proportiones ibi jam contemplantur Geometræ, dum utuntur methodis indivisibilium.*"

This, I think is not true. For the passage, to which the quotation relates, is manifestly the following. "Sed notandum est quod unitas ista quæ pro momento ponitur est superficies cum de solinis, & linea cum de superficiebus, & punctum cum de lineis (ut in hoc exemplo) agitur." In which passage the moment o is understood, though not express'd, as is declared in the account of the *Comm. Epistolicum.*

XXXIII. The next passage alledged by Mr. *Robins* is this, "because we have no ideas of infinitely little quantities, he (Sir *Isaac Newton*) introduced fluxions into his method, that it might proceed by finite quantities as much as possible."

I do not apprehend what Mr. *Robins* would infer from this passage. If he means that Sir *Isaac Newton* introduced fluxions on purpose to avoid indivisibles, I agree with him. But then I must observe, that Sir *Isaac* has already invented the method of fluxions, when he wrote the *Analysis*, as Mr. *Robins* himself * acknowledges. Consequently, he had then found a way of avoiding indivisibles; and when he had invented fluxions on purpose to avoid indivisibles, it cannot reasonably be supposed he would still use indivisibles.

So far was he from this, that he was for *proceeding by finite quantities as much as possible.* And accordingly we find him using no other than finite quantities, except that he supposes these finite quantities to diminish *ad infinitum* and vanish, in order to find their last proportions.

XXXIV. But we are told, "he had not at the first discover'd his doctrine of prime and ultimate ratios, which entirely rejects indivisibles."

I am glad with all my heart to hear, that *the doctrine of prime and ultimate ratios entirely rejects indivisibles.* Surely, it will not now be pretended, that the second *lemma* of the second book of the *Principia*, or any part of it, is written *in the sense of indivisibles*; much less that any part of the first section of the *Principia*, is written in such a sense. As the only design of that *lemma* is to determine the first proportion of the nascent moments, and as the sole business of the first section of the *Principia* is to deliver and explain the doctrine of first and last ratio's in general, indivisibles must in both these by *entirely rejected.*

But is it true, that, *at first*, when the *Analysis* of 1666 was written, Sir *Isaac Newton had not discovered his doctrine of prime and ultimate ratios*? I believe not.

Mr. *Robins* tells us a few pages farther, "We are sure he had a part of it in 1669, on account of a demonstration added at the end of his *Analysis.*" This is something. But I will tell Mr. *Robins* how he may be *sure*, very sure, that Sir *Isaac* had the whole of this method, at the time he wrote that *Analysis.*

* Rep. of Let. for Apr. p. 328.

That Mr. *Robins* is possessed of the whole of this doctrine, no doubt can be made, after he has so *easily and elegantly* explained it in his *Discourse* upon that subject, *and that by a method not yielding in brevity to the artless inconclusive process by indivisibles.* I would therefore be glad to know of him, what course he would take, to determine the prime and ultimate ratio of the *momenta* mentioned in the *Analysis* in question.

I suppose he will tell me, his answer is already written. He will take the same course, as Sir *Isaac* has taken * *in his treatise of Quadratures*, where *he* has *freed his demonstrations from this defect*, (the use of indivisibles) *under which they first laboured.* He will suppose his moments to be at first finite, and having finished his calculation, he will then suppose the moments to be diminished *ad infinitum* and vanish into nothing, and will thereby obtain the last proportion of those moments.

But is not this the very method taken in the *Analysis* of 1666? Does not the Writer of the account of the *Commercium Epistolicum*, that is, Sir *Isaac Newton* himself, as Mr. *Robins* acquaints us, does he not, I say, distinctly inform us, in the passage quoted from that account by Mr. *Robins* in the next † page, that this very method was taken in that *Analysis*? Does not he also say in the same account, p. 182? "This was Mr. *Newton*'s way of working in those days, when he wrote this compendium of his *Analysis*? And the SAME way of working he used in his book of Quadratures, and STILL uses to this day." Do we not accordingly see, that, in the demonstration of the first rule of that *Analysis*, Sir *Isaac* takes exactly the same method, and almost in the same words, as in the demonstration of the first Proposition of the *Quadratures*? If it be said, there is no mention of prime and ultimate ratios in the first of these demonstrations; we answer, neither are they mentioned in the other. But the method is the same. Sir *Isaac* could find the first and last proportions of variable quantities, without deafning our ears with the sound of prime and ultimate ratio's a thousand times repeated. This is easily seen not only in his *Analysis*, but in the tenth proposition of the second book of the *Principia*, by those who attend more to sense than sound, more to the meaning than the form of expression.

XXXV. Had Mr. *Robins* done thus, he would not have fallen into the mistake that follows, of thinking Sir *Isaac Newton* used the word moment in ‡ two senses, because, in demonstrating, he at first supposes a moment to be finite, and afterwards to decrease *in infinitum* and vanish; and in investigation, for making dispatch, he at once supposes his moment to be infinitely little. For this infinitely little moment is no other, than the former moment considered as diminished *in infinitum* and vanishing, as has been shown above more than once.

XXXVI. Mr. *Robins* informs us, that Sir *Isaac Newton* § "thought fit for the sake of brevity to introduce the term moment in the second book of his *Principia Philosophiæ*." The place here meant is the second *lemma* of the second book; where the doctrine of fluxions is likewise introduced, but for a different reason, according to Mr. *Robins.* ¶ "It seems, says

* Rep. of Lett. for *October*, p. 265.
† Rep. of Lett. for *April*, p. 324.
‡ Repub. of Let. for Apr. p. 324.
§ Ibid.
¶ Rep. of Let. for *April*, p. 330.

this Gentleman, as if he took notice of them chiefly, that a cypher, he thought fit to explain relating to them, might be understood."

I apprehend, Sir *Isaac* introduced the mention of fluxions here, in order to explain and demonstrate the foundation of that method, and that he introduced moments in order to serve for that foundation, his method of fluxions being built upon the proportions of those moments.

But let it be so, that moments were introduced *for the sake of brevity*, and fluxions for the sake of *a cypher*, what are we next to believe?

We are told, * Sir *Isaac Newton* "did not scruple there to describe moments according to the sense of that doctrine" (of indivisibles.) But, "he shews how to correct the idea arising from this description of moments."

Let us therefore for once, notwithstanding all that has been said to the contrary, admit this to be true, that Sir *Isaac*'s *momenta*, his *incrementa vel decrementa momentanea*, are *described according to the sense of the doctrine of indivisibles.* If the words, *incrementa momentanea* give us the idea of indivisibles, how is this idea to be *corrected*?

Mr. *Robins* gives the corrective to this idea in two parts.

The first part of the corrective is, "You must never consider their magnitudes, but their ultimate ratio," Now, how this corrective is to operate, in order to amend the idea already conceived of indivisible, or infinitely little quantities, I can by no means understand. If I must never consider the magnitude of the moments, how is it possible for me to know, that their magnitude is either finite, or infinitely little? This first part of the corrective seems to be good for nothing; I shall never get rid of my idea of infinitely little quantities by these means.

The second dose of Mr. *Robins*'s corrective looks much more promising. It runs thus. "It would come to the same thing, if instead of these moments you used the velocities of increase or decrease of quantities, which he (Sir *Isaac Newton*) is wont to call fluxions, or if you used any OTHER finite quantities proportional to these fluxions."

The efficacy of this remedy depends entirely upon a right use of the word, *other*. If this word is to be understood in contradistinction to the word velocities, that we may use velocities, or fluxions, (which are finite quantities) or any *other* finite quantities proportional to these fluxions; the corrective is of no manner of use, it will never rectify the idea I have entertain'd of moments being indivisibles. But it happily falls out, that this is by no means the natural purport of the words, any more than that it can at all suit Mr. *Robins*'s design.

Must we not therefore interpret the passage thus, that instead of these moments we may use any *other* finite quantities proportional to the fluxions? This indeed is clear and decisive, and manifestly implies that moments are finite quantities. Otherwise, it were absurd to say, instead of moments, any *other* finite quantities may be used. Consequently, since moments are thus shewn to be finite quantities, they cannot be infinitely little quantities, the idea of indivisibles is wholly removed, abolished, eradicated and extirpated.

But, alas! it unfortunately happens, that this word *other*, this efficacious, this important word is no where to be found in Sir *Isaac Newton*'s Prescription. Though he has the honour of it given him, yet, it is certain, it never entered the good old gentleman's thoughts, at least it never dropp'd from his pen. His words are, not *finitæ quævis* ALIÆ *quantitates*, but *finitæ*

* Ibid. p. 324.

quævis quantitates. And consequently the moments must still continue infinitely little, we may be *perpetually obnoxious to the absurdities of the doctrine of indivisibles*, notwithstanding any assistance from this corrective.

The reader perhaps will be curious to know, how this word came to be slipt into the quotation from Sir *Isaac Newton.* And possibly, if he is not throughly acquainted with Mr. *Robins*'s integrity and candor, he may be apt to suspect that Gentleman of some foul play. If so, I must undeceive him; though, I am sensible, it will be hard to bring him to believe, that the insertion of this word *other*, which so totally alters Sir *Isaac*'s sense to Mr. *Robins*'s purpose, is owing only to the zeal of an officious Compositor for the press. And yet I cannot but acquaint him, that, no sooner was notice given by a friend of this extraordinary proceeding, which was like to be interpreted very much to Mr. *Robins*'s disadvantage, than care was taken to rectify it, among the *Errata* of the press, in the *Republick of Letters* for the following month; where any one, who doubts of my veracity, may with his own eyes see this correction made, p. 325. lin. 8. *dele* other.

And when I have apprised that Gentleman, that there is a paragraph towards the bottom of the same page, which calls the above-mentioned permission of Sir *Isaac Newton* by the name of a *caution*, and pretends that by a and b he meant finite quantities, that is, as I understand, finite quantities only; which paragraph manifestly proceeds from the same hand, as had before inserted the word *other*, and is grounded solely upon that insertion; no doubt ought to be made, but that he will take care, for his own reputation, to place this whole paragraph likewise among the *errata.*

If any one should object how incredible it is, that a whole paragraph, which cannot possibly pass for an *Erratum*, and such a paragraph too, should be put in by a Compositor; I must desire him to observe, that this is no common Compositor: He must be a considerable Mathematician, as well as a Compositor; else he could never have been able to put in so pertinent a word, and so cleverly suited to his Author's purpose, as the word OTHER. And I must likewise observe, that his calling that a caution, which any man of learning would rather call a permission, or liberty given us by Sir *Isaac Newton*, is much more like a Compositor, than a person so *exact in the choice of his expressions* as Mr. *Robins.* For in that passage Sir *Isaac* does not *caution* us to use fluxions, or finite quantities proportional to them, instead of moments; he does not tell us we must use them; but gives us to understand we may use them; tells us it will come to the same thing, if we do use them.

I wish this same Compositor may not have imposed upon us in other places, particularly in the words *literal translation, coincidence* and *coinciding* in *December*, and *literally import* in this present *Dissertation*, p. 302. as likewise in the word *always*, p. 321. but especially in the words, *or necessarily imply*, p. 306. *cum multis aliis.*

XXXVII. Mr. *Robins* comes next to object to my demonstration, as he is pleased to call it, of the method for finding the *momentum* of a rectangle.

He says, I "have endeavoured to prove, that the moment of the rectangle is an arithmetick mean proportional between the contemporaneous increment and decrement of the same rectangle. But it has been shewn, that this is only true, when the sides augment in the same constant proportion."

I do not call to mind, that I have any where spoke of *the increment and decrement of the same rectangle*, as being *contemporaneous.* If I have done so, I was certainly not very *exact* *in the choice of* my *expressions.* The increment of one rectangle may be contemporaneous with the increment of another rectangle: But the increment of a rectangle cannot possibly be contemporaneous with the decrement of the same rectangle. However, let that pass.

I imagine, Mr. *Robins*'s meaning is, that supposing the moment, increment and decrement to be severally generated in equal particles of time, I have endeavoured to prove, that in this case, *the moment is an arithmetick mean proportional between the increment and decrement.*

But, says he, *it has been shewn, that this is only true, when the sides augment in the same constant proportion.*

Let it be so; at least, in the case of finite moments. But, in that *supposed demonstration*, I neve considered any case, where *the sides* do not *augment in the same constant proportion.* I there considered only that one case, in which the sides augment uniformly.

I was not then writing a *Discourse upon Fluxions*; but an answer to the *Author of the Analyst.* And that Author did not then appear to me, nor does he yet appear, after his reply to me, and his several answers to Mr. *Walton*, ever to have considered any other case, in his two objections to Sir *Isaac Newton*, than that only, in which the sides are uniformly augmented.

It told him therefore, the moment of the rectangle AB was different from the increment of the rectangle AB. And from his own positions, or such as by parity of reason he was obliged to acknowledge equally with his own, I * shewed him, that the increment of the rectangle AB added to the decrement of the same rectangle, made twice the moment of the rectangle; and consequently that the moment of the rectangle AB was $aB + bA$, the same as Sir *Isaac Newton* had determined. But throughout that argument here is no such word as demonstrate, or demonstration to be found; and the whole tenour of the passage shews it to be addressed to the *Author of the Analyst*, and to him only.

In the *Minute Mathematician* published a year afterwards, I have indeed called it a demonstration, nay a demonstration as strong as any in *Euclid*: but pray, good reader, mind the words. † "It is;" said I, addressing myself to the *Author of the Analyst*, "a demonstration AGAINST YOU as strong as any in *Euclid*, that the moment of the rectangle AB is a middle arithmetical proportional between the increment and decrement of the same rectangle AB."

By the words, *against you*, it is easy to see, this was not intended for a demonstration, strictly speaking; but for an *argumentum ad hominem*, and nothing else. And, if I am not greatly misinformed, Mr. *Robins* himself was once of this opinion; insomuch that, after he had inserted this very objection into his paper of *October* last, upon shewing it to a friend of his, who is likewise a lover of truth, which friend told him of these words and that they manifestly imported a meer *argumentum ad hominem*, he struck the whole objection out of his copy. How comes it now to be put in? Does he think the words, *against you*, were flipt in by the compositor? He will not find them among the *Errata.*

This proceeding of Mr. *Robins* seems to be the more extraordinary, because a person of his penetration might easily see, that, although in this *supposed demonstration* I had considered only one case, yet I had the other cases in my eye, and particularly was prepared to deduce the second case from the first by the help of the method of last proportions, whenever I

* Geometry no Friend to Infidelity, p. 46.

† P. 51.

should be called upon so to do, from what I had written in page 50, 53 of the same *Letter*; but especially after the declaration I had made in the *Minute Mathematician* p. 81., that *this was to guard against future objections.* At least, as the defect of this demonstration, if there were any in it, has since been supplied by a fuller and clearer demonstration published in *November* last, the former might well have been spared by Mr. *Robins.*

XXXVIII. In this last demonstration I have made use of finite moments, and hope therefore it is entirely to Mr. *Robins*'s mind. I see he has carefully perused it, but his nibbling at two or three passages, which, though no way material to the truth or clearness of that demonstration, I shall nevertheless, out of regard to the Objector, particularly consider.

First, Mr. *Robins* charges me with saying, * *that Sir* Isaac Newton *considers these moments as the differences of* Leibnitz.

I have said no such thing. What I have said is, † "There is likewise a third acceptation, in which we may take the quantities represented by a, b, c. For these LETTERS may be understood to represent the differences of Monsieur *Leibnitz.*" I had before taken notice, that those letters were by Sir *Isaac Newton put to signify either the moments, or the velocities, of the flowing quantities* A, B, C.

But if I had really done what Mr. *Robins* here imputes to me; all that could be meant by it is, that these *moments* are considered as analogous to the differences of Monsieur *Leibnitz*; not that they are exactly the same with those differences. And here, were it necessary, I could shelter myself under the authority of a writer, who makes no little figure in the *Republick of Letters*, who has likewise given the name of *momenta* to Monsieur *Leibnitz*'s differences. "In ‡ that method," (of indivisibles) says he, "these *momenta* are considered absolutely as parts, whereof their respective magnitudes are actually composed." Where it is observable, that, in three sentences, immediately following one another, the very same words, *these momenta*, are made use of. And yet in the first sentence, by *these momenta*, he means the *momenta* of Sir *Isaac Newton*; in the second, by these *momenta*, he means the differences of Mr. *Leibnitz*; and in the third, by *these momenta*, he again means the *momenta* of Sir *Isaac Newton.*

How other persons may be affected with such passages, I know not. For my self, I can never read them without joining in the lamentations of Mr. *Robins*, while, like another *Heraclitus*, or rather like a second *Jeremy*, he weeps over the *Geometrical Writers* and Sir *Isaac Newton.*

> —*Quis talia fando*
> Leibnitius, Grenius*ve, aut duræ Præsul* Iernes
> *Temperet a lachrymis! Manibus date lilia plenis;*
> *Tu* Newtonus *eras.*

How mournfully does he bewail their *negligence in regard to their style and diction*!

> *Qualis populeâ moerens Philomela sub umbrâ*
> *Amissos queritur foetus, quos durus arator*

* Republick of Letters *for* April, p. 326.

† Republick of Letters for *November*, p. 386.

‡ Discourse upon Fluxions, p. 77.

Observans nido implumes detraxit, at illa
Flet noctem, ramoque sedens miserabile carmen
Integrat, & moestis latè loca questibus implet.
Mirandum est unde ille oculis suffecerit humor!

XXXIX. The second thing Mr. *Robins* excepts against, is my asserting, that * "The Course taken by Sir *Isaac Newton* to find the *difference* of variable quantities, though not rigorously geometrical in the higher cases, yet approaches nearer to geometrick rigour than the method used by Mons. *Leibnitz*."

Here Mr. *Robins* does not dispute, that Sir *Isaac Newton's computation may come nearer the value of any quantity sought after than that of* Leibnitz. But he says, that *considered as a medium of demonstration to determine the* ABSOLUTE *value of such quantity, both will be totally, and therefore equally void of geometric rigour.*

Very true. But I never considered this course taken by Sir *Isaac Newton, as a medium of demonstration to determine the* ABSOLUTE *value*; all that is pretended by it, is to determine the value, not absolutely, nor in geometrick rigour, but *quam proximè*; to come nearer to the absolute, geometrick, rigorous value, that one can come, by following the method of Mons. *Leibnitz.*

In considering the letters a, b and c in this acceptation, as representing the differences of Mons. *Leibnitz*, I had an eye not only to the passage then quoted, *Utiusque* (*sc. methodi*) *fundamentum continetur in hoc lemmate*; but to the following passage of the Introduction to the *Quadratures. Fluxiones sunt* QUAM PROXIME *ut fluentium augmenta æqualibus temporis particulis quam minimis genita; &, ut* ACCURATE *loquar, sunt in prima ratione augmentorum nascentium.* The *quam proximè* has a view to Mr. *Leibnitz*'s method, and *accuratè* to his own.

XL. The next and last thing objected against, is my saying that † Sir *Isaac*'s Cas. 1. of this *lemma* is naturally to be understood in one of these two senses, that the sides of the rectangle flow either uniformly, or proportionably.

This, Mr. *Robins* says, ‡ "cannot be the natural interpretation of that case, because it is immediately quoted to prove the second case, where the augmentation is confessedly different."

But I take the natural sense of the first case to be that sense it will appear to a careful reader, upon perusing that case alone, though he never reads the second case at all. Mr. *Robins* seems to think Sir *Isaac Newton* so bad a writer, that his first case is not to be understood, till we have read the second; not his propositions to be intelligible, till we read the demonstration. I think otherwise; but I must own, *it often requires careful attention.* How to apply the first case, when understood in what I take to be its natural sense, to case the second, *where the augmentation is confessedly different*, has been clearly shown in the *Republick of Letters* for *November* last.

* Republick of Letters *for* April, p. 327.
† Republick of Letters *for* November, p. 392.
‡ Repub. of Let. for *Apr.* p. 327.

Mr. *Robins* goes on thus, "Nor can any reason be assigned to shew, why it should be thus understood."

But I apprehend the reader will find clear and sufficient reasons for thus understanding it, if he is pleased to peruse what I have said upon this *lemma*, in the *Republick of Letters* for *November* last, where he will find Sir *Isaac Newton*'s demonstration very distinctly and particularly explained, not only when the moments are taken in the sense I understand them in, but also in the sense Mr. *Robins* contends for, in taking moments for finite quantities.

To what is there delivered, it may, for Mr. *Robins*'s farther satisfaction, be added, that, in this demonstration, Sir *Isaac Newton* manifestly considers three different points of time: One, in which the sides of the rectangle are of the magnitudes $\mathrm{A}-\frac{1}{2}a$ and $\mathrm{B}-\frac{1}{2}b$; A second, in which they are of the magnitudes A and B; And a third, in which they are of the magnitudes $\mathrm{A}+\frac{1}{2}a$ and $\mathrm{B}+\frac{1}{2}b$.

Between the first and second point of time, he supposes one half of the moment a, and one half of the moment b to be gained: And between the second and third point of time he likewise supposes $\frac{1}{2}a$ and $\frac{1}{2}b$ to be gained.

Now these suppositions may be taken in two different senses: One, which arises from the words of the demonstration, only, without introducing any foreign consideration not expressed in the demonstration; and this may be called the natural sense: The other, besides the words of the demonstration, requires the introducing the method of first and last proportions, which though delivered before by Sir *Isaac Newton*, yet is not mentioned in this demonstration.

If we take the suppositions in the first sense, each half of the moment a will be actually contemporaneous with each half of the moment b, *i. e.* the sides of the rectangle must flow either uniformly or proportionably.

If we take them for the second sense, the sides may flow in any manner whatsoever, and yet, by the help of the method of first and last proportions, we may conceive the halves of the moments to be contemporaneous.

But perhaps Mr. *Robins* would have liked my reasoning better, if I had said, we must take $a\mathrm{B}+b\mathrm{A}$ for the moments, *because so much only is required for determining the ultimate ratio*; Sir *Isaac Newton* has taken such a method, *because by this means the superfluous rectangle is sooner disengaged from the demonstration: It is convenient, by neglecting some superfluous part of the increments, or by a proper addition to them, to form new quantities; the superfluous terms are rejected out of the increment of the power*, &c.

I must needs say, this is a far easier way of reasoning, than that I have taken: But Ἀιδέομαι Τρῶας χὶ Τρωάδας, and most of all I should fear the *Author of the Analyst.* For he, though he would readily grant me, that *it is* CONVENIENT *to neglect some superfluous part of the increments, to reject the superfluous terms*; though he might be candid enough to say, *The case indeed is difficult; There can be nothing done till you have got rid of* them; yet, I apprehend, he would not stick to tell me, that, although *the final cause or motive to this proceeding is very obvious*; yet, *it is not so obvious or easy to explain a just and legitimate reason for it, or shew it to be geometrical.*

I must confess, I should not know what to reply. But it matters not. It is time for us two to give over our weak and fruitless contention. Mr. *Robins* now appears in the field:

Mox ambos sua fata manent majore sub hoste.

XLII. Having now replied to all that concerns my self, in this *Dissertation* of Mr. *Robins*, I shall only take notice, that the latter part of it, for about eight pages, is taken up with a vindication, shall I say, or accusation? censure, or encomium of Sir *Isaac Newton*? Mr. *Robins* shews that respect to the memory of this great man, that he will suffer no body to attack his reputation, but himself. He defends him against the *Author of the Analyst*; but, at the same time, he points out other faults in Sir *Isaac*, which that *Author* never thought of. How happy was it that these two Writers did not attack him at the same time, were not joined in alliance against him. It must have been equally dangerous, as if *Philip* had joined his arms to those of *Hannibal*, and not waited the event of the battle at *Zama*.

XLIII. Mr. *Robins* gives us to understand, that Sir *Isaac Newton* was at first an errant Indivisibleist: Afterwards indeed he saw and renounced his errors. *He had sufficiently seen the abuses, that had been made of the doctrine of infinitely little quantities. He looked on no demonstration as valid, that was built on those absurd principles. He was fully apprised of the real imperfections of indivisibles: Yet he scarce condemned them himself, and frequently made use of expressions peculiar to them: He still allowed himself some use of infinitely little quantities*: He has not *demonstrated the proportions of fluxions according to the accurate methods of the ancients.*

XLIV. I must needs say, after this heavy charge, Mr. *Robins* has the goodness to make some excuses for Sir *Isaac Newton.* He was *very young*: He was too *modest*, and *as it were fearful*: He had not *read the ancients.* Very kind and tender truly! I should be glad I had as much to say for Mr. *Robins*: But for Sir *Isaac Newton* these apologies may as well be let alone.

That he was once *very young*, is not denied. But the youth of that great man was superior to the mature age of the rest of mankind. He had made the greater part of his amazing discoveries by the year 1666, at the end of which year he was four and twenty.

That Sir *Isaac Newton*'s *modesty is surprizing* to Mr. *Robins*, I do not wonder. But this *surprizing modesty, and as it were fearfulness*, did not hinder him from *censuring the method of indivisibles*, though *in the softest manner.* Notwithstanding this *modesty and fearfulness, he discovered his dislike to indivisibles*, he *condemned them*; he *shewed a way to avoid their imperfections*; he *has given us a caution in order to prevent mistakes about many of these forms of speech.*

If he did not *read the ancients*, and consequently could not *from thence be enabled to have demonstrated the proportions of fluxions according to their accurate methods*; yet Mr. *Robins* tells us, *he did much more, by finding out one of his own, more compendious than theirs, and equally geometrical*; one that is *an abbreviation and improvement of the form of demonstrating used by the ancients on the like occasions*; one, *which is more concise than the method of demonstrating used in these cases by the ancients, yet is equally distinct and conclusive.* So that he seems to have stood in no great need of the ancients, since he so much improved and outdid *their accurate methods.*

But why does Mr. *Robins* think, that Sir *Isaac Newton* had not read the ancients? I find him often mentioning the geometry of the ancients, as, *consonum est geometriæ veterum, more veterum geometrarum &c.* And the character he gives in the *Scholium* to the first section of the *Principia*, of these same *accurate methods*, seems to be so just, that I think he must be well acquainted with them. *Præmisi hæc lemmata, ut* EFFUGEREM TAEDIUM *deducendi* PERPLEXAS *demonstrationes ad absurdum more veterum Geometrarum*. To avoid cavil, I must take notice, that Sir *Isaac* was by some means or other prevailed upon, to change the word *perplexas* into *longas*, in the last edition of the *Principia*. This alteration seems to be copied in the following passage of Mr. *Robins*. "Sir *Isaac Newton* by his doctrine of prime and ultimate ratios, has found out the proper medium, whereby to avoid the impossible notion of indivisibles on the one hand, and the *length* of exhaustions on the other."

Now let all those, who had the honour of Sir *Isaac Newton*'s acquaintance, and were witnesses to the modesty, integrity and veracity of that Great Man, who as so singularly *Vir antiqui moris*, let them I say judge, whether he would have talked in this manner, to make the world believe he had read the works of the ancient Geometricians, if in reality he had not read them. But why do I say, let them judge? They all know, from a thousand instances, that he had carefully read them, and was perfectly master of them.

XLV. To conclude, if Mr. *Robins will not pretend to defend* Sir *Isaac Newton* without making such sort of excuses and apologies for him; if without *interpreting Sir Isaac*'s *expressions suitably to* his own *representation of this doctrine*, he *acknowledges himself totally incapable of reconciling the method of prime and ultimate ratios with the character, the author himself has given of it*; I think it might be as well to let that defence alone, to suffer Sir *Isaac Newton* to stand or fall by the merit of his own works. For my part, I am verily persuaded, to use once more the words of Mr. *Robins*, that *all the absurdity of expression, and all the inconsistence with himself charged on Sir* Isaac Newton *by the Author of the Analyst*, or any body else, *arises wholly from misinterpretation*, or misunderstanding him.

E tenebris tantis tam clarum extollere lumen
Qui primus potuisti, & fluxu expendere curvas;
Te sequor, O Britonum gentis decus, inq; tuis nunc
Fixa pedum pono pressis vestigia signis,
Certandi cupidus nequaquam, at propter amorem,
Quod te imitari aveo. Quid enim contendat hirundo
Cycnis? Aut quidnam tremulis facere artubus hoedi
Consimile in cursu possint, ac fortis equi vis?
Tu Pater, & rerum Inventor: Tu patria nobis
Suppeditas præcepta; tuisque ex, Inclute, chartis,
Floriferis ut apes in saltibus omnia libant,
Omnia nos itidem depascimur aurea dicta,
Aurea, perpetuâ semper dignissima vitâ.

My Readers may very justly expect an Apology for this Number, wherein they will find no Account of Books, which ought indeed to be the principal Subject, nor any thing but what's peculiar to the Mathematicians. As I assure them they shall hereafter have no Occasion to complain upon this Head, so I beg leave to lay before them the Reason of my present Conduct, which I intreat 'em to excuse without pretending to justify. They are all apprised of a Controversy concerning Sir *Isaac Newton*'s Doctrine of Fluxions, which two very learned Antagonists have carried on for some time past in these Papers: It is upon a very sublime and useful Topic, and the Disputants, however they happen to disagree and mistake one another, are both of them eminently Masters of it: and as their Discourses have afforded a noble Entertainment to many Gentlemen of the most distinguished Skill in the Science; so their chusing to communicate their several Sentiments to the Public in this Work, has done me a singular Honour, and laid me under great and equal Obligations to them both: and nothing but the Hazard of a general Offence, which I hope I shall for this time escape, could prevail on me to refuse either of their Requests. It happened that some Expressions of the anonymous Writer in his *Considerations, &c.* published in our last Month, were thought injurious by the ingenious Mr. *Robins*, who regarded them as personal Reflections, to which he could not refuse himself the Justice of an immediate Reply, and as he drew one up with the utmost expedition, so he demanded a Place for it in my next *Republick*.