CONSIDERATIONS UPON SOME PASSAGES OF A DISSERTATION CONCERNING THE DOCTRINE OF FLUXIONS, PUBLISHED BY MR. ROBINS IN THE REPUBLICK OF LETTERS FOR APRIL LAST.

By

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NOTE ON THE TEXT

This text is transcribed from *The present state of the republick of letters* for July 1736.

The following spelling, differing from modern British English, are employed in the original 1735 text: exprest.

The following erratum, noted in *Republick of Letters*, August 1736, has been corrected:

‘*momentis genita*’ corrected to ‘*momentis genitæ*’.

David R. Wilkins
Dublin, June 2002

Considerations *upon some passages of a* Dissertation *concerning the Doctrine of* Fluxions*, published by Mr.* Robins *in the* Republick of Letters *for* April *last. By* Philalethes Cantabrigiensis.

[*The Present State of the Republick of Letters*, July 1736, pp. 45–82.]

I. To enter into controversy for any other cause than that of Truth, is beneath the character of a man of letters: But to persist in dispute, when he is clearly shown his error, when his mind is under strong and invincible conviction, were much more unworthy of such a character. Flatter himself as he may, whoever acts so disingenuous a part, can never pass for what he would appear to be. The eye of the judicious and attentive reader will easily discern him; nor can it be difficult even for the less intelligent, to distinguish between him and the sincere defender of Truth alone. The advocate for Truth will always be known by the following particulars of his conduct.

II. He will keep close to the points in dispute, with as little mixture as possible of foreign matter.

He will not begin *de novo* every time he writes, to embarrass and perplex his reader; but will resume the dispute just where his adversary left it.

He will study plainness and perspicuity, endeavouring always to set the point in the clearest and strongest light; and will be careful to avoid ambiguity as much as possible.

He will talk as little as may be in general terms: On the contrary, whenever it shall be necessary to clearing up the point in hand, he will descend to the minutest particulars, and the most circumstantial examination. More especially, if his opponent have already done so.

He will at all times be ready to give satisfaction, in case of any question, or challenge, from the person he disputes with.

He will not omit taking notice of any argument brought against him; much less will he pass by in silence, such as are the strongest and most forcible.

He will be so far from lessening the strength of any objection, by representing it unfairly and imperfectly, that he will, if possible, set it in a stronger and fuller light than it was urged in by his antagonist.

He will quote the words of his opponent, or of other writers, fairly and exactly, not giving his own paraphrase as if it were their expression; nor leaving out part of their words, or adding others of his own, in order to change or disguise the meaning; nor will he, with the same intent, omit citing any material passage, following and explaining the passage cited by him.

He will not impute any opinions to his antagonist, which he does not hold, much less when he manifestly holds the contrary opinions.

He will take the words of the Authors he quotes, in that sense which is agreeable to their constant doctrine, and the general tenour of their writings, if the words are fairly capable of such a sense; and will not wrest them to another meaning more for his purpose.

He will not endeavour to impose upon his reader, by confident assertions instead of proof, by saying a thing is most evident, most manifest, most evidently appears, is expresly declared or affirmed, where there is no foundation for such asservations, but rather the contrary.

III. How far my behaviour, in the dispute I have unwillingly been engaged in by Mr. *Robins*, has been agreeable to these rules, is submitted to my reader's judgment. I can truly say, I have endeavoured it should be so, and shall continue to endeavour it. And as a farther mark of my regard to Truth, I shall divide what I have to say, into distinct sections, the particulars being *now* grown too numerous for a division into heads only, in the method I used in *November* last; that if Mr. *Robins* thinks fit to take the same course, his sections may be compared with mine, in order to make the truth more evidently appear. I am afraid, the reader will find it highly necessary so to compare us.

IV. In the first sixteen pages of this *Dissertation*, I apprehend myself to have, directly at least, little or no concern; and may therefore leave them to the followers of Messieurs *Cavalieri*, *Leibnitz*, *Bernoulli*, *Parent* and others, who are all very severely, though perhaps not altogether unjustly handled in those pages. But I must needs say, my learned friend, the *Author of the Analyst*, is much too hardly dealt with. I have had, it is true, some little difference with that Gentleman; notwithstanding which, I have still good nature enough left, to defend him against the unjust reproaches of Mr. *Robins*, but that I am sensible he is very capable of doing it himself, and I have at present too much other business upon my hands. Only, as in two or three places I apprehend a design to wound me through his sides, self-preservation will there oblige me to vindicate both him and myself; and if he will do me the honour to stand behind my shield, I shall use my best endeavours to bring him off safe and unhurt.

V. Mr. *Robins* tells us,* "This writer for the support of his objections against this doctrine, (*of Fluxions*) found it necessary to represent the idea of fluxions as inseparably connected with the doctrine of prime and ultimate ratios, intermixing this plain and simple description of fluxions with the terms used in that other doctrine, to which the idea of fluxions has no relation: and at the same time by confounding this latter doctrine with the method of *Leibnitz* and the foreigners, has proved himself totally unskill'd in both.

"These two methods of Sir *Isaac Newton* are so absolutely distinct, that their Author had formed his idea of Fluxions before his other method was invented, and that method is no otherwise made use of in the doctrine of fluxions, than for demonstrating the proportion between different fluxions."

How fairly the conduct of my Friend and Correspondent is here represented,† I shall leave to him or others to consider. The question with me is, whether the *two methods*, as Mr. *Robins* calls them, of Sir *Isaac Newton*, are *absolutely distinct*; whether *the idea of fluxions has no relation* to the method of prime and ultimate ratios.

And here, I suppose, I may take it for granted, that by *the idea of fluxions* Mr. *Robins* means the doctrine or method of fluxions. For if he meant anything else, he would not have

* Pag. 294.

† *Vid.* Anal. p. 6, 7.

used the words, *this latter doctrine, these two methods*: nor could he have inferred, that, because Sir *Isaac Newton* had *formed his idea of fluxions before his method* of first and last ratios *was invented*, that therefore his method of fluxions was *absolutely distinct* from his doctrine of first and last ratios.

This therefore being allow'd me, that by *the idea of fluxions* I am to understand the method of fluxions, I proceed to enquire whether the method of fluxions *has no relation to* the method of first and last ratios, whether the first of these methods be *absolutely distinct* from the last.

And here Mr. *Robins* himself gives me great assistance. In this very passage I learn from him, that *the method of first and last ratios is made use of in the doctrine of fluxions, for demonstrating the proportion between different fluxions.*

From him likewise I understand, that * *it is by means of this proportion* ONLY*, that fluxions are applied to geometrical uses.*

If so; it should seem, that the method of fluxions should *have* some *relation to* that of first and last ratios, that it should not be *absolutely distinct* from it; nay, that the former could not possibly be *formed before* the latter *was invented.*

By why so much pother about this distinction? I see no other use of it than to vindicate the Title page of Mr. *Robins*'s *Discourse concerning Fluxions*, where thro' inadvertence there happens to be an S too much. Were it not for that, we might as well suppose the doctrine of first and last proportions to be a part, or indeed to be the foundation of the method of fluxions.

It has been said indeed, that † *after what had been written*, (by Mr. *Robins* I suppose) *it seemed scarce possible any longer to confound these two methods together.* But to this the *Author of the Analyst* and I may plead in excuse, that our pieces were publish'd before Mr. *Robins*'s *Discourse.* His instruction came too late.

Ay, but ‡ *Sir* Isaac Newton *himself has distinguished them*, and when Mr. *Robins considers how expressly* he has done so, *he owns himself surprised, that this mistake should ever have been made.*

As Mr. *Robins* does not tell us where this express distinction of Sir *Isaac*'s is to be met with, I have been at the pains to turn over all his works, in order to find it, not forgetting the account of the *Commercium Epistolicum* in *Philos. Tr.* N° 342. which Mr. *Robins*, I know not upon what foundation, ascribes to that Great Man. The result of my enquiry is as follows.

Sir *Isaac*, in his Treatise of the *Quadrature of Curves* and the Introduction to it, and *Lemm.* 2. *L.* II. *Princip.* does not only *intermix his plain and simple description of fluxions with the terms used in that other doctrine*, as moments, momentaneous increments, nascent and evanescent augments or increments, and their first and last proportions, but says towards the end of the Introduction, *Similibus augmentis per methodum rationum primarum & ultimarum colligi possunt fluxiones in casibus quibuscunque.* From which words it seems to me, that the method of fluxions *has* some *relation to that other doctrine*, that the *two methods are* not *absolutely distinct.*

* Discourse upon Fluxons, *p.* 6.

† Republick of Letters for *October*, p. 262.

‡ Ibidem.

But farther in the account of the *Commercium Epistolicum*, which, whether written by Sir *Isaac Newton* or not, I make no doubt was agreeable to his sentiments, as having been some years afterwards republish'd in Latin with his consent and approbation, that method is called * *the Method of Fluxions and Moments*, † *the method of moments*, ‡ *his method of moments*, and again § *the method of fluxions and moments*, with other passages to the same purpose. Now ¶ *momentum is a term appertaining to the doctrine of prime and ultimate ratios only*, according to Mr. *Robins*. It should seem therefore to be no great mistake, if my Friend and I should take the method of fluxions, and the method of prime and ultimate ratios, to be one and the same method. But enough of this.

VI. Mr. *Robins* goes on to charge this learned Person with *a* ‖ *twofold mistake*. The first of these mistakes appears to me to be of so little consequence, at least to relate so little to me, that I shall say nothing about it.

But in the second, though it is not directly charged upon me, I apprehend myself to be not a little concern'd.

"He, it is said, always represents these *augmenta nascentia*, not as finite indeterminate quantities, the nearest limit of whose continually varying proportions is here called their first ratio, but as quantities just starting out from non-existence, and not yet arrived at any magnitude, like the infinitesimals of the differential calculus."

That this Gentleman has represented the *augmenta nascentia like the infinitesimals of the differential calculus*, which are fixed, determinate, invariable quantities, I no where find; nor will the Reader find that I have any where done so; nor that either of us have ever spoke of them as *determinate quantities*. But that we have represented them as indeterminate *quantities just starting out from non-existence, and yet not arrived at any magnitude, and not as finite quantities*, I am very free to own, and apprehend we are both justified in so doing, by the *express words of Sir* Isaac Newton. *Cave* **, says that great Man, in speaking of the *augmenta nascentia*, by the name of *momenta*, or *incrementa momentanea, intellexeris particulas finitas. Particulæ finitæ non sunt momenta, sed quantitates ipsæ ex momentis genitæ. Intelligenda sunt principia jamjam nascentia finitarum magnitudinum.*

And in the account of the *Commercium Epistolicum*, to which Mr. *Robins* so often appeals, it is said, †† *Mr.* Newton *represented moments by the rectangles under the fluxions and the moment o, when he wrote his Analysis*; and ‡‡ *in his calculus there is but one infinitely little quantity represented by a symbol, the symbol o*; it is also said, §§ *Prick'd letters*

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* Philos. Trans. *p.* 173.
† Ibid. *p.* 176.
‡ Ibid. *p.* 177.
§ Ibid. *p.* 179.
¶ Republick of Letters for *Octob. p.* 263.
‖ Republick of Letters for *April*, *p.* 294.
** Lemm. 2. L. II. Princip.
†† Philos. Trans. N°. 342. *p.* 205.
‡‡ Ibid. *p.* 205.
§§ Ibid. *p.* 204.

never signify moments, unless when they are multiplied by the moment o either exprest or understood to make them infinitely little, and then the rectangles are put for moments.

Moments therefore are not finite quantities, but infinitely little in the sense of Sir *Isaac Newton*. And if it be said that, when Sir *Isaac Newton* wrote his *Analysis*, he proceeded upon the principles of indivisibles, I reply, that this is not true: For the quantities, which Sir *Isaac Newton* calls infinitely little, are widely different from the infinitely small quantities in the method of indivisibles, as I shall have occasion particularly to shew by and by. In the mean time I would observe, that though this *Analysis* was written in or before the year 1669, yet the passages I have just now quoted, were written in the year 1715, long after the method of fluxions and of first and last proportions was perfected by Sir *Isaac Newton*. And I must observe farther, we are told in the account so often quoted, that * *Mr.* Newton *used the letter o in his Analysis written in or before the year* 1669*, and in his book of Quadratures, and in his Principia Philosophiæ, and* STILL *uses it in the very same sense as at first.* If therefore we would know in what sense Sir *Isaac Newton* used the letter *o*, which represents the only infinitely small quantity used in his calculus, we need only turn to the *Principia* or his book of *Quadratures*, to be informed of it.

But Mr. *Robins* says, our notion of the *augmenta nascentia* † *is contrary to the express words of Sir* Isaac Newton, and quotes the following passages against us. ‡ *In finitis quantitatibus analysin sic instituere, & finitarum nascentium vel evanescentium rationes primas vel ultimas investigare consonum est geometriæ veterum: & volui ostendere, quod in methodo fluxionum non opus sit figuras infinite parvas in geometriam introducere.*

In answer to this I would observe, that Sir *Isaac Newton* is not here purposely describing or defining his moments or augments, as in the passages I have above quoted: but his design in this passage is to defend his method of Fluxions, and to shew it to be agreeable to the Geometry of the Ancients; and he only speaks of his *quantitates nascentes vel evanescentes* in a transient manner. Therefore this passage is by no means proper to shew his sentiment about the magnitude of those quantities, as the other passages I have quoted, where they are purposely described.

However, since Mr. *Robins* has thought fit to quote this passage against us, it will be necessary to consider it. I apprehend therefore, that by the words *finitarum nascentium vel evanescentium*, it was not meant that nascent or evanescent quantities were finite at the instant of their origin, or at the instant of their vanishing, when their first or last proportions are determined: but that a quantity just beginning to exist, or as Mr. *Robins* expresses it, *just starting out from non-existence*, might by flowing or growing, become finite; and likewise that a finite quantity by decreasing *sine fine, sine limite, ad infinitum*, may at last vanish into nothing, or cease to exist.

Agreeable to this explication is the following passage, taken from the account of the Com. Ep. "When § he is demonstrating any proposition, he uses the letter *o* for an finite moment of time, or of its exponent, or of any quantity flowing uniformly, and performs the whole calculation by the Geometry of the Ancients in finite figures or schemes without any

* Philos. Trans. N° 342. *p.* 204.

† Republick of Letters for *April*, 1736. p. 295.

‡ Introduct. ad Quadratur.

§ Philos. Trans. N°. 342. *p.* 179.

approximation: and so soon as the calculation is at an end, and the equation is reduced, he supposes that the moment *o* decreases in infinitum and vanishes."

This seems to me to be a passage parallel to that quoted by Mr. *Robins*, and, being more explicit and plain, may serve to explain the other, and a very clear and full example suited to this explication is to be found in the † *Analysis* of 1669, and other such examples in that very Introduction to the Quadratures, from which Mr. *Robins*'s passage is taken. I must add a word of two about the conclusion of this passage, quoted by Mr. *Robins, Volui ostendere, quod in methodo fluxionum non opus sit figuras infinite parvas in geometriam introducere.* This will be easily understood, by a little reflexion upon the several examples Sir *Isaac* has just been assigning, of the proportion between different fluxions, and upon the other passage just now quoted from the account of the *Commercium Epistolicum.*

For in those examples, the augments that Sir *Isaac* sets before us, that he *introduces into* his *geometry*, are all finite, and represented *in finite figures* agreeable to the *Geometry of the Ancients.* He has no occasion to introduce any figures *infinitely small*, as is done by the followers of Mons. *Leibnitz.* For when he comes to find the proportion of the fluxions, he has no more to do, but to suppose that the finite augments decrease *in infinitum* and vanish; by which means he finds their last proportion, which is the same with that of the fluxions, and may be expounded by any lines whatsoever which are proportional to them.

But though, *non opus sit*, there by no necessity of introducing figures infinitely small into Geometry, yet Sir *Isaac* tells us immediately after, * the Analysis may be performed in any figures whatsoever, whether finite, or infinitely small, provided they are supposed similar to the evanescent figures; as also in such figures as by the methods of indivisibles are usually taken for infinitely small, provided you proceed with caution. Parallel to which is the following passage, which comes immediately after that already quoted from the *Commercium Epistolicum.* "When he is not demonstrating, but only investigating a proposition, for making dispatch, he supposes the moment *o* to be infinitely little, and forbears to write it down, and uses all manner of approximations, which he conceives will produce no error in the conclusion."

These two passages need no comment, they are manifestly written with a view to one another, and explain one another. In one we have Sir *Isaac Newton*'s own practice *for making dispatch*, with his caution for avoiding error in the conclusion. In the other he tells us what we may do, if we please; but hints, that when we leave the method he observes, in supposing his infinitely small figures to be similar to the evanescent ones, and come to make use of the infinitely small figures of the method of indivisibles, we must proceed with caution. Of the want of this caution Mr. *Robins* ‡ has collected several examples. But it is now time to come to what more immediately concerns myself.

VII. After taking notice that I have interpreted the first *Lemma* of the *Principia* after

† Demonstr. Reg. I.

* Peragi tamen potest analysis in figuris quibuscumq; seu finitis seu infinite parvis quæ figuris evanescentibus finguntur similes, ut & in figuris quæ per methodos indivisibilium pro infinite parvis haberi solent, modo caute procedas.

‡ Repub. of Lett. for *April*, 1736. p. 303.

a manner something different from himself, Mr. *Robins* says that my * "Interpretation does not ascribe to the word *given*, used by Sir *Isaac Newton* in this lemma, the true sense of that word in *Geometry*, but supposes it to stand for *assignable*; whereas it properly signifies only what is actually assigned."

I am very free to acknowledge, that the word *given*, in Geometry does properly signify what is actually assigned. If a Geometrician speaks of a given quantity, he means a quantity actually assigned, one certain, determinate quantity, and no other. Thus far we are agreed.

But if he uses the words *any given quantity*, does he then mean a quantity actually assigned? One certain, determinate quantity, and no other? Or does he intend any quantity whatever, any quantity that may be given, or assigned?

Sir *Isaac Newton* in this *lemma* uses the words *data quavis differentia*, *any given difference*, by which, I apprehend, he intends any assignable difference, any difference that can or may be assigned; for otherwise the demonstration Mr. *Robins* so often appeals to, cannot be conclusive. The difference represented by D in that demonstration, is manifestly any difference that can be assigned by the objector, and to have such a difference is said to be contrary to the hypothesis, by which the quantities were to come nearer together than any given difference. Consequently, any difference that can be assigned, is with Sir *Isaac Newton* an equivalent expression to any given difference; and surely himself must be the best interpreter of his own words.

If this authority be not satisfactory, I must have recourse to that of a person, whom nobody should suspect of any inaccuracy in style, as having so laudably signalized his zeal *to preserve propriety of expression and perspicuity of conception in mathematical matters*; who *freely acknowledges, that he has not vindicated this doctrine, unless he shall be found to have accommodated to it a clear and unexceptionable mode of expression*; who *has often lamented the negligence of geometrical writers in regard to their style and diction*, and who has more than once shown himself dissatisfied, even with Sir *Isaac Newton*, at being *less* † *exact in the choice of his expressions*, as sometimes *deviating* ‡ *from the utmost propriety of expression*, as § *using some loose and indistinct expressions resembling those of indivisibles*.

This Gentleman, in explaining this very *Lemma*, and illustrating it by examples, has with great variety of diction given the sense of these words, *data quavis differentia*, in manner following.

Any [a] line that can be named. Any [b] quantity how minute soever, that CAN BE ASSIGNED. Some [c] ASSIGNABLE distance. Any [d] other ASSIGNABLE proportion. The [e] nearest limit, that

* Ibid. p. 307.

† Republick of Letters for *October*, p. 270.

‡ Ibid. *p.* 264.

§ Ibid. *p.* 258.

[a] Discourse upon Fluxions, *p.* 46.

[b] Ibid. *p.* 48.

[c] Ibid.

[d] P. 49.

[e] P. 51.

CAN BE ASSIGNED. Any [f] degree of nearness whatever. Any [g] space that can be proposed. Any [h] difference that can be named. Any [i] difference that can be proposed. Any [k] ASSIGNABLE difference. Any [l] other that CAN BE ASSIGNED. Any [m] space that SHALL BE ASSIGNED. Any [n] difference whatever that may be proposed. Any [o] magnitude that shall be proposed. Any [p] ASSIGNABLE magnitude. Any [q] difference that CAN BE ASSIGNED. Any [r] difference that CAN BE ASSIGNED. Any [s] whatever that should be proposed.

Many more passages of like nature might be produced from the *Discourse* abovemention'd, and from the Defences of it published in the *Republick of Letters* for *October*, and for *December* last.

And in this very *Dissertation*, in the page immediately preceding that which contains the animadversion upon me, after preparing the mind of the Reader for a *short representation of the true sense, in which Sir* Isaac Newton*'s phraseology ought to be understood*, Mr. *Robins* is pleased to expound the words in question by *any difference, how minute soever, that* CAN BE ASSIGNED.*

Lastly, to remove all doubt and scruple, in the following sentence he gives us to understand, that the expression, *any difference how minute soever being given*, is *consonant to* the abovesaid *representation of Sir* Isaac Newton*'s meaning. Any given difference* is therefore, in Mr. *Robins*'s opinion, *consonant to any difference that can be assigned.*

If this defence be not satisfactory, I shall not pretend to offer a fuller vindication.

VIII. Mr. *Robins* goes on thus. "*Philalethes* insinuates that by our interpretation, and the forementioned remark upon it, Sir *Isaac Newton* is rendred obnoxious to the charge of first supposing what he would prove, and with proving only what he has before supposed. But our interpretation cannot possibly mean less than this, that those quantities and ratios will have no last difference," *&c.*

Which interpretation is Mr. *Robins* here speaking of? Before this insinuation of *Philalethes* in *Jan.* last, Mr. *Robins* had already published three different interpretations of the *Lemma* in question, which *Lemma* it may therefore be proper once more to set before the Reader, together with the suppositions therein contained, as we published them in the

[f] P. 53.
[g] Ibid.
[h] P. 54.
[i] P. 55.
[k] P. 56.
[l] P. 57.
[m] P. 61.
[n] Ibid.
[o] Ibid.
[p] Ibid.
[q] P. 62.
[r] Ibid.
[s] P. 63.
* Pag. 306.

Republick of *Letters* for *November* last.*

LEMMA.

Quantitates, ut & quantitatum rationes, quæ ad æqualitatem tempore quovis finito constanter tendunt, & ante finem temporis illius propius ad invicem accedunt quam pro data quavis differentia, fiunt ultimo æquales.

In this *Lemma* are contained the four following suppositions.

1. That the quantities, or ratios, constantly tend to equality, *ad æqualitatem constanter tendunt*,

2. During some finite time, that either happens to be determined in any particular case, or else may be proposed and assumed at pleasure, *tempore quovis finito*,

3. And come nearer together than to have any given difference, *& propius ad invicem accedunt quam pro data quavis differentia*,

4. Before the end of that finite time, *ante finem temporis illius.*

If any one of these suppositions be omitted; much more if the second and fourth be left our; or if, with the author of the *Analyst*, we neglect the first, second and fourth; we can never justly come at Sir *Isaac Newton*'s conclusion, That, at last, *i. e.* at the end of the given time, during which the quantities, or ratio's, were supposed to tend constantly to equality, they become equal, *fiunt ultimo æquales.*

I come now to consider Mr. *Robins*'s several interpretations of this *Lemma.*

The first interpretation is contained in his *Discourse upon Fluxions*, p. 48. and is repeated in this *Dissertation*, being introduced as *a short representation of the true sense, in which Mr.* Robins *apprehended Sir* Isaac Newton*'s phraseology ought to be understood.* It runs thus:

"In this method any fix'd quantity, with some varying quantity, by a continual augmentation or diminution, shall perpetually approach, but never pass, is considered as the quantity, to which the varying quantity will at last or ultimately become equal; provided the varying quantity can be made in its approach to the other to differ from it by less than by any quantity how minute soever, that can be assigned."†

In this interpretation the second and fourth of Sir *Isaac*'s suppositions are entirely omitted; and instead of approaching *during a finite time*, the quantities are supposed to approach *perpetually*, that is, I suppose, to all eternity. Likewise instead of Sir *Isaac*'s conclusion, they *at last become equal*, we are told, they are *considered as at last or ultimately becoming equal*, and we are left to find out as well as we can, what is meant by the words *at last*, or the term *ultimately*, which I own I can no way discover, in case of a *perpetual*, or eternal approach.

To supply the defect of this first interpretation, Mr. *Robins* gave us a second interpretation in the *Republick of Letters* for *October* last, *p.* 254. where we are told "the genuine menaing of the Lemma is, That those quantities are to be esteemed ultimately equal, and those ratio's ultimately the same, which are perpetually approaching each other in such a manner, that any difference how minute soever being given, a finite time may be assigned, before the end of which the difference of those quantities or ratio's shall become less than that given difference."

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* Pag. 371.

† Princ. Philos. Lib. I. Lemm. 1.

Here likewise the second supposition of Sir *Isaac Newton* is omitted, and instead thereof the quantities or ratio's are supposed to approach *perpetually*, as in the first interpretation; and tho' mention is now made of a finite time and of the end thereof, yet it is not done in the same manner as in the *Lemma* itself. In the *Lemma* the finite time precedes the difference; but Mr. *Robins*, in order to change the sense, has thought fit to place the difference first, and afterwards to assign a finite time. And his conclusion here is equally faulty with the conclusion of the first interpretation, the word *considered* being only altered into the word *esteeemed*.

Immediately after this second interpretation Mr. *Robins* acquaints us, that Sir *Isaac*'s meaning *in this Lemma will be best known from the demonstration annexed to it*. Does Mr. *Robins* mean, that to read the demonstration first, and the proposition afterwards, is the best way to know the meaning of the proposition?

Mr. *Robins* goes on, "By that it appears, Sir *Isaac Newton* did not mean, that any point of time was assignable, wherein these varying magnitudes would become actually equal, or the ratio's really the same; but ONLY that no difference whatever could be named, which they should not pass."

To this interpretation and this remark I gave a full and distinct answer in the *Republick of Letters* for *November*, p. 370, *&c.* in consequence whereof Mr. *Robins* gave us a third interpretation in the *Republick of Letters* for *December*, p. 442. and has since greatly altered his remark in the *Dissertation* which I am now replying to.

The third interpretation, which is said to be *the true interpretation*, runs thus:

"If, according to the must usual and authentick signification of this phrase, there is meant by the *given difference*, (N.B. Sir *Isaac*'s phrase is *any given difference*) in this *Lemma*, a difference first assign'd, according to which the degree of approach of these quantities may be afterwards regulated; then variable quantities or ratios, and their limits, tho' they do never actually coincide, will come within the description of this *Lemma*; since the difference being once assign'd, the approach of these quantities may be so accelerated, that in less than any given time the variable quantity, and its limit, shall differ by less than the assign'd difference.

"Now that" this "is the true interpretation, will appear from the demonstration and application of this *Lemma*."

Here it is observable, that Sir *Isaac*'s second supposition is omitted, the quantities not being supposed to approach one another *during a given time*, nor yet *perpetually*, as in the two former interpretations. Likewise the difference is again made to precede the time of approach, contrary to the order observ'd by Sir *Isaac Newton*. Likewise, instead of any difference that can be assign'd, as in the first interpretation, or of any given difference, as in the second interpretation, Mr. *Robins* here substitutes *a difference first assign'd*, or a *difference once assign'd*. Likewise instead of *a finite time may be assign'd*, as in the second interpretation, he here substitutes *any given time*. Likewise he tells us *the degree of approach may be afterwards regulated*, and *the approach may be so accelerated*, of which *regulation* and *acceleration* there is no mention in Sir *Isaac Newton*'s *Lemma*.

Lastly, his conclusion, That *the variable quantity, and its limit, shall differ by less than the assign'd difference*, is widely different from Sir *Isaac*'s conclusion, *fiunt ultimo æquales*; and is indeed no other than his third supposition, *That the quantities approach nearer than any given difference*, as I had more than *insinuated* in the *Republick of Letters* for *January* last, p. 78, 82. where this *Lemma* of Sir *Isaac Newton*, and particularly the conclusion of it, is very particularly and minutely considered.

Having finished this third interpretation, Mr. *Robins*, in order to convince us *that this is the true interpretation*, refers us to *the demonstration and application of this Lemma.* He gave us the same good advice at the end of his second interpretation.

Let us therefore at once comply with Mr. *Robins* so far, as to suppose that this *Lemma* of Sir *Isaac Newton* is not of itself intelligible; but that it is necessary to consult the demonstration, not only to be satisfied that the *Lemma* is true, but to find out the meaning and *true interpretation* of the *Lemma.*

The principal part of Mr. *Robins*'s *true interpretation*, and that upon which all the rest depends is, that by the words *any given difference, is meant a difference first assigned, according to which the degree of approach of these quantities may be afterwards regulated.* Let us see how far this interpretation will be warranted by the words of the demonstration.

Dem. *Si negas; siant ultimo inæquales, & sit earum ultima differentia* D. *Ergo nequeunt propius ad æqualitatem accedere quam pro data differentia* D*: contra hypothesin.*

Here the difference D is any difference that can be assigned by the objector, not *at first*, nor yet during the tendency to equality; but *at last, ultimo*, when the tendency to equality is entirely over, on which account it is called *ultima differentia*: but this difference D is ultimately the same as *any given difference* in the *Lemma*; otherwise the having this difference D would not be *contra hypothesin.* Therefore *any given difference* in the *Lemma* does not mean a difference *first assigned*, but any difference that may be assigned *at last, after the celerity or degree of approach of these quantities is in every part determined.*

But now comes out a fourth interpretation, to tell us, "Our interpretation" (some one, or more, I presume, of the three former interpretations) "cannot possibly mean less than this." No truly, I think not. For I cannot find that this has any meaning at all; at least it has no one, clear, determinate meaning, as every good interpretation ought to have. For what, in the name of perspicuity, is the meaning of these words, *That those quantities and ratio's will have no last difference*? Is it that *at last* they will have no difference? If so; Mr. *Robins* and I are agreed, *it being indubitable, that those things are equal, which have no difference.* Or is it meant, that they will always have some difference or other, but none of these differences can be called the *last*? Then I must ask, how this can be the meaning of the words *fiunt ultimo æquales*? Equal quantities have no difference at all.

I must observe farther, that if the quantities do not become perfectly equal, if after the expiration of the finite time, they have any difference at all, they must of necessity have a *last difference.* For since their tendency to equality continues only to the end of the finite time, it is plain that whatever difference they have at the instant the time is expired, can never grow less, but must from that instant continue their *last* and only *difference* to all eternity.

I must likewise ask another question, before I can understand this interpretation. What am I to understand by the word *perpetually*? Does it signify the same as constantly during a finite time? Or does it stand for *endlessly*, without end of time, to all eternity?

As the rest of this interpretation is the same with the second, the faults of which I have already specified, I shall here forebear to mention them; and think it hardly worth while to take notice, that the *regulation* and *acceleration* talked of in the third interpretation, are now omitted.

I am persuaded, it would puzzle even a *W—n*, to write a good *Harmony* upon these four interpretations of Mr. *Robins.* For surely, one shall hardly meet with a plain text more tortured by diversity of explications, in the *Synopsis Criticorum* it self. But that is the work of many heads; these are the product of a single Commentator. Possibly, so fertile a Genius may some time or other oblige us with a fifth interpretation; and if thereby it may be clearly and particularly shown, either that any two of these interpretations agree together, or that any one of them agrees with the *Lemma* they are designed to interpret, I, for my part, shall set a greater value upon such fifth interpretation, than upon all the four former interpretations put together.

IX. Mr. *Robins* goes on to speak * of "two ways whereby good writers explain the use of terms they introduce: one is by expressly defining them; another, when, to avoid that formality, they convey the sense of such terms by their manner of using them."

What the terms here hinted at are, I cannot imagine. I find no new term in the whole *Lemma*, at least the words *fiunt ultimo æquales* are not new terms.

† "And to make appear, says Mr. *Robins*, that Sir *Isaac Newton* by the demonstration annexed to this lemma, has sufficiently evinced, in what sense the lemma itself must be understood, and at the same time to prove what that sense is, it was shewn, that this demonstration is no less applicable to quantities, which only approach without limit to the ratio of equality, than it will agree to such quantities, as at last become actually equal."

Here we are again referred to the demonstration, in order to understand the sense of the *Lemma*. Surely Sir *Isaac* could be none of the *good writers* mentioned above; else we might understand what he was about to prove, before we read the proof itself. But let us follow Mr. *Robins* in his own way.

If I understand the paragraph last quoted, Mr. *Robins* is NOW of opinion, that Sir *Isaac*'s demonstration is applicable *to such quantities, as at last become actually equal*, as well as *to quantities, which only approach without limit to the ratio of equality.*

And in this sentiment I am confirmed by what I find in the next page, "that Sir *Isaac Newton* has neither demonstrated the actual equality of ALL quantities capable of being brought under this lemma, nor that he intended to do so."

"Whenever the quantities or ratio's compared in this lemma are capable of an actual eqality, they must really become so."

It seems therefore to be allowed me, that Sir *Isaac Newton has demonstrated the actual equality of* SOME quantities, that such quantities as *are capable of an actual equality, must really become so.* And if they must really become so, they must become so at the end of a finite time. Therefore Sir *Isaac*'s *lemma* and demonstration, may be taken in the sense I have always understood it in.

Consequently, that Gentleman was much overseen, when in the *Republick of Letters* for *October*, p. 255. he was pleased to say, "By the demonstration annexed to the lemma it appears, Sir *Issac Newton* did NOT mean, that any point of time was assignable, wherein these varying magnitudes would become actually equal, or the ratios really the same; but ONLY that no difference whatever could be named, which they should not pass." As likewise in telling us soon after, "It is evident, that NO point of time can be assigned, wherein they are actually equal; for to suppose this were to assert, that the variation ascribed to these

* Page 308.

† Page 308.

figures, tho' endless, could be brought to a period, and be perfectly accomplished; and thus we would return to" (the usual scarecrow) "the unintelligible language of indivisibles." It was easy to see, that in the month of *December* he was grown sensible he had gone too far; and now this more plainly appears, not only by the passages above quoted, wherein he allows of my interpretation, but by the new * edition of his remark, where with great dexterity, after the words, *this lemma did not mean*, the words, OR NECESSARILY IMPLY, ar tacitly flipt in, as if they had always made a part of this remark.

X. But though Mr. *Robins* allows of the sense I give to the *lemma* and demonstration, yet he strongly contends that they may be taken in another sense; though Sir *Isaac Newton* did mean as I say, yet it is not NECESSARY to confine the *lemma* to that meaning only.

† "It was shewn," says he, "that this demonstration is no less applicable to quantities, which only approach without limit to the ratio of equality, than it will agree to such quantities, as at least become actually equal. For this purpose this demonstration was applied to the ordinate of an hyperbola, compared with the same continued to the asymptote, which do approach without limit to the ratio of equality, though they never become actually equal."

Here I apprehend the *phraseology* is far from being exact. I beg leave to correct it thus: Instead of the words, *it was shewn*, read, *it was said*; and instead of *this demonstration was applied*, put in, *it was said that this demonstration might be applied*, or *could be applied.*

For in the *Republick of Letters* for *October*, P. 255, it was *said*, but was not offered to be *shown*, that *demonstration can be* so *applied, without changing a single word.*

This I denied in the *Republick of Letters* for *November*, p. 374, 375, and gave unanswerable, at least as yet unanswered reasons for my opinion.

Mr. *Robins*, in his reply, in the *Republick of Letters* for *December*, p. 442. again *said*, but never offered to *shew*, that *the demonstration, without the change of a single word, may be applied*, and took not the least notice of the arguments I had brought against him.

In the *Republick of Letters* for *January*, p. 83, 84. I repeated my denial, and gave a farther unanswerable argument, *shewing it to be utterly impossible* so *to apply the lemma, or the demonstration of it*, and concluded with a desire, that *if any one thought otherwise, such application might be particularly and distinctly made.*

And now, in the month of *April*, Mr. *Robins* is not pleased either to answer any of my arguments, or to gratify me in this desire, but contents himself with telling us, *it was shewn, the demonstration was applied.*

It should seem therefore, that *Philalethes has* justly *taken exception to this instance*, not only as *not conceiving how to regulate this approach so as to bound it within a finite time*, but as *conceiving* it to be utterly impossible so to do.

But, says Mr. *Robins, without enquiring how far that limitation was necessary to our purpose, we shewed a method of adding this circumstance.* It seems therefore to be still matter of doubt with Mr. *Robins*, whether the *limitation* of a finite time be *necessary* to the *purpose* of interpreting Sir *Isaac*'s *lemma*; though we have so often shown it to be one of the suppositions expressed in that *lemma.*

* Republick of Letters *for* April, *p.* 306.

† Republick of Letters for *April, p.* 308.

But, says he, *we shewed a method of adding this circumstance.* Is this true? Can *this circumstance* be really *added* by the *method* Mr. *Robins* has proposed? No. Mr. *Robins* himself has shown this pretended method to be * *absurd, fallacious* and *inconclusive*, and to depend upon an *impossible operation*, and has thereby saved *Philalethes* the trouble of shewing it for him.

XI. After acknowledging † that, "Whenever the quantities or ratios compared in this lemma are capable of an actual equality, they must really become so," Mr. *Robins* proceeds to tell us that, "when they are incapable of such equality, the phrase of ultimately equal must of necessity be interpreted in a somewhat laxer sense." But till he shews me, that any quantities or ratios *incapable of an actual equality*, are *compared in this lemma*, which he has not yet offered to do, I cannot see the *necessity* of *interpreting* the word *equal* in any *laxer sense* than that of an actual, perfect and absolute equality.

XII. And though I were inclined to admit of a laxer sense, yet I can by no means think the example Mr. *Robins* would furnish me with from *Prop.* 71. *Libr.* I. *Principiorum*, is either a proper one for his purpose, or fairly quoted, or truly rendered into *English*, or rightly interpreted.

For the quantities Sir *Isaac* there speaks of, are such as vanish with an actual ratio of equality, as appears by his adding, immediately after the words quoted by Mr. *Robins, pro æqualibus habeantur*, the following words, *quippe quarum ratio ultima est æqualitates*, which last words ought likewise to have been quoted by Mr. *Robins.* Consequently this example is neither a proper one for Mr. *Robins*'s purpose of *a laxer sense*, nor is it fairly quoted.

And the words, *pro æqualibus habeantur*, must not be rendered, *are to be esteemed equal*; but *let them be taken for equal*, or *let them be esteemed equal.* The reason why Sir *Isaac Newton* desires they may be taken for equal, is not, that he *means only, that they approach without limit*; but because, though they are at present unequal, as being drawn *in a finite scheme* according to the Geometry of the Ancients, yet, when they come to vanish, they will arrive at the ratio of an absolute equality. Consequently, this passage is neither truly rendered into *English* by Mr. *Robins*, nor rightly interpreted.

Nor can I see that, of the two expressions, *ultimo in ratione æqualitatis*, and *ultimo æquales*, the one will admit of a laxer interpretation any more than the other. I readily allow them to be synonymous. Nor has Mr. *Robins* yet shown, that *Sir* Isaac Newton *himself has applied this lemma to quantities and ratios incapable of an actual equality or agreement.*

For though the second *Lemma, in ‡ the account of Mr.* Robin*'s discourse, was produced as an example of this*, we § have manifestly shown that *Lemma* to be an example quite contrary to Mr. *Robins*'s purpose, and we shall present have occasion further to consider it.

In the *Republick of Letters* for *October*, and likewise in that for *December*, Mr. *Robins* had strongly insisted upon another example, namely that of two *lines increasing together by*

* Republick of Letters for *December, p.* 444, 445.

† Republick of Letters for *April*, 1736. p. 309.

‡ Republick of Letters for *October*, 1735. p. 255.

§ Republick of Letters for *November*, 1735. p. 375 & seq. & Rep. of Lett. for *January*, 1736. p. 84, & seq.

equal additions, and having from the first a given difference; and had made a little too free with Sir *Isaac Newton*'s name upon that occasion. But in the present *Dissertation* I find no mention made of this example. Probably, he has now better considered of what was said in the *Republick of Letters* for *November* last.

XIII. It is said to have been *there also observed, that vanishing quantities may never actually have that proportion, which, according to this lemma, is said ultimately to belong to them.*

Here I must beg leave to take notice of a great variation in Mr. *Robins*'s doctrine, or at least in his *style and diction.*

In his *Discourse*, p. 50. we are told, *the ultimate proportion is the proportion which the* vanishing *quadrilaterals* CAN *never actually have to each other.*

In the account of his *Dicourse*, Republick of Letters for *October*, p. 257. it is said, *the quantities called by Sir* Isaac Newton*, vanishing,* MAY *never subsist under that proportion here esteemed their ultimate.*

And in the same account, a little after, he acquaints us, that *these lines* MUST *not be conceived, by the name of evanescent or any other appellation, ever to subsist under that proportion.*

And in the next page we find, that *the quantities themselves* CAN *never attain* that proportion.

And in the subsequent acocunt of the same *Discourse*, we are told, * that *the ultimate proportion was not a proportion that these varying quantities* COULD *ever subsist under, during their variation.*

And soon after we learn *that the decrements* CANNOT*, in any circumstance whatever, bear to each other that proportion.*

And now in the *Republick of Letters* for *April*, after telling us in the passage above quoted, that the quantities MAY *never actually have that proportion*; it is said in another place, *nothing is † more evident, than that their diminution* WILL *never bring them actually to bear that ratio to one another.* And in a third ‡ we are given to understand, that we ought not to *seek after any state or condition, at which these quantities* CAN *actually arrive, wherein to be the subjects of this proportion.* And a little after we are told, § *the quantities are incapable of being converted by the variation ascribed to them into any condition, wherein they* WILL *be the subjects of that ratio.*

In other places we are told, it is a mistake to think, that ¶ *the ultimate ratio is a ratio that these quantities* DO *at sometime or other exist under*; or MUST *sometime or other exist under*; that it *can be the ratio, which those quantities* THEMSELVES *at any time* MUST *actually have.*

I freely own, this great variety of *phraseology* confounds me. I can neither find, what is Mr. *Robins*'s present opinion about the *ratio ultima* of vanishing quantities, nor whom

* Republick of Letters for *December*, p. 438.

† Pag. 315.

‡ Pag. 316.

§ Ibid.

¶ Pag. 319.

all his artillery is levelled against. Not against me, I hope; for in my last, I very explicitly declared my sentiment upon this head, introducing it with this Preamble, * *To prevent all mistakes as much as possible, I shall here once for all explain myself in such a manner, as I am persuaded this learned Gentleman will not except against.* In fact this learned Gentleman has not ventured to except against it: Tho' I must needs say, some of the passages above quoted seem to look asquint at it, and the word THEMSELVES, in the last of those passages, shews plainly that it has been under consideration.

XIV. Mr. *Robins* comes not to consider the second lemma, in which, says † "Sir *Isaac Newton* directs, that the number of these parallelograms should be augmented *in infinitum.* This must not be interpreted, till the number become infinitely great, for this is the express language of indivisibles. We render the words *in infinitum, endlessly,*"

I am very unwilling to dispute about words, and yet I must needs say I cannot like this word *endlessly.* It is not the novelty of the word that gives me offence; but it is an ambiguous term, and I remember it led Mr. *Robins* into a grievous mistake ‡ once before, by his not attending to the two senses of which it is capable.

One of these senses regards the magnitude of the number of the parallelograms; the other, the time of their augmentation.

If I say, let a number be augmented *endlessly*, I may mean, let the number be augmented without limit; be it already never so great; yet let it still become greater.

I may likewise mean by the same expression, let the number continue to be augmented to all eternity, or as Mr. *Robins* expresses it in another place, *perpetually and without end.*

The first of these senses I take to be Sir *Isaac Newton*'s meaning; and the latter, I suppose is Mr. *Robins*'s. For in this very sense I find him using the word *endless* upon this subject, in the *Republick of Letters* for *October* last, *p.* 255. "Here the first lemma is applied to prove, that by multiplying the number, and diminishing the breadth of these parallelograms *in infinitum*, that is, *perpetually and without end*, the inscribed and circumscribed figures become ultimately equal to the curvilinear space, and to each other; whereas it is evident, that no point of time can be assigned, wherein they are actually equal; to suppose this, were to assert, that the variation ascribed to these figures, though ENDLESS, could be brought to a period, and be perfectly accomplish'd; and thus we should return to the unintelligible language of indivisibles."

And here I cannot but observe, that it is with great prudence, Mr. *Robins* has chosen to understand the words in this latter sense. For, grant him that the words, *in infinitum*, are to be rendered *endlessly*, i.e. *perpetually and without end*, and he may then justly say, *it is evident that no point of time can be assigned, wherein the figures are actually equal; it is manifest, that the subdivision can never be actually finished, or brought to a period.* But this, I think, is what some censorious people call *petitio principii.*

I must likewise, in justice to Mr. *Robins*, take notice of the singular art and skill he has shown, in preparing us for this his interpretation of the words *in infinitum.* Sir *Isaac Newton*"s direction is, *parallelogrammorum latitudo minuatur, & numerus augeatur in infinitum*; the

* Republick of Letters for *January*, p. 76.

† Rep. of Lett. for *April*, p. 310.

‡ *V.* infra.

words, *in infinitum*, belong equally to the former part of the direction, *latitudo minuatur*, and to the latter, *numerus augeatur*; and must therefore be interpreted in such manner, as to suit them both. But with Sir *Isaac Newton* to diminish a quantity *ad infinitum*, is to diminish it till it vanishes into nothing; and this necessarily implies a finite time, (for if it does not vanish in a finite time, it cannot vanish at all) and not a diminution proceeding *perpetually and without end*. And since the number of the parallelograms always increases as the breadth decreases, and no otherwise; it is plain that in whatever time the diminution of the breadths is compleated, in that very same time, and no other, the increase of the number must likewise be compleated, that is in a finite time. My Reader will hence observe, that it did not at all suit Mr. *Robins*'s purpose, to deliver the whole direction, and in the manner Sir *Isaac* gave it. Accordingly in the month of *October*, in the passage last quoted, the order of the precept is inverted; the former part, *latitudo minuatur*, is placed hindmost, that it might be taken less notice of; and I having winked at that trespass, it is now in the month of *April* entirely omitted, as if it had been no part of Sir *Isaac Newton*'s direction, though the stress of his demonstration is principally founded upon it; and the other part, *numerus augeatur in infinitum*, is left alone, which being more easily capable of two senses, is interpreted in that sense which Mr. *Robins* finds most convenient.

I do not remember that I have ever used the expression, *infinitely great*, in speaking of this lemma; but I must confess I see no great harm in it, if any body should happen to do so, provided he gave his Reader to understand that all he meant by it, was only, *let the number increase without end, or limit of its magnitude*; just as Sir *Isaac Newton* has done at the end of the general *Scholium* to his first Section; where the expression, *in infinite maguis*, is explained in the next line, *si quantitates augeantur in infinitum*; and we are presently after given to understand, that there can be no *quantitates maximæ*, or *ultimæ*, but that by *in infinitum* is to be understood *sine limite*.

But why so great fear of any term that has ever been made use of in the language of indivisibles? May not an Orthodox Writer and a Heretick sometimes use the same term, without embracing one another's opinions? If this bugbear of indivisibles is always to be brought in, *in terrorem* to those who presume to differ from Mr. *Robins*, not only in sentiment, but even in *mode of expression*, possibly it may have a better effect, were it dignified with a termination in *ism*. What if we should call it *Indivisibleism*? But to proceed.

XV. "We, says Mr. *Robins*, perform what is here directed, by that simple and obvious method practiced by the ancient geometers, of continually subdividing the base of the curve. And it is manifest, that such subdivision can never be actually finished."

I apprehend our present enquiry is, not how Mr. *Robins* performs what is here directed; but how Sir *Isaac Newton* intended it should be performed; and if I have rightly * shown this, it will not be very material to consider how Mr. *Robins* performs it, nor indeed can I well understand in what manner he performs it.

There are two ways, whereby we may conceive the base of the curve to be continually subdivided. One is the method practiced by *Euclid* and the other *ancient Geometers*, which consists in continually repeating the operation directed in the tenth proposition of the first

* Republick of Letters for *November*, p. 375, *& seq.* and for *January*, p. 84, *& seq.*

book, or in the tenth proposition of the sixth book of the Elements. And it is very true, *that such subdivision can never be actually finished.*

The other method is agreeable to Sir *Isaac Newton*'s constant practice, in supposing a line to be described by a point. If a given line, as for example, the breadth of this page, be described by a point moving uniformly, in a given time, as for instance, the time of an hour; it is manifest that in half an hour the point will arrive at the middle of the line, *i. e.* the point will at that instant bisect the line. And in a quarter of an hour more the point will bisect the remaining half of the line: And in an eighth of an hour more the point will bisect the remaining quarter of the line: And in a sixteenth part of an hour more the point will bisect the remaining eighth part of the line, *&c., in infinitum.* That is, all the possible bisections or subdivisions of the line will happen exactly at those points of time, in which the hour is alike bisected or subdivided. But all the possible subdivisions of the hour will *be actually finished*, and *brought to a period*, at the end of the hour. Consequently all the possible subdivisions of the line will likewise *be actually finished*, and *brought to a period*, at the end of the hour. Consequently, the *multiplication of the parallelograms* conceived to be erected upon the subdivided parts of the line, will be *actually finished*, and *brought to a period*, at the end of the hour. So that, although *the bases of these parallelograms be constantly equal, and each some aliquot part of the whole base*, yet *such description by continued motion is* not *necessarily excluded.* So that the second *lemma* may have a distinct *demonstration compleat within itself*, altho' the subdivision of the line by continued motion is suited both to that and the third *lemma.*

Now if Mr. *Robins* will *perform what is directed*, by the first of these methods, it will be proper for him to shew, that this was the design of Sir *Isaac*'s direction; and here I must desire him to consider, that Sir *Isaac*'s direction of diminishing the breadths and increasing the number of the parallelograms *in infinitum*, extends to the third *lemma* equally with the second, though in that *lemma* Mr. *Robins* will not pretend to *perform what is directed*, by the method of the ancients.

And if he chuses the second method, we must ask, Whether any, and how much of the subdivision of the line wants to *be actually finished* at the end of the hour?

But to cut off all dispute about what method Mr. *Robins*, or I, shall chuse to take of performing what is here directed, I must beg leave to repeat what I have already urged against that Gentleman in *January* last. * "Sir *Isaac Newton* says not one word of continual division, or subdivision into parts, of the base of the figure, in order to describe the parallelograms. And his words of diminishing the bases, and increasing their number *ad infinitum*, certainly are no more applicable to a continual division, than to lessening the bases by a continued motion. But this latter method is agreeable to his constant practice, and fully answers his purpose, which the former will not. Equity therefore requires, that his words should be taken in the latter sense."

XVI. By this one plain and simple motion, indeed the plainest and simplest possible, of a point describing a line, the line described may be conceived not only to be continually bisected in the manner just now mentioned; but also to be divided into any other sort of *aliquot* parts; as likewise into parts not *aliquot*, after the manner we described in *January*

* Republick of Letters for *January*, p. 90.

* last, where all are equal except one less than all the rest; and also into unequal parts described in equal particles of time, where the velocity is not uniform, as when this *lemma* is applied by Sir *Isaac Newton in the first proposition of the second section of the first book of the Principia*; and all these divisions of every sort will *be actually finished* at the end of the hour. And in which soever of these manners we chuse, at any instant of time, to consider the line as being divided, the contemplation of the parts, into which it is supposed to be divided, is alike easy to the mind. I see nothing *complex*, nor *intricate*, nor *subtle*, nor *involved and perplexed to strain our imagination*, much less *confused*, either in the idea of this motion, or in the contemplation of the parts, into which the line by this motion is supposed to be divided. And it is as general, as it is easy: One and the same motion will serve *for every variation of these inscriptions and circumscriptions.*

It will perhaps be said, that the perplexity arises, not from the consideration of the parts into which the line is divided, but from the contemplation of the parallelograms which are conceived to be erected upon those parts. But is the contemplation more difficult or intricate, when the line is conceived to be divided by motion, than when it is conceived to be perpetually bisected, or trisected, after the *method practiced by the ancient Geometers*? If after ever bisection in *Euclide*'s method, Mr. *Robins* can conceive parallelograms erected upon the two parts of the line last bisected, and upon every part of the given line equal to those two; may not I likewise, after every bisection by motion, conceive the same parallelograms erected upon the very same parts? Certainly I may. When once I have represented to my mind the parts, into which a line is divided, I can with equal facility conceive parallelograms erected upon those parts, by whatsoever method, and indeed without entering at all into the consideration by what method the line became so divided. And by what we have above delivered, it is manifest, that all the possible subdivisions both of the hour, and of the line described during the hour, and consequently all possible *multiplication of the parallelograms* conceived to be erected upon the subdivided parts of the line, will *be brought to a period* by the end of the hour.

If Mr. *Robins* will tell me, that the imagination cannot pursue these parallelograms to the very end of the hour; I may ask him, whether the imagination can any better pursue the subdivision of the line, or even of the hour itself, to the end of the hour, which subdivisions he must own to *be brought to a period* by the end of the hour.

But there is no need *to strain our imagination, to labour in any case*, or indeed in any case, *after some idea of motion however intricate*; no *subtle enquiry is at all necessary*, since we are obliged to own the conclusion to be true and certain, either by the proof given above, or by the demonstration of Sir *Isaac Newton*'s first *lemma*, which being general comprehends all particular cases, without ever troubling our imagination about them.

The Remainder of this Piece will be publish'd in our next.

* Republick of Letters for *January*, p. 90.