ON THE INSCRIPTION OF CERTAIN “GAUCHE” POLYGONS IN SURFACES OF THE SECOND DEGREE

By

William Rowan Hamilton

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On the Inscription of certain “Gauche” Polygons in Surfaces of the Second Degree.

By Sir William R. Hamilton.

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In the same note to the Secretary, it was requested by Sir William R. Hamilton that the Academy might be informed of a theorem respecting the inscription of certain gauche polygons, in surfaces of the second degree, which he had lately communicated to the Council. This theorem was obtained by the method of quaternions, and included, as a particular case, the following:—“If the first, second, third, and fourth sides of a gauche nonagon, inscribed in a surface of the second order, be respectively parallel to the fifth, sixth, seventh, and eighth sides of that nonagon, and also to the first, second, third, and fourth sides of a gauche quadrilateral, inscribed in the same surface; then the plane containing the first, fifth and ninth corners of the nonagon will be parallel to the plane which touches the surface at the first corner of the quadrilateral.”

More generally the theorem here referred to shews that for the inscribed quadrilateral we may substitute a gauche polygon with any even number, $2n$, of sides; and for the nonagon, another gauche polygon, with $4n + 1$ sides, connected with that polygon of $2n$ sides, by the same law of construction as that which had connected the nonagon with the quadrilateral; and that then the tangent plane to the surface at the first corner of the polygon of $2n$ sides, will be parallel to the plane through the first, middle, and last corners $(1, 2n + 1, 4n + 1)$ of the polygon of $4n + 1$ sides.