CKM matrix with lattice QCD

– the full determination using recent results –

Masataka Okamoto (KEK)

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \end{pmatrix}$$

Prologue

CKM matrix — parameter in Standard Model

$$V_{\rm CKM} = \begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

weak eigenstates $\overset{V_{\rm CKM}}{\longleftrightarrow}$ mass eigenstates

$VV^\dagger = 1 \quad \Longrightarrow$ 4 independent parameters $\{\lambda, A, \rho, \eta\}$.

role of Lattice QCD:

calculate hadronic amplitudes (nonpertubative effects) from 1st principles

"CKM matrix with lattice QCD" $V_{\mathrm{CKM}}^{\mathrm{LQCD}}$

For each CKM element, there exists hadronic processes whose amplitudes can be *reliably* calculated from LQCD —

gold-plated quantities: at most one hadron in initial/final states.

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\ D \to l\nu & D_s \to l\nu & \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$

$$\langle B|\overline{B}\rangle, \langle K|\overline{K}\rangle, \sin(2\beta) \Longrightarrow \{\rho, \eta\}$$

Given recent developments (unquenching, improved actions, machines etc.), we are now in a good position for the *full determination of* $V_{\mathrm{CKM}}^{\mathrm{LQCD}}$.

First result in unquenched QCD (M.O, hep-lat/0412044)

$$V_{\rm CKM}^{\rm Lat} = \begin{pmatrix} |\mathbf{V_{ud}}| & |\mathbf{V_{us}}| & |\mathbf{V_{ub}}| \\ 0.9744(5)(3) & 0.225(2)(1) & 3.5(5)(5)\times 10^{-3} \\ |\mathbf{V_{cd}}| & |\mathbf{V_{cs}}| & |\mathbf{V_{cb}}| \\ 0.24(3)(2) & 0.97(10)(2) & 3.9(1)(3)\times 10^{-2} \\ |\mathbf{V_{td}}| & |\mathbf{V_{ts}}| & |\mathbf{V_{tb}}| \\ 8.1(2.7)\times 10^{-3} & 3.8(4)(3)\times 10^{-2} & 0.9992(0)(1) \end{pmatrix}$$

value(lat.err)(exp.err)

$$\lambda = 0.225(2)(1) \quad , \quad A = 0.77(2)(7) \quad , \quad \rho = 0.16(28) \quad , \quad \eta = 0.36(11)$$

- $|V_{us}|, |V_{ub}|, |V_{cd}|, |V_{cs}|, |V_{cb}|$ and $\{\lambda, A\}$ obtained from 5 semileptonic decays with $n_f = 2+1$ LQCD calc.(by FNAL/MILC)
- rest of $|V_{qQ}|$ and $\{\rho, \eta\}$ obtained with CKM unitarity & $\sin(2\beta)_{B\to\psi K}$ result

NOTE: theory inputs (for nonperturbative QCD effects) are LQCD *only*.

A purpose of this talk

Present a World Average (WA) of $V_{\mathrm{CKM}}^{\mathrm{Lat}}$ in 2005
(along a review on recent results)
using current best results for gold-plated quantities.

For WA, I exclude:

- non-gold-plated quantities (ε'/ε etc.)
- quenched ($n_f = 0$) lattice QCD
- non-lattice model calculations (Light-cone sum rule etc.)
- inclusive decays ($B \to X l \nu$ etc.)

This is a *biased* WA, but (I believe) such an average is useful, at least, to announce the status/ability of LQCD.

Outline of this talk

CKM magnitude $|V_{qQ}|$

- D meson decays $\Longrightarrow$ $|V_{cd}|,\ |V_{cs}|$
- B meson decays $\Longrightarrow$ $|V_{ub}|,\ |V_{cb}|$
- (K meson decays $\Longrightarrow$ $|V_{us}|$)

CKM phase $\{\rho,\eta\}$

- B meson mixing, $f_{B_q}^2 B_{B_q}$
- (K meson mixing, B_K)
- Unitarity Triangle analysis $\Longrightarrow$ $\{\rho,\eta\}$

Summary/Outlook

Topics *not* covered in this talk

I apologize for not covering topics (in HQ session) below: topic (presenter)

Heavy hadron spectra and heavy quark masses

- $Q\bar{Q}$ mass (Gottlieb), Heavy-light meson (Koponen, Foley)
- m_c (Nobes), m_b and HQET (Sommer, Garron, and Negishi)

Other developments

- Renormalon (Pineda), Pionic coupling (Becirevic)
- B mixing in P.T (Palombi) and in quenched QCD (Blossier)
- N.P tuning of action (Lin), Radiative transitions in $c\bar{c}$ (Richards)
- J/ψ-Hadron interaction (Yokokawa), heavy quark potential (Koma)

CKM magnitude from D meson decays

$$D \to \pi(K) l\nu, \; D_{(s)} \to l\nu$$

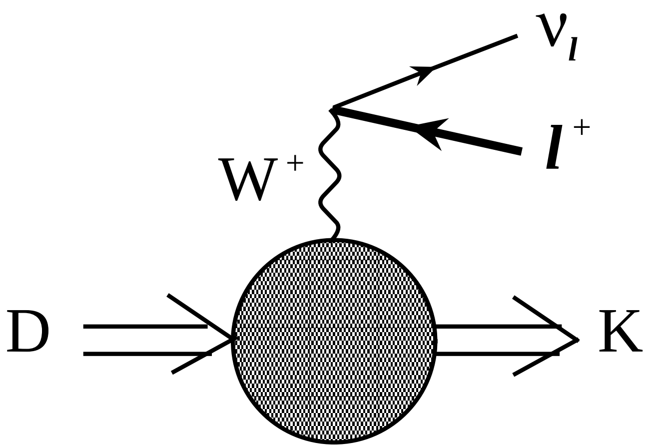
$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \end{pmatrix}$$

- Check LQCD (for B physics)
  by comparing LQCD results ($f_+^{D\to\pi/K}$, $f_{D_{(s)}}$) with Exp't ($|V_{cq}|$ input)
- Determine $|V_{cq}|$ by combining LQCD+Exp't

Semileptonic decay (one example) :

$|\mathbf{V_{cs}}|$ from semileptonic $D \to Kl\nu$ decay

Experiment



$$\equiv \Gamma(D \to Kl\nu) \propto \int_0^{q^2_{\max}} dq^2 \, |f_+(q^2)|^2 |\mathbf{V_{cs}}|^2$$

$$(q = p_D - p_K)$$

Lattice

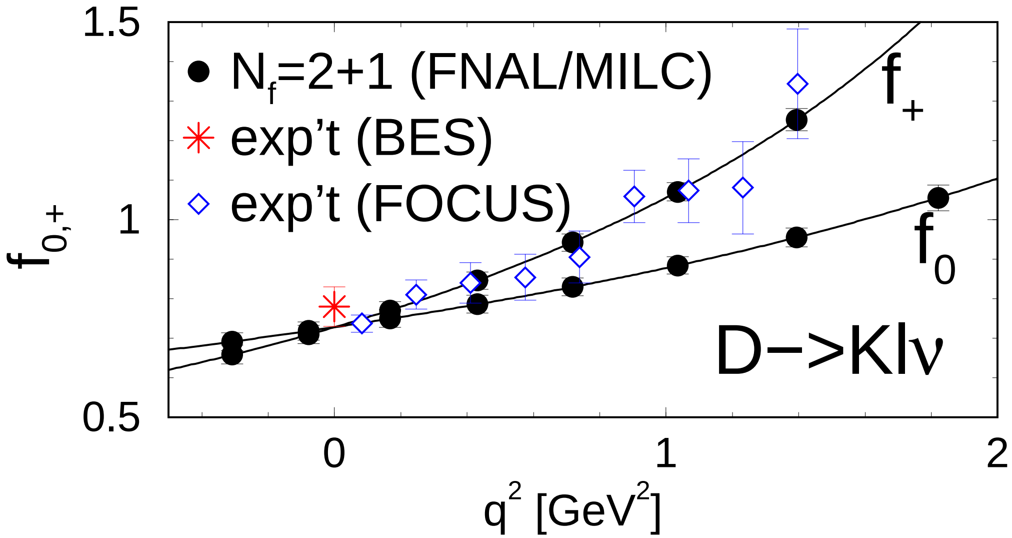
$$\langle K(p_K)|V^\mu|D(p_D)\rangle = f_+(q^2)\left[p_D + p_K - \frac{m_D^2 - m_K^2}{q^2}\, q\right]^\mu + f_0(q^2)\,\frac{m_D^2 - m_K^2}{q^2}\, q^\mu$$

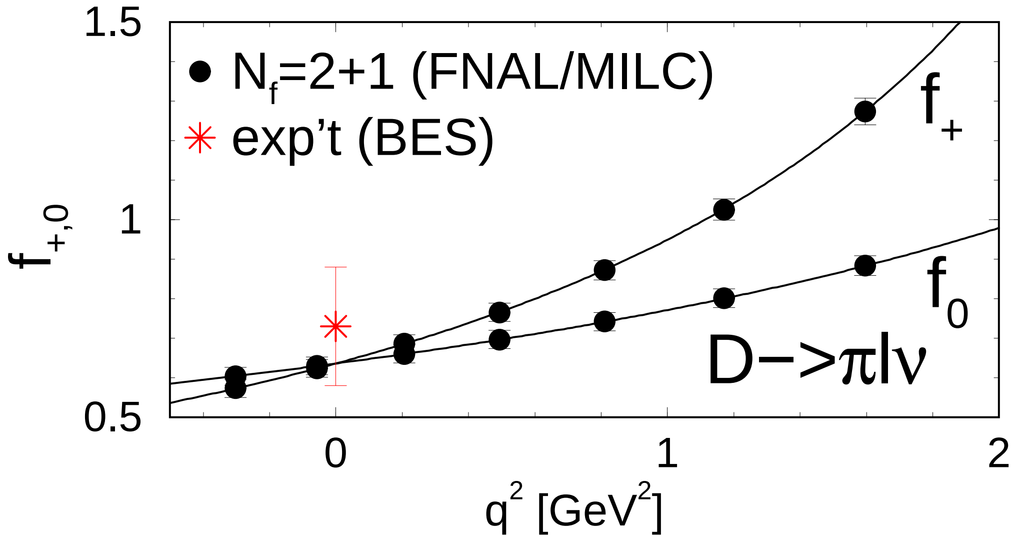
$D \to \pi l\nu$ and $D \to Kl\nu$ results

FNAL/MILC, hep-ph/0408306;PRL (Mackenzie's poster)

$n_f = 2+1$ (MILC "coarse"), improved staggered light + FNAL(clover) heavy

small χfit error,3%; finite-a,9% $\Longrightarrow$ total 10% error





Agree with Exp't for $D \to K(\pi)l\nu$ (both norm & shape)

$\Longrightarrow$ credibility of $B \to \pi l\nu$ result

See Kronfeld's poster for more comparisons with Exp't.

D, B physics with staggered u, d, s + Wilson(NRQCD) c, b

(M. Wingate *et.al.*, 2001)

- compute staggered propagator $\langle \bar{\chi}(x)\chi(y)\rangle$ ($\Longrightarrow$ smaller m_q accesible!)
- convert stag quark (1-comp) prop $\longrightarrow$ “naive” quark (4-comp) prop:

$$\Omega(x)^\dagger \langle \bar{\chi}(x)\chi(y)\rangle \Omega(y) = \langle \bar{\psi}(x)\psi(y)\rangle \quad \text{with} \quad \Omega(x) = \gamma_0^{x_0}\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}$$

- combine light naive + heavy Wilson in 2-pt and 3-pt functions:

$$M_D \approx m_u + m_c,\ m_u + m_c + \frac{2}{a},\ m_u + m_c + \frac{4}{a}, \cdots$$

$$C^D_{2,3}(t) \propto e^{-M_D\, t} \longrightarrow e^{-(m_u+m_c)t}$$

$\Longrightarrow$ No doubling in 2-pt/3-pt functions!

Succesfully applied to FNAL/MILC,HPQCD calculations of

$f_{D_{(s)}}, f_{B_{(s)}}, D \longrightarrow \pi(K), B \longrightarrow \pi$

Chiral exptrapolation ($m_l \to m_{ud}$) for $D \to \pi$ form factor

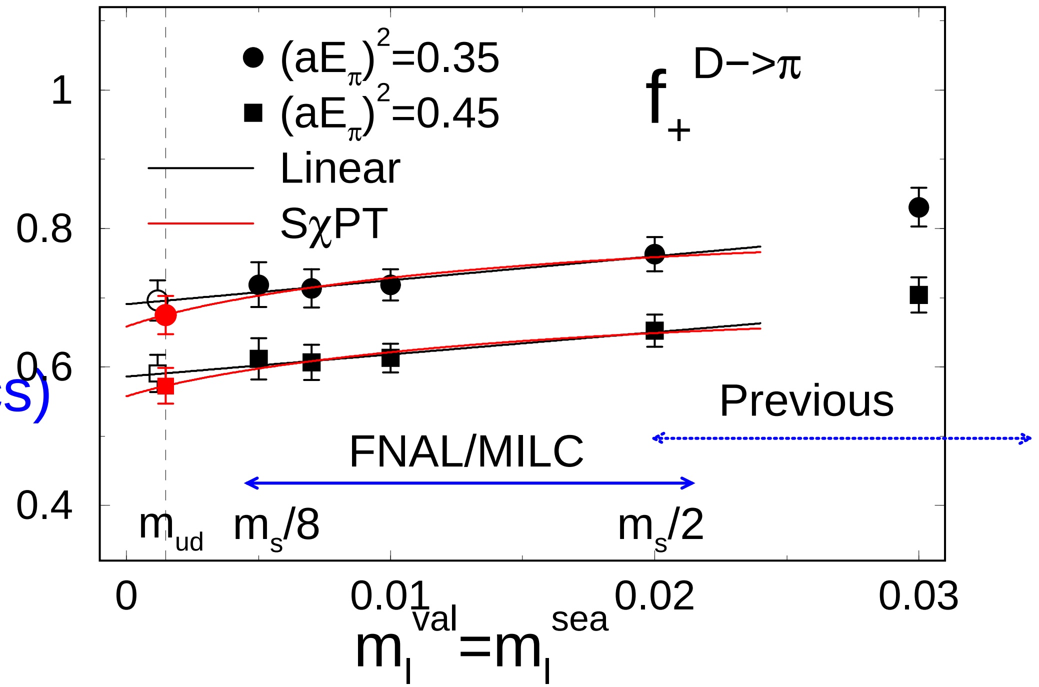
(1) χ fit with Staggered χPT (Aubin&Bernard)

$$f = A(1 + \delta f^{S\chi PT}) + Bm_l$$

$\delta f^{S\chi PT}$ contains (staggered) χ-log:

$\frac{1}{16}\sum_{\xi}^{P,A,T,V,I} M_{\pi,\xi}^2 \log(M_{\pi,\xi}^2) \times n_\xi$

$M_{\pi,\xi}^2 = 2\mu m_l + a^2\delta_\xi$ ($\Longrightarrow$ χ-log diluted)



Constants in SχPT fit:

$f_\pi, g_{DD^*\pi}^2$ & $\delta_\xi, \delta'_{A,V}$ (*fixed* from light physics)

Free parameters: A, B *only*

(2) Linear χ fit: $f = A' + B' m_l$

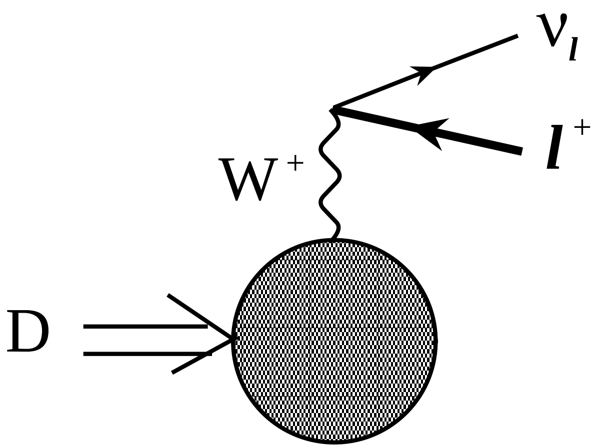
Fit (2) agrees with (1) within $\approx$3%

$\Longrightarrow$ *insensitive* to fit form with $m_l \le m_s/2$

Leptonic decay (one example):

$|\mathbf{V_{cd}}|$ from leptonic $D \to l\nu$ decay

Experiment



$$\equiv \quad \Gamma(D \to l\nu) \quad \propto \quad |f_D|^2 |\mathbf{V_{cd}}|^2$$

CLEO-c will measure them to 2–3% accuracy;   8% now (Lepton-Photon'05)

Lattice

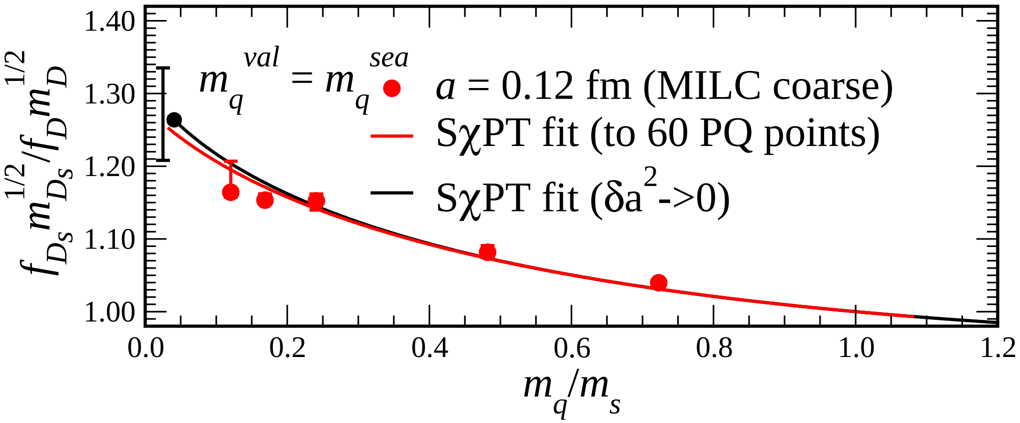
$$\langle 0|A^\mu|D(p)\rangle \quad = \quad f_D \, p^\mu$$

$D \to l\nu$ (f_D) **results**

FNAL/MILC, hep-lat/0506030

(Simone's poster) $n_f = 2{+}1$, stag light

SχPT fit to Partially Quenched data



$$f_D = 201(03)_{\rm sta}(17)_{\rm sys}\ \text{MeV} \quad \Longrightarrow$$

$\sim$9% error (finite-a& χfit dominate)

$$f_D^{n_f=2} = 202(12)_{\rm sta}(^{+20}_{-25})_{\rm sys}\ \text{MeV}$$ **(CP-PACS, prelim, next slide)**

CLEO-c Exp't,
updated@Lepton-Photon'05

$$f_D = 223(16)_{\rm sta}(^{+07}_{-09})_{\rm sys}\ \text{MeV}$$

(with $|V_{cd}| = |V_{us}| = 0.225$)

$$\mathcal{B}(D \to \mu\nu) =$$
$$4.45(67)(^{+29}_{-36}) \times 10^{-4}$$

$$\Downarrow$$

$$|V_{cd}| = 0.250(22)_{\rm lat}(21)_{\rm exp}$$

new!

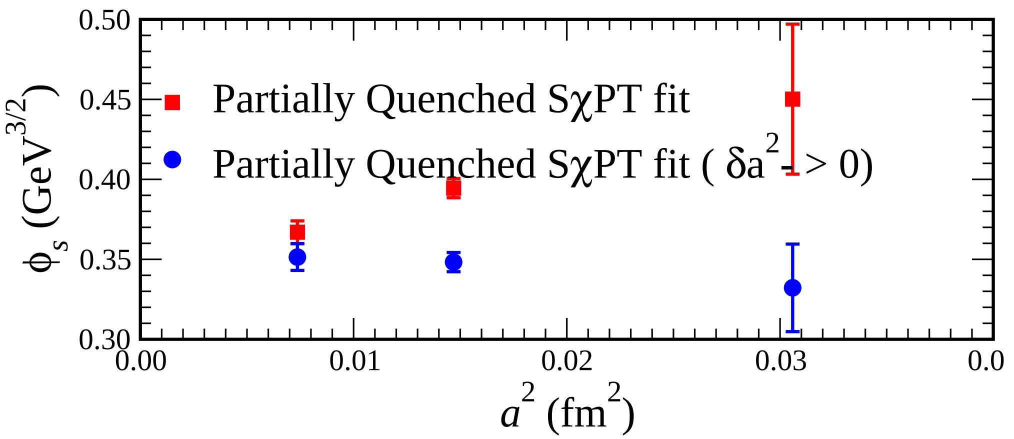
Agree with exp't for f_D $\Longrightarrow$ credibility for f_B

$D_s \to l\nu \ \ (f_{D_s})$ **results**

FNAL/MILC, hep-lat/0506030

(Simone's poster)

$n_f = 2{+}1$, stag light + FNAL heavy



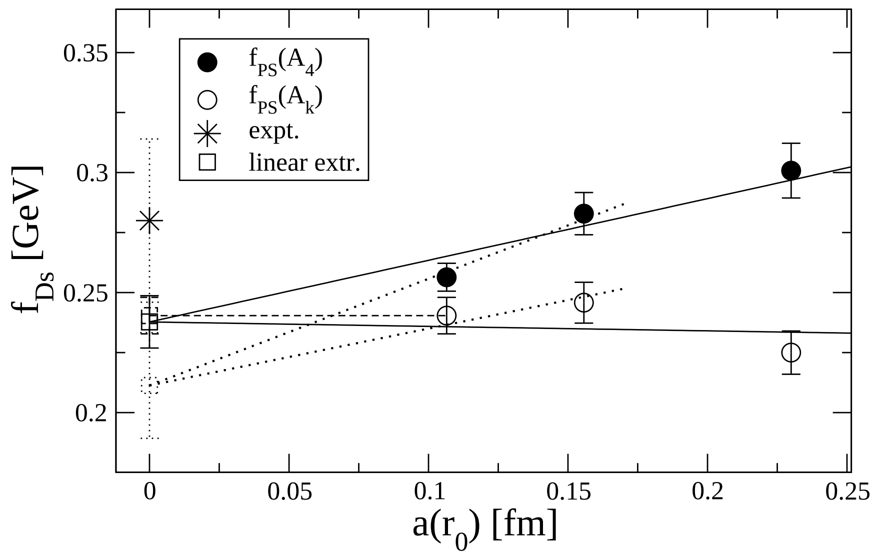
$f_{D_s} = 249(03)_{\rm sta}(16)_{\rm sys}$ MeV

$\sim$7% error (finite-a error largest)

CP-PACS, preliminary

(Kuramashi's talk)

$n_f = 2$, clover light + Tsukuba heavy



$f_{D_s} = 238(11)_{\rm sta}\left({}^{+07}_{-27}\right)_{\rm sys}$ MeV

$\sim$12% error (finite-a error largest)

n_f=0 results with Overlap/DW:**(Dong's talk / Chiu *et al*, hep-ph/0506266)**

CLEO-c will measure f_{D_s}.

Leptonic/Semileptonic ratio

CKM factor $|V_{cq}|$ canceled in the ratio ($\Longrightarrow$ a good test of LQCD):

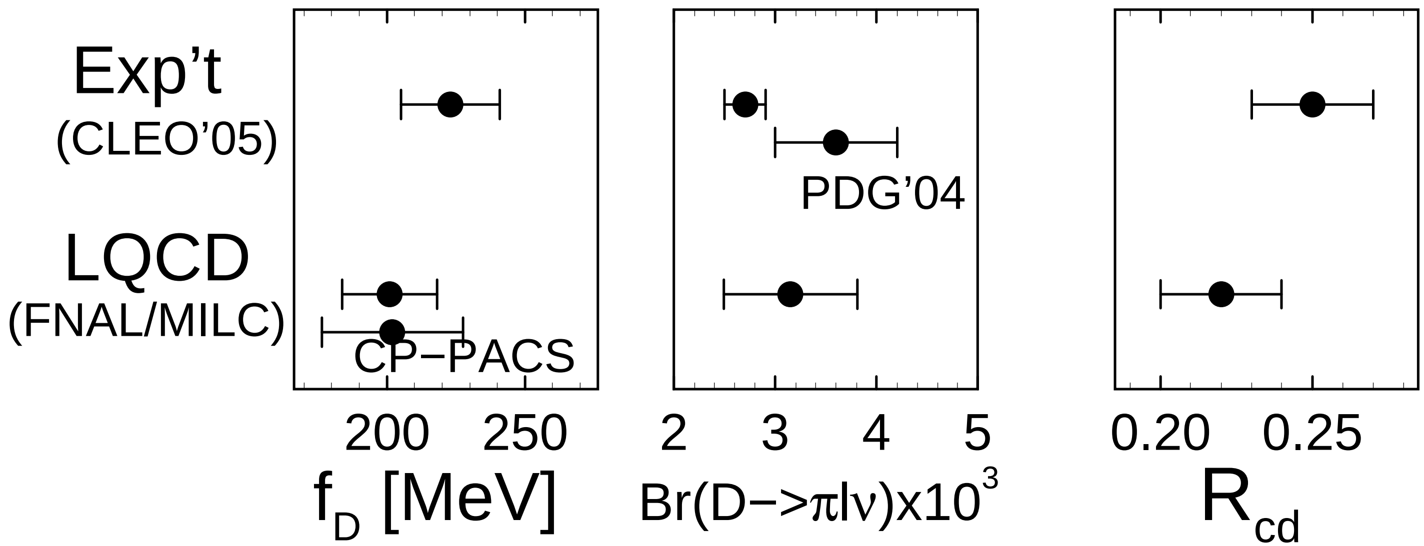
$$R_{cd} \equiv \sqrt{\frac{\mathcal{B}(D \to l\nu)}{\mathcal{B}(D \to \pi l\nu)}} \propto \frac{f_D}{f_+^{D\to\pi}(0)} \cdot \frac{|V_{cd}|}{|V_{cd}|}$$

LQCD($n_f = 2+1$), FNAL/MILC

$R_{cd} = 0.22(2)$

Exp't, CLEO-c'05 etc

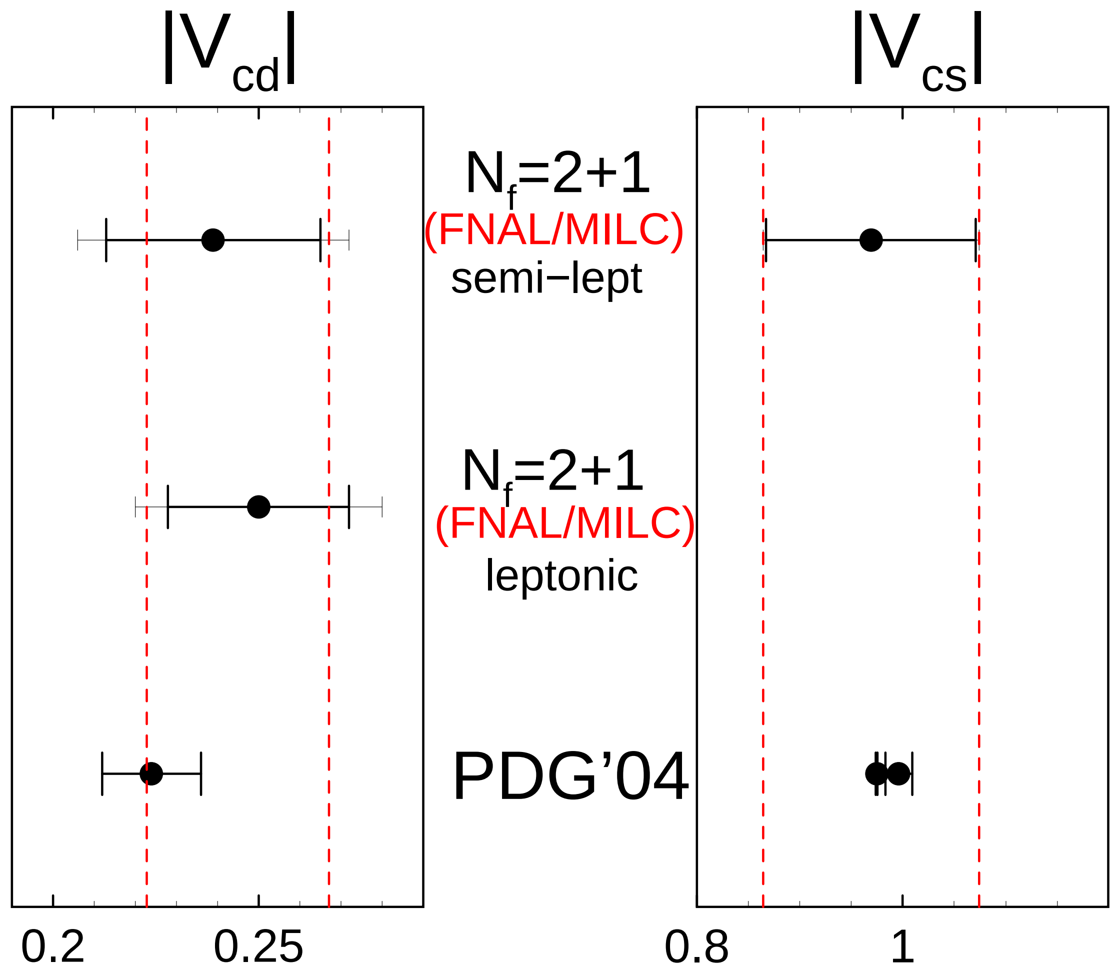
$R_{cd} = 0.25(2)$



Agree with Exp't for D physics. $\Longrightarrow$ credibility for B physics

CKM magnitude from D decays

$$|V_{cd}|^2_{\rm semi-lep} \propto \frac{\mathcal{B}(D\to\pi l\nu)}{\int dq^2 |f_+(q^2)|^2}, \qquad \text{or} \qquad |V_{cd}|^2_{\rm lep} \propto \frac{\mathcal{B}(D\to l\nu)}{|f_D|^2}$$



$$|V_{cd}|_{\rm Lat05} = 0.245(22) \quad , \quad |V_{cs}|_{\rm Lat05} = 0.97(10)$$

CKM magnitude from B meson decays

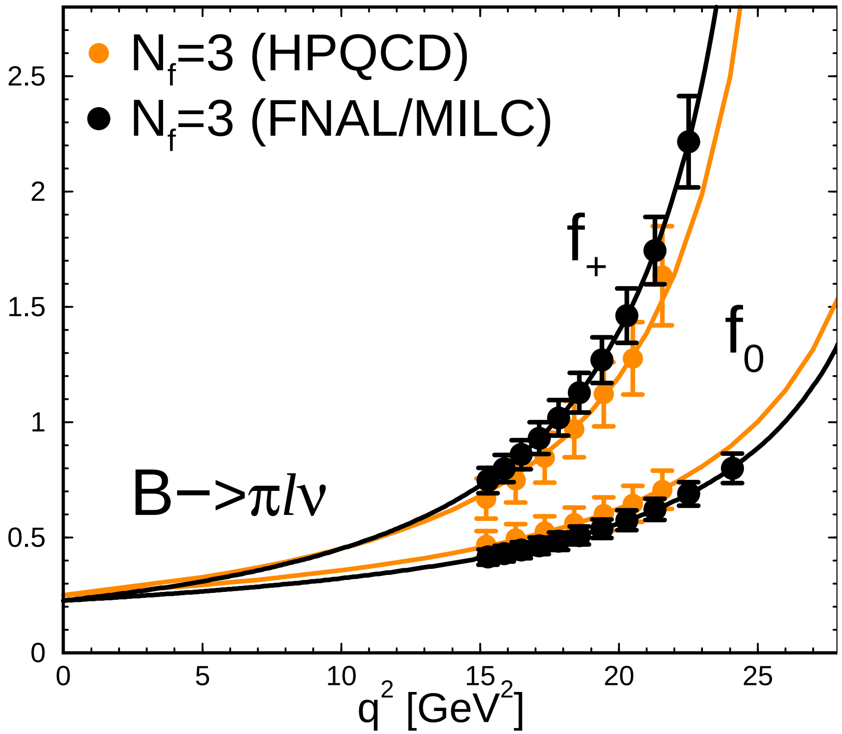
$$B \to \pi(D) l\nu,\ B_{(s)} \to l\nu$$

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \end{pmatrix}$$

- Agreements with Exp't for D decays
  give us confidence in similar quantities for B decays ($f_+^{B\to\pi}$, $f_{B_{(s)}}$ ...)
- Determine $|V_{xb}|$ by combining LQCD+Exp't

$B \to \pi l \nu$ with $n_f = 2+1$ LQCD

FNAL/MILC	stag light + FNAL heavy	$m_l^{\rm val} = m_l^{\rm sea}$	Mackenzie's poster
HPQCD	stag light + NRQCD heavy	fixed $m_l^{\rm sea}$	Gulez's talk



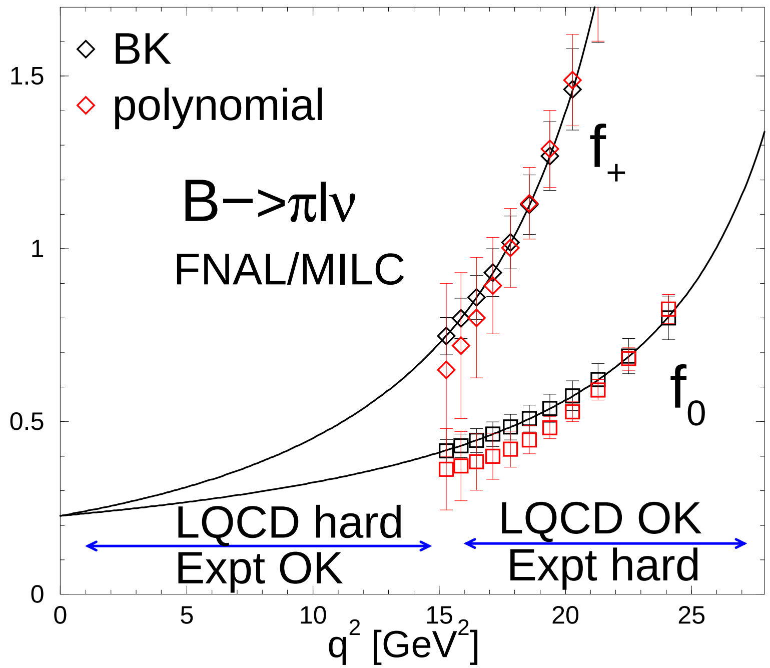
Systematic error	FNAL/MILC	HPQCD
m_l extrapolation	4%	4%
current matching	1%	9%
q^2 dependence	4%	-
finite-a, $1/m_Q$	9%	5%
Total syst	11%	11%

Using branching ratio $\mathcal{B}(q^2 \geq 16~\mathrm{GeV}^2)$ by CLEO'03+Belle'04,

$$|V_{ub}| \times 10^3 = 3.48~(29)_{\rm sta}(38)_{\rm sys}(47)_{\rm exp}$$ [FNAL/MILC; (8+11+13)%=19% error]

$$|V_{ub}| \times 10^3 = 4.04~(20)_{\rm sta}(44)_{\rm sys}(53)_{\rm exp}$$ [HPQCD; (5+11+13)%=18% error]

q^2 dependence of $B \to \pi$ form factors



Method-1 Becirevic&Kaidalov (BK) ansatz (central value)

$$f_+(q^2) = \frac{F}{(1-q^2/M_{D^*})(1-Aq^2)}, \quad f_0(q^2) = \frac{F}{(1-Bq^2)}$$

Method-2 polynomial ansatz (for error estimate)

$$f_{+,0}(q^2) = c^0_{+,0} + c^2_{+,0}q^2 + c^4_{+,0}q^4 + \cdots$$

Difference betw. **M-1** and **M-2** for $|V_{ub}|$ $(\propto \int dq^2 f_+{}^2)^{-1/2}$ is:

4% with $\mathcal{B}(16\ \mathrm{GeV}^2 \le q^2 \le q^2_{\max})$ [mostly interpolation]

11% with $\mathcal{B}(0\ \mathrm{GeV}^2 \le q^2 \le q^2_{\max})$ [long extrapolation]

Q. How can we reach lower q^2 ?

Solution-1. combine LQCD (higher q^2) + non-LQCD (lower q^2)

(1) Arnesen *et.al.*, hep-ph/0504209

LQCD(higher q^2) + disp relation(q^2 dep) + SCET($q^2 = 0$)

$\Longrightarrow$ $\delta|V_{ub}| \approx 13\%$

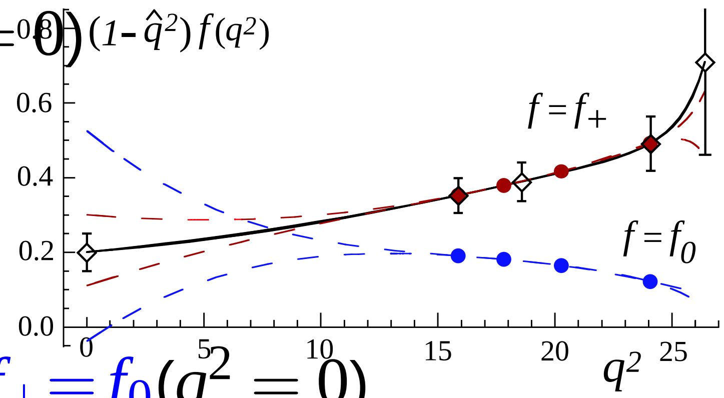
disp relation is *not* a model, but an analyticity bound.

Mackenzie's poster: similar attempt

(2) Flynn's poster (Albertus *et.al.*, hep-ph/0506048)

LQCD(higher q^2) + disp relation (q^2 dep) + LCSR($q^2 = 0$)

$\Longrightarrow$ $\delta|V_{ub}| \approx 11\%$ (prelim.)



(3) Fukunaga&Onogi, hep-lat/0408037

LQCD(higher q^2) + disp relation + $\frac{d\Gamma_{\rm exp}}{dq^2}$ (q^2 dep) + $f_+ = f_0(q^2 = 0)$

$\Longrightarrow$ $\delta|V_{ub}| \approx 14\%$

$|V_{ub}|$ more accurately determined than LQCD alone ($\delta|V_{ub}|_{\rm Lat05} \approx 18\%$)

Solution-2. direct LQCD simulation using "moving NRQCD"

Moving NRQCD is a generalized version of NRQCD
in B meson moving frame $(\mathbf{u} = \mathbf{p}_B/M_B \neq \mathbf{0})$:

$$\mathcal{L}_{\mathrm{mNRQCD}} = \psi^\dagger \left(iD_t + i\mathbf{v}\cdot\mathbf{D} + \frac{\mathbf{D}^2}{2\gamma m} - \frac{\mathbf{v}\cdot\mathbf{D}^2}{2\gamma m} + \cdots \right) \psi$$

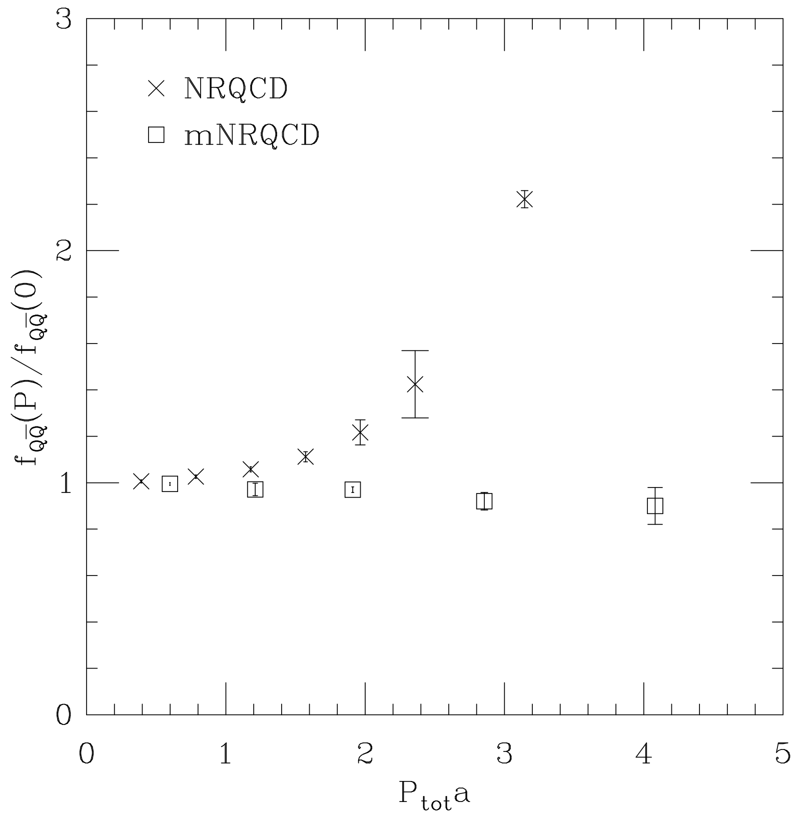
where $u_\mu = \gamma(1,\mathbf{v})$, $\gamma^{-1} = \sqrt{1-\mathbf{v}^2}$.

mNRQCD allows $B \to \pi l \nu$ calc at lower q^2 with smaller $\mathbf{p}_\pi$:
For $\mathbf{v} \approx 0.75$, $\mathbf{p}_\pi = 1\mathrm{GeV} \implies q^2 = (p_B - p_\pi)^2 = 0$

- proposed $>$5 years ago (Hashimoto&Matsufuru'96, Sloan'98 etc.), but suffered from large statistical error.
- Foley *et al* (@Lat'04) showed stat error can be reduced using a special smearing function $\implies$ $\mathbf{v} \approx 0.7 - 0.8$ works

Solution-2. direct LQCD simulation using “moving NRQCD” (cont’d)

Quenched test for $f_{Q\bar{Q}}$ looks encouraging. (Dougall’s talk)



Expect $B \rightarrow \pi l \nu$ (and $B \rightarrow K^*(\rho)\gamma$) calc at $q^2 \sim 0$ will follow.

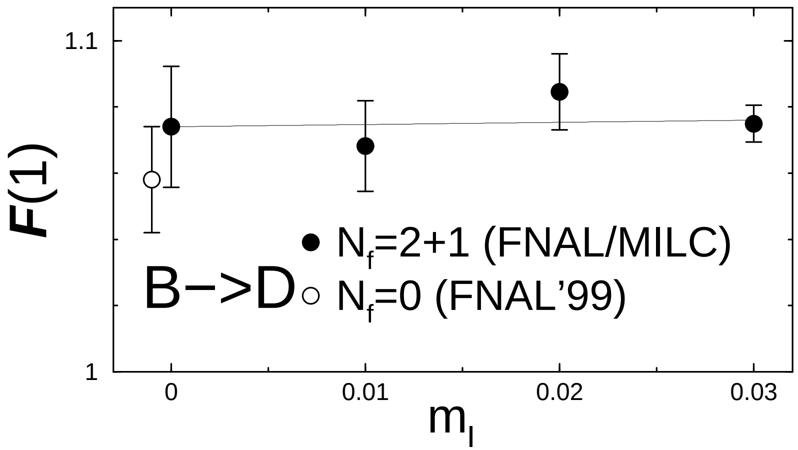
$B \to D^{(*)} l\nu$ decay

$$\mathcal{B}(B \to D^{(*)} l\nu) \;\propto\; |V_{cb}|^2 |\mathcal{F}_{B\to D^{(*)}}(1)|^2 \int dw\, f^{(*)}(w)$$

where $w = v_B \cdot v_D$. Use double ratio (FNAL'99): $\frac{C^{DV_0B}(t)C^{BV_0D}(t)}{C^{DV_0D}(t)C^{BV_0B}(t)} \longrightarrow \frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle}$

$B \to Dl\nu$

$n_f = 2+1$, FNAL/MILC



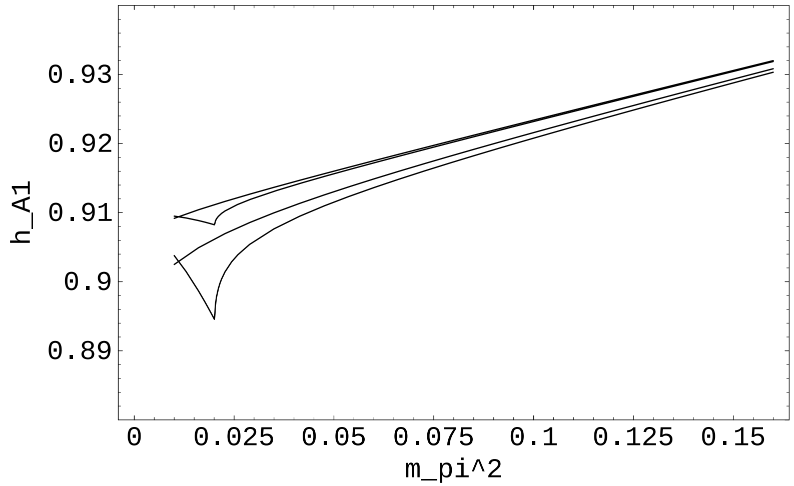
$\mathcal{F}_{B\to D}(1) = 1.074\,(18)_{sta}(15)_{sys}$

Using HFAG'04 avg for $|V_{cb}|\mathcal{F}(1)$,

$|V_{cb}|_{Lat05} = 3.91(09)_{lat}(34)_{exp} \times 10^{-2}$

$B \to D^* l\nu$

SχPT calc. completed (Laiho's talk)



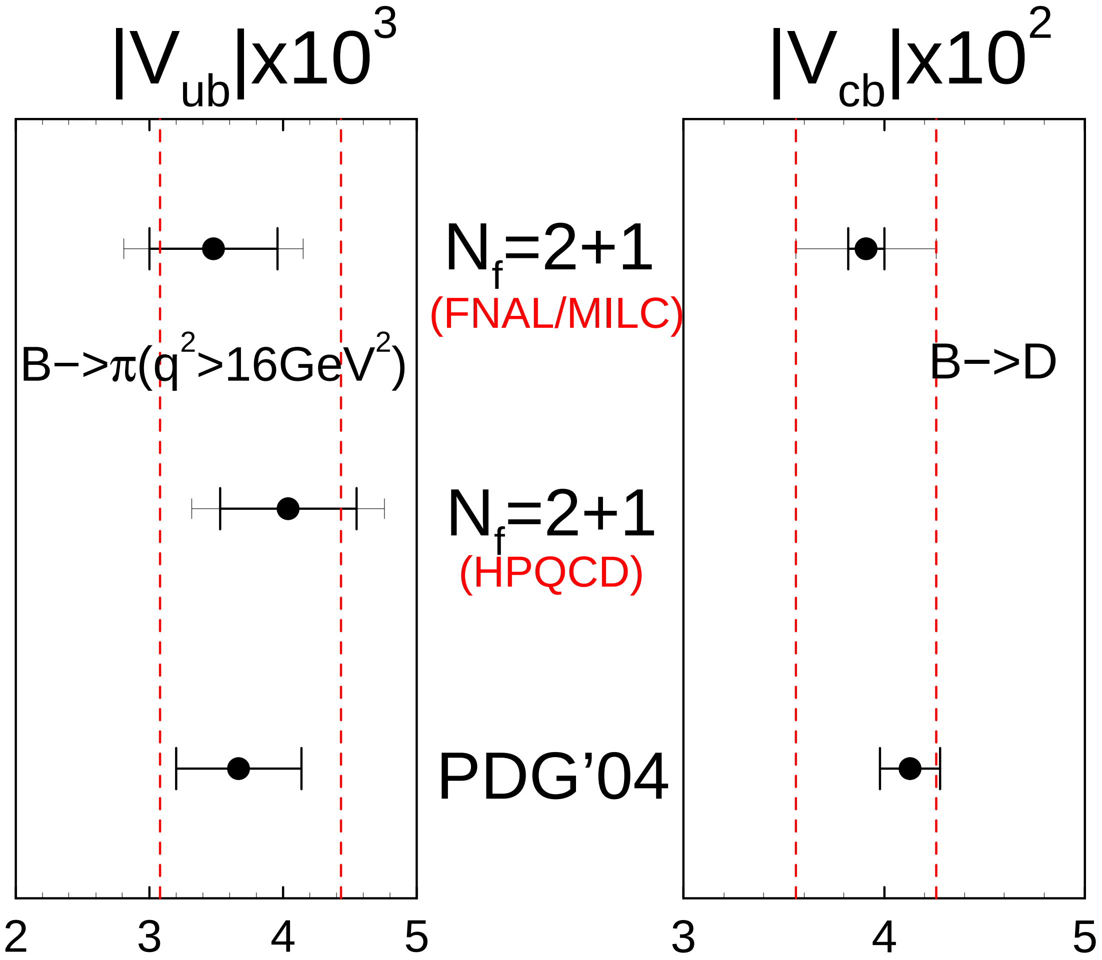
Cusp (in χPT) disappears in SχPT

$n_f = 2+1$ calc. underway (FNAL)

$\Longrightarrow$ more precise $|V_{cb}|$

CKM magnitude from B decays

$$|V_{ub}|^2_{\text{semi-lep}} \propto \frac{\mathcal{B}(B\to\pi l\nu)}{\int dq^2 |f_+(q^2)|^2}, \quad \text{and} \quad |V_{cb}|_{\text{semi-lep}} \propto \frac{[|V_{cb}| * \mathcal{F}(1)]_{\text{exp}}}{\mathcal{F}(1)}$$



$$|V_{ub}|_{\text{Lat05}} = 3.76(68)\times10^{-3} \quad , \quad |V_{cb}|_{\text{Lat05}} = 3.91(35)\times10^{-2}$$

CKM magnitude $|V_{us}|$ from K decays (see Dawson's review for details)

$K \to \pi l \nu$ (use double ratio, as in $B \to D l \nu$)

preliminary unquenched results with Wilson-light:

$$f_+(0) = \begin{cases} 0.954(9) & (n_f = 2\text{, JLQCD, Tsutsui's talk}) \\ 0.962(6)(9) & (n_f = 2+1\text{, FNAL, hep-lat/0412044}) \\ 0.960(5)(7) & (n_f = 0\text{, Becirevic et al, hep-lat/0403217}) \end{cases} ,$$

$n_f = 2$ calc. with DWF (RBC, Kaneko's poster)

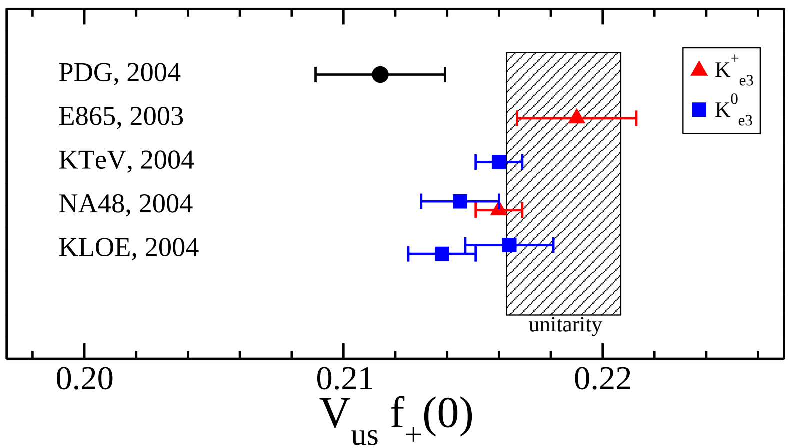
calc. with stag quarks is interesting (needs SχPT)

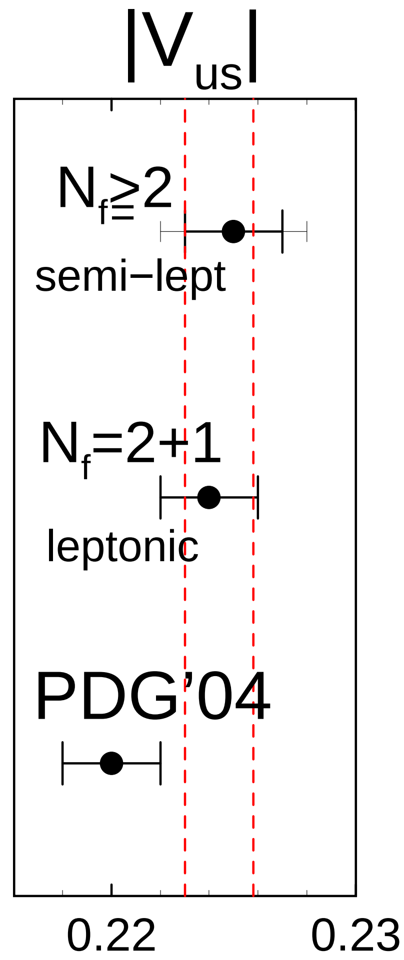
$$\begin{aligned} |V_{us}|_{\text{semi-lep}} &= [|V_{us}| * f_+(0)]_{\text{KTeV}} / f_+(0)^{n_f \geq 2} \\ &= 0.2250\ (24)_{\text{lat}}(12)_{\text{exp}} \end{aligned}$$

$\underline{K \to l\nu}$

$f_K/f_\pi = 1.200(4)\left(^{+17}_{-05}\right)$ ($n_f = 2+1$, MILC)

$\Longrightarrow \quad |V_{us}|_{\rm lep} = 0.2238\left(^{+12}_{-32}\right)$ (Bernard's poster)





$|V_{us}|_{\rm Lat05} = 0.2244(14)$

CKM matrix with LQCD (2005)

(from leptonic/semileptonic decays)

$$\begin{pmatrix} |\mathbf{V_{ud}}| & |\mathbf{V_{us}}| & |\mathbf{V_{ub}}| \\ & 0.2244(14) & 3.76(68)\times10^{-3} \\ |\mathbf{V_{cd}}| & |\mathbf{V_{cs}}| & |\mathbf{V_{cb}}| \\ 0.245(22) & 0.97(10) & 3.91(35)\times10^{-2} \\ |\mathbf{V_{td}}| & |\mathbf{V_{ts}}| & |\mathbf{V_{tb}}| \end{pmatrix}$$

5/9 determined with LQCD($n_f \geq 2$)+Exp't.

CKM unitarity check with LQCD

$(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(10)$ $\longrightarrow$ using CKM unitarity...

CKM matrix with LQCD (2005)

(from leptonic/semileptonic decays + unitarity)

$$\begin{pmatrix} |\mathbf{V_{ud}}| & |\mathbf{V_{us}}| & |\mathbf{V_{ub}}| \\ 0.9745(3) & 0.2244(14) & 3.76(68)\times10^{-3} \\ |\mathbf{V_{cd}}| & |\mathbf{V_{cs}}| & |\mathbf{V_{cb}}| \\ 0.245(22) & 0.97(10) & 3.91(35)\times10^{-2} \\ |\mathbf{V_{td}}| & |\mathbf{V_{ts}}| & |\mathbf{V_{tb}}| \\ & 3.79(53)\times10^{-2} & 0.9992(1) \end{pmatrix}$$

8/9 determined.

Wolfenstein parameters: $\lambda = 0.2244(14)$, $A = 0.78(7)$

To extract (ρ, η) and $|V_{td}|$, use LQCD results for $f_B^2 B_B$ & B_K

CKM phase from
B and *K* meson mixings

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \end{pmatrix}$$

$$\{\lambda\, ,\, A\, ,\, \rho\, ,\, \eta\}$$

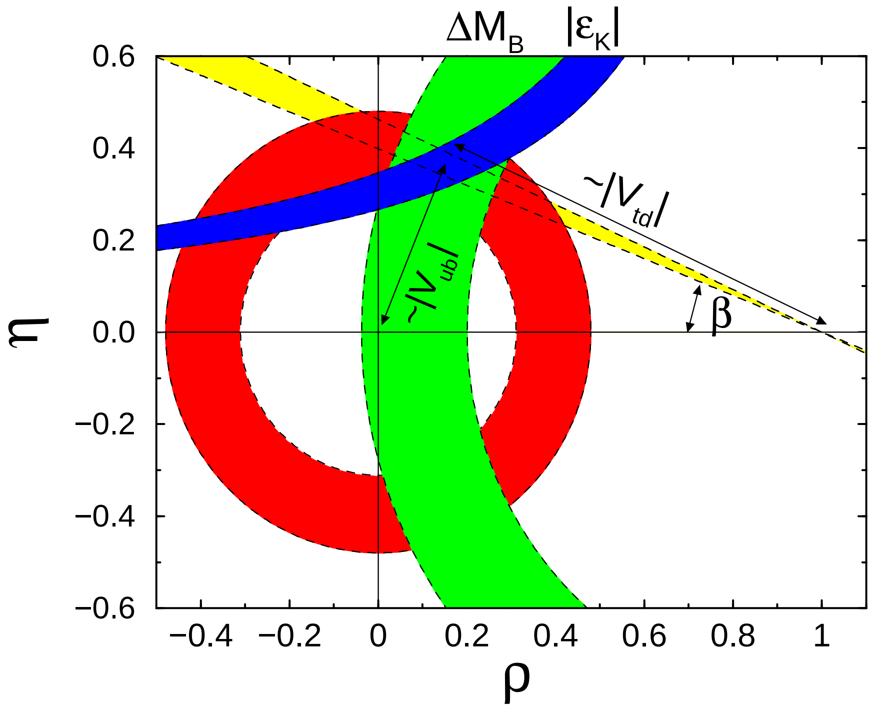
- Determine $\{\rho, \eta\}$ and $|V_{td}|$
- Test Standard Model, if good precision achieved

(1) $B - \bar{B}$ mixing

$$\Delta M_{B_{d(s)}} \propto B_{B_{d(s)}} f^2_{B_{d(s)}} |V^*_{tb} V_{td(s)}|^2$$

lattice:

$$\langle \bar{B}^0 | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B^0 \rangle \propto B_{B_q} f^2_{B_q}$$



(2) $K - \bar{K}$ mixing

$$|\varepsilon_K| = B_K \, \eta [(1-\rho) c_1 + c_2]$$

lattice:

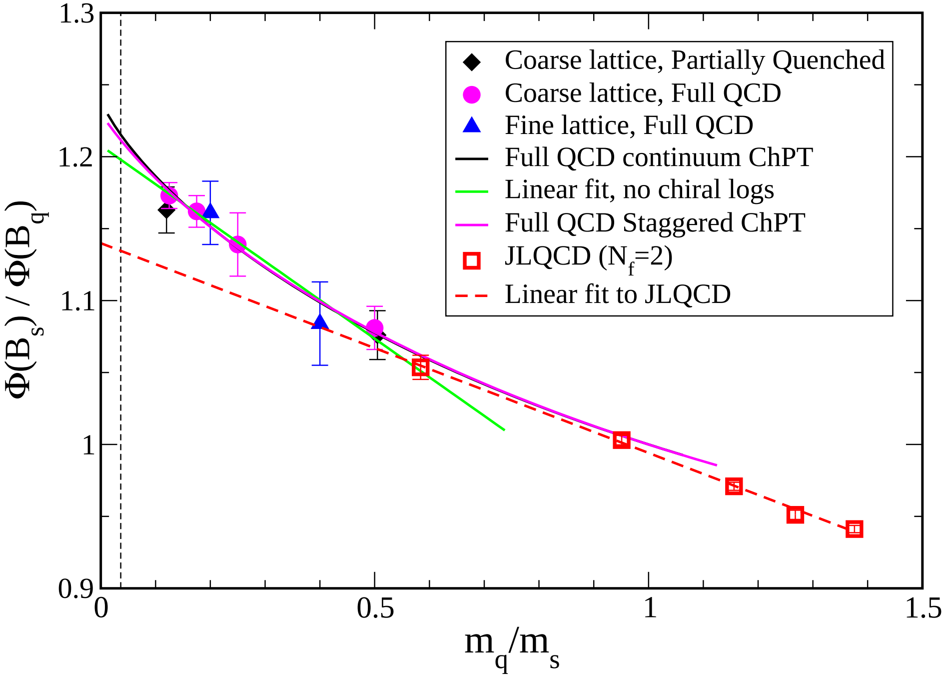
$$\langle \bar{K}^0 | O_{\Delta S=2} | K^0 \rangle \propto B_K f^2_K$$

$f_{B_{(s)}}$ result

$f_{B_{(s)}}$ is similar to f_D, for which we have seen an agreement with Exp't

HPQCD, hep-lat/0507015 (Shigemitsu&Allison's talks)

$n_f = 2{+}1$ (MILC conf), impr stag light + NRQCD heavy



$f_{B_s}/f_B\sqrt{M_{B_s}/M_B}$ vs. m_q/m_s

χ fits with SχPT, χPT, linear ansatz

$\Longrightarrow$ only 3% difference

insensitive to fit form with $m_q < m_s/2$

deviation from JLQCD($n_f=2$) linear fit

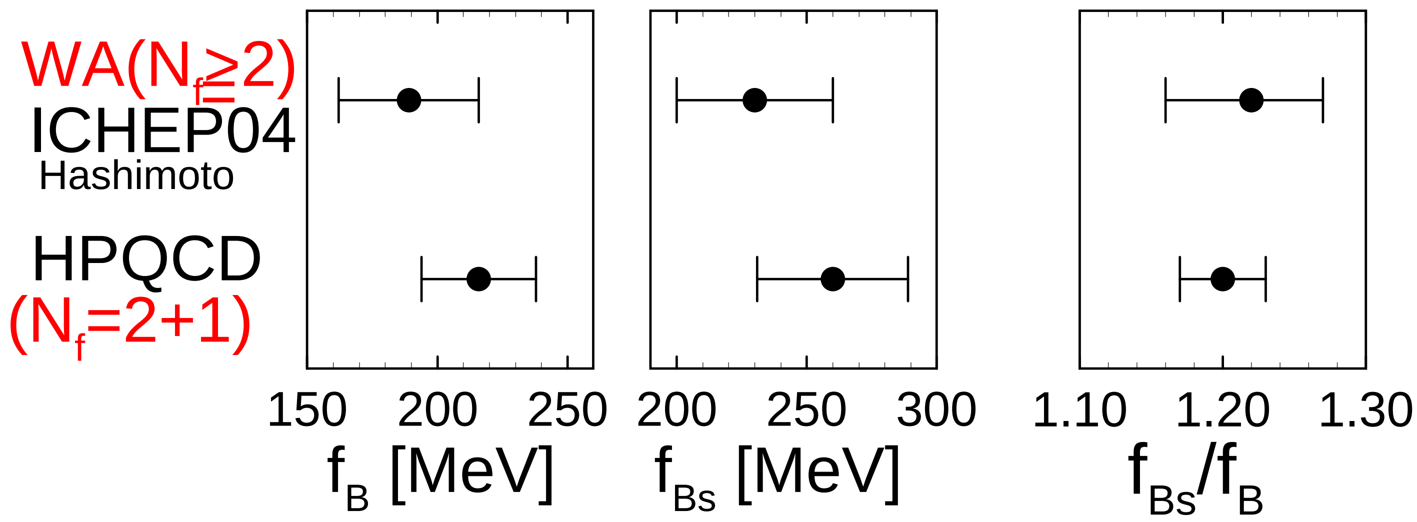
$$f_B = 216(9)_{\mathrm{sta}+\chi\mathrm{fit}}(19)_{\mathrm{PT}}(7)_{\mathrm{others}}\mathrm{MeV}$$

total 10% (PT $O(\alpha^2)$ error largest)

$$f_{B_s}/f_B = 1.20(3)_{\mathrm{sta}+\chi\mathrm{fit}}(1)_{\mathrm{others}}$$

PT error cancel $\Longrightarrow$ total 3%

$f_{B_{(s)}}$ result (cont'd)



- reasonable agreement with previous averages for f_B and f_{B_s}
- good agreement and better accuracy for the ratio f_{B_s}/f_B
  (smaller χfit error with staggered quarks)
- χ-log effect *included* in Hashimoto's ICHEP'04 avg

$B_{B_{(s)}}$

No new/updated unquenched $B_{B_{(s)}}$ result this year.
($n_f = 0$ study with Overlap light, Blossier's talk)

Best result: JLQCD'03

$n_f = 2$, clover light + NRQCD heavy

$$B(m_b) = 0.836(27)(^{+56}_{-62}) \quad , \quad \hat{B}_s/\hat{B} = 1.017(16)(^{+56}_{-17})$$

$$\Downarrow$$

With HPQCD's $f_{B_{(s)}}^{n_f=2+1}$,

$$f_B\sqrt{\hat{B}_B} = 244(26)\text{MeV}, \quad \Longrightarrow \quad |V_{td}|_{\text{Lat05}} = 7.4(0.8)\times 10^{-3})$$

$$(|V_{td}|_{\text{PDG04}} = 8.3(1.6)\times 10^{-3})$$

$$f_{B_s}/f_B\sqrt{\hat{B}_{B_s}/\hat{B}_B} = 1.210(^{+47}_{-35})$$ $\delta(|V_{td}|/|V_{ts}|) = 3-4\%$ with forthcoming ΔM_{B_s}

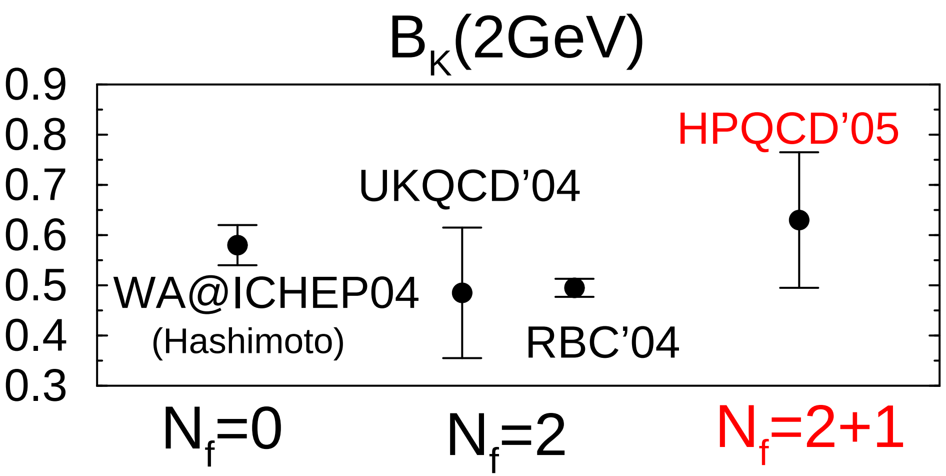
B_K (See Dawson's review for details)

new preliminary $n_f = 2{+}1$ result with improved staggered quark

(HPQCD, Gamiz's talks)

$$B_K^{\overline{MS}}(2\mathrm{GeV}) = 0.630(18)_{\mathrm{sta}}(15)_{\chi\mathrm{fit}}(30)_{\mathrm{disc}}(130)_{\mathrm{PT}}$$

- MILC "coarse" lattice ($a^{-1} = 1.6$ GeV)
- large error from 1-loop PT matching; $\alpha(1/a) \approx 0.4,\ \alpha^2 \approx 0.2$
  $\Longrightarrow$ need 2-loop/NP and smaller a
- linear χfit $\Longrightarrow$ need χfit using SχPT formula (Van de Water's talk)



Two more $n_f = 2{+}1$ studies

(Cohen's talk, Lee's poster)

Below I use HPQCD's in UT analysis

Unitary Triangle analysis with LQCD (2005)

Theory inputs (LQCD *only* as a rule)

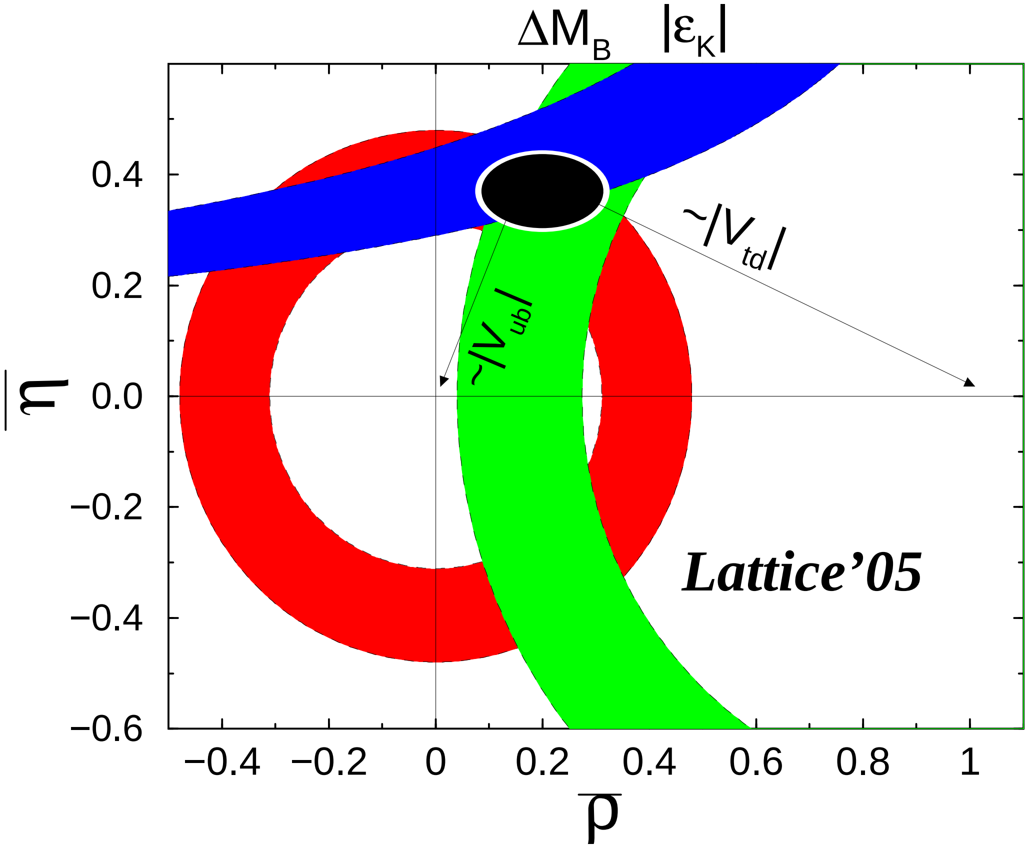
- $B \to \pi l \nu$ form factor ($n_f = 2+1$, FNAL/MILC+HPQCD, preliminary)
  $\Longrightarrow \quad |V_{ub}| \propto \sqrt{\rho^2 + \eta^2}$
- f_B ($n_f = 2+1$, HPQCD) + B_B ($n_f = 2$, JLQCD'03)
  $\Longrightarrow \quad |V_{td}| \propto \sqrt{(1-\rho)^2 + \eta^2}$
- B_K ($n_f = 2+1$, HPQCD, preliminary)
  $\Longrightarrow \quad \eta(1-\rho)$

Exp't inputs

- $\mathcal{B}(B \to \pi l \nu)$, ΔM_B, ε_K (waiting for ΔM_{B_s}...)
- $B \to \psi K$ (Belle, Babar) $\Longrightarrow \quad \sin(2\beta)$

Unitary Triangle analysis with LQCD (2005)

Result without $\sin(2\beta)_{B\to\psi K}$

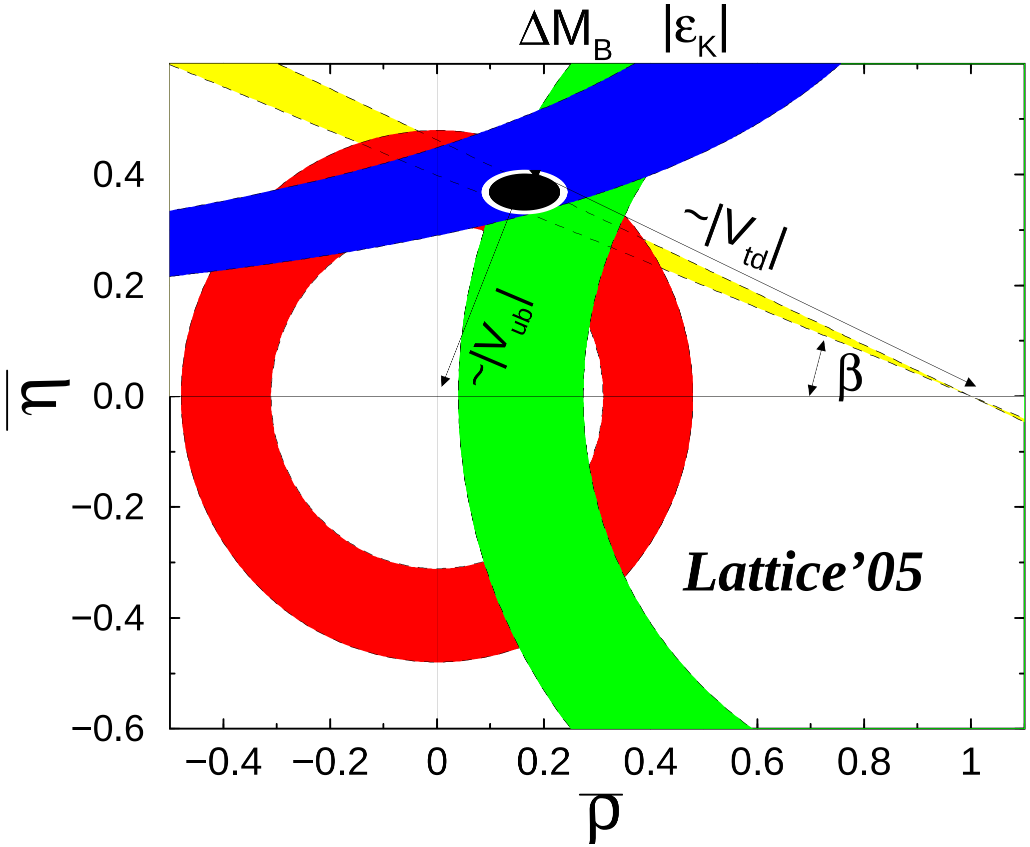


$$\rho = 0.20(12) \qquad [\,\text{PDG}: \; 0.20(9)\,]$$

$$\eta = 0.37(07) \qquad [\,\text{PDG}: \; 0.33(5)\,]$$

Unitary Triangle analysis with LQCD (2005)

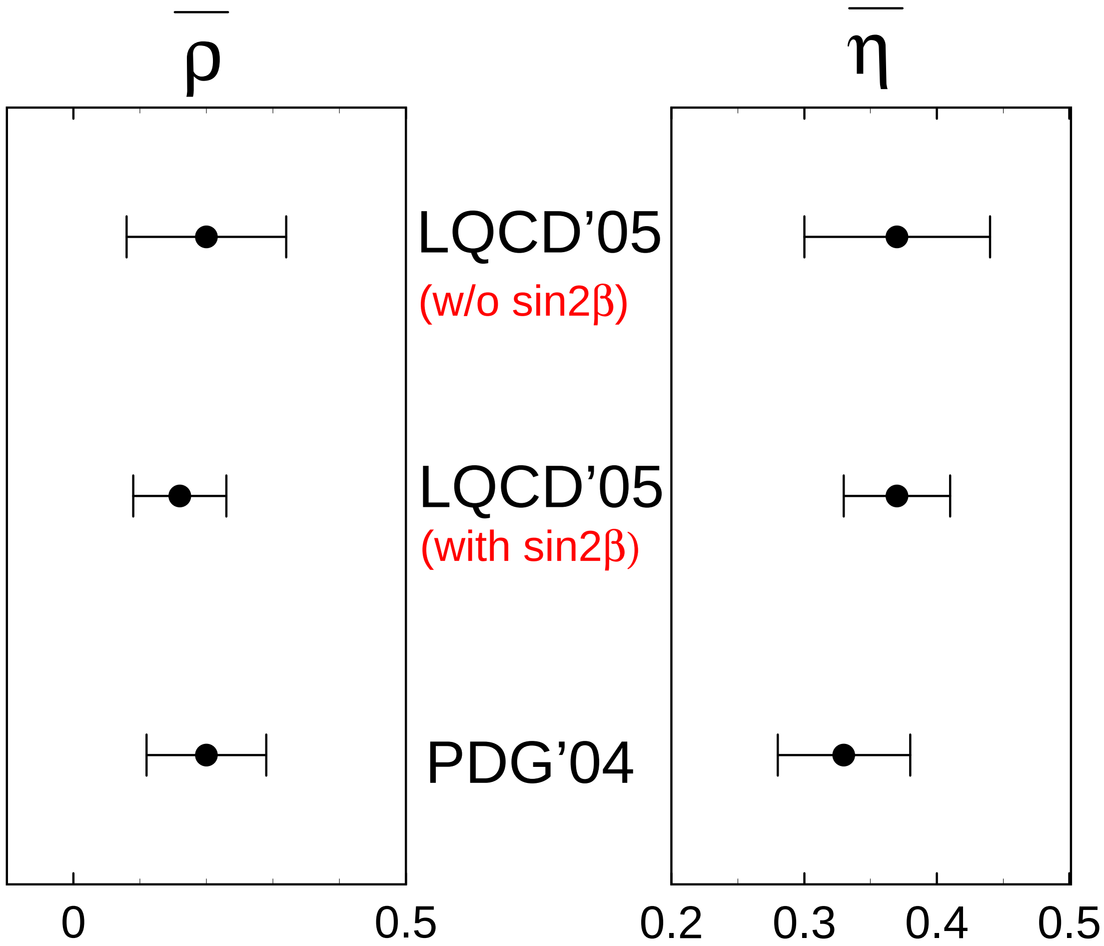
Result with $\sin(2\beta)_{B\to\psi K}$



$$\rho = 0.16(7) \qquad [\ \mathrm{PDG}: \ 0.20(9)\ ]$$

$$\eta = 0.37(4) \qquad [\ \mathrm{PDG}: \ 0.33(5)\ ]$$

CKM phase $\{\rho, \eta\}$ from UT analysis



$$\rho_{\mathrm{Lat05}} = 0.16(7) \quad , \quad \eta_{\mathrm{Lat05}} = 0.37(4)$$

CKM matrix with LQCD (2005)
(full determination)

$$V_{\rm CKM}^{\rm Lat05} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ 0.9745(3) & 0.2244(14) & 3.76(68)\times10^{-3} \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ 0.245(22) & 0.97(10) & 3.91(35)\times10^{-2} \\ |V_{td}| & |V_{ts}| & |V_{tb}| \\ 7.40(79)\times10^{-3} & 3.79(53)\times10^{-2} & 0.9992(1) \end{pmatrix}$$

$$\{\lambda , A , \rho , \eta\}_{\rm Lat05} = \{ 0.2244(14) , 0.78(7) , 0.16(7) , 0.37(4)\}$$

9/**9** CKM elements and **4**/**4** Wolfenstein parameters **fully** determined with LQCD($n_f \geq 2$)+Exp't!

Summary

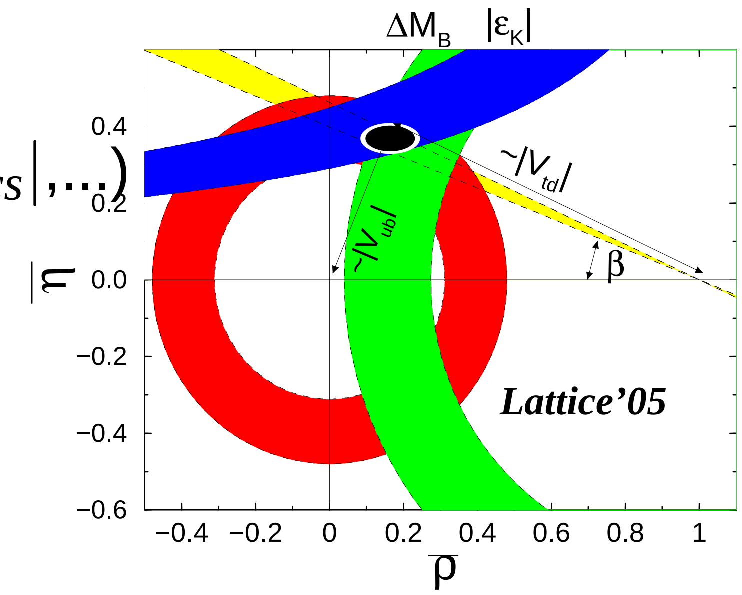
It is now possbile to fully determine CKM matrix with LQCD($n_f \geq 2$)

At present (2005)

Many unquenched results with staggered light quarks using MILC configs (FNAL/MILC,HPQCD).

an agreement with Exp't for D physics $\Longrightarrow$ credibility for B physics

More unquenched results using other fermions (Wilson,DWF,overlap,...) needed for checks!



Typical accuracy of $V_{\rm CKM}^{\rm Lat}$ is $O(10\%)$ $(|V_{ub}|,\ |V_{cd}|,\ |V_{cs}|,\ldots)$

$f_B\sqrt{B_B},\ B_K$ to $O(10\%)$ $\Longrightarrow$ $\{\rho,\eta\}$ to $\approx$10%

dominated by PT or discretization errors.

good exception: new $f_{B_s}/f_B\sqrt{B_{B_s}/B_B}$ to $\approx 3\%$

$\Longrightarrow$ *more* precise $\{\rho,\eta\}$, once ΔM_{B_s} is measured.

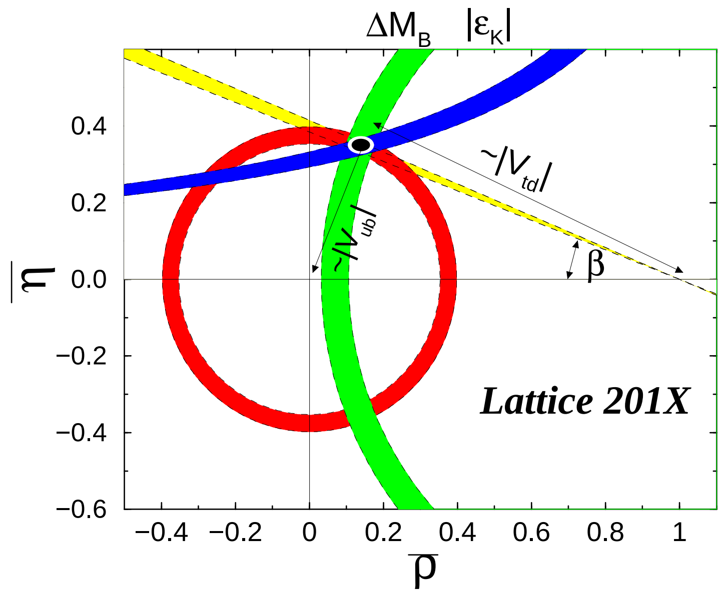
Outlook

I hope “CKM matrix with LQCD” will be updated every year...

In next 5 years (before 2010)

I *expect/hope*:

- Many unquenched results from many groups
- 2-loop/NP matching done
- finer lattices and/or highly improved actions done
- ΔM_{B_s} is measured by Exp’t



Typical accuracy of $V_{\rm CKM}^{\rm Lat}$ will be $\approx$ 5% or better;
$|V_{td}|/|V_{ts}|$ to $\approx$ 3% $\Longrightarrow$ $\{\rho,\eta\}$ to $\approx$5%

Ready for New Physics !?

Acknowledgment

Thanks to:

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Special thanks to:

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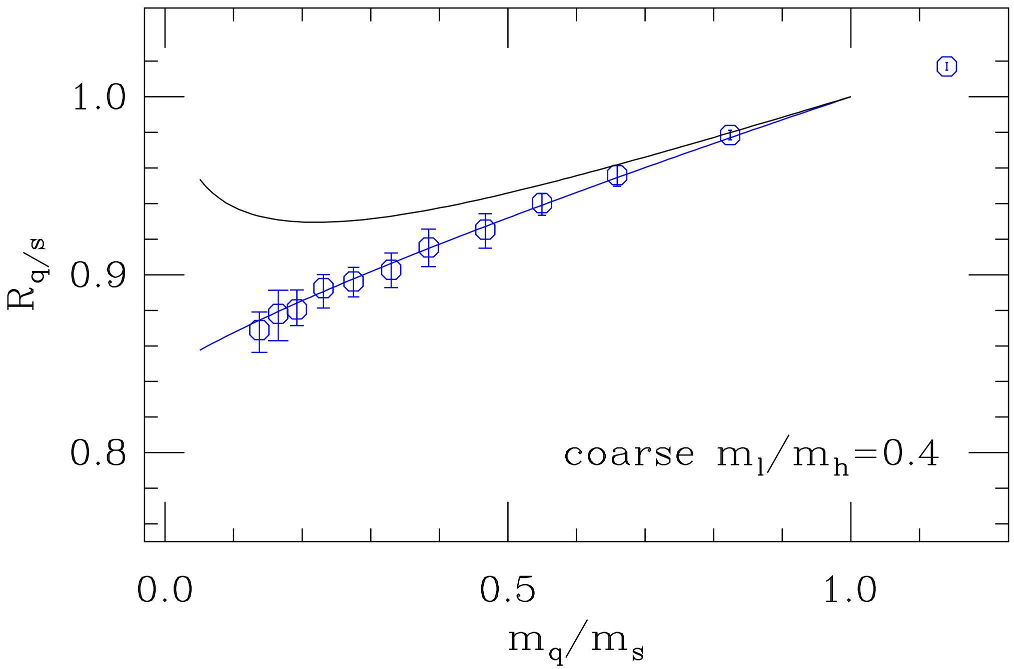
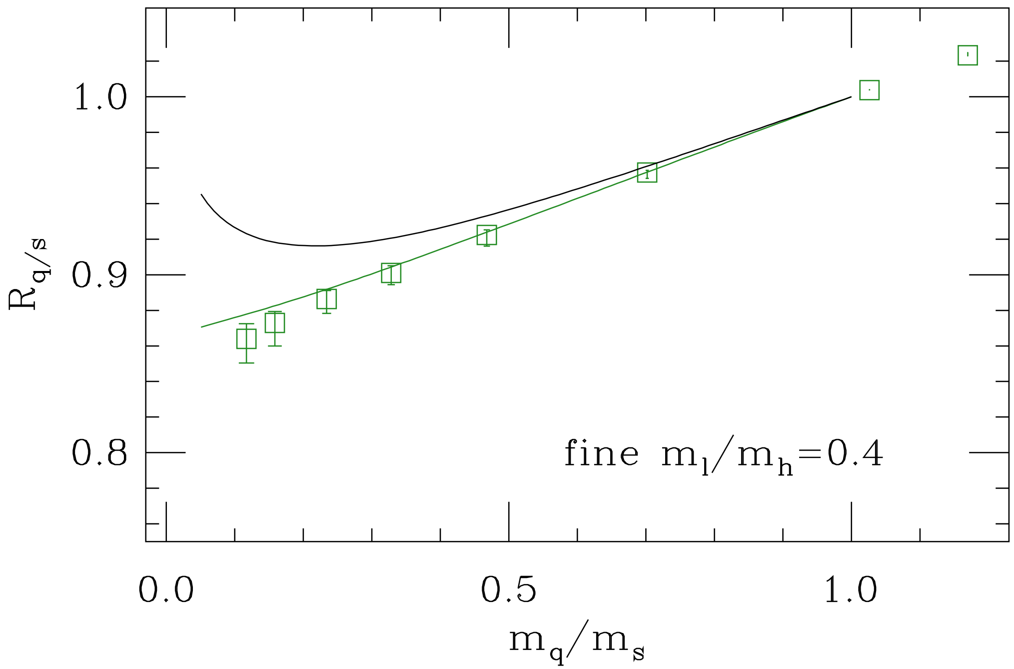
Backup slides

Partially Quenched f_D fits





Matching between “lattice” $\leftrightarrow$ “continuum” heavy-light current

$$\langle \pi | O^{\rm cont} | D \rangle \;=\; Z_O^{hl} \, \langle \pi | O^{\rm lat} | D \rangle \quad (O = V_\mu \text{ or } A_\mu)$$

Quasi-non-perturbative determination for Z_O^{hl} (FNAL’01)

$$Z_O^{hl} \;\equiv\; \rho_O \sqrt{Z_V^{hh}\, Z_V^{ll}}$$

- compute Z_V^{hh}, Z_V^{ll} non-perturbatively from:

$$Z_V^{qq} \, \langle D | V_4^{qq} | D \rangle = 1 \quad (qq = hh, ll)$$

- compute $\rho_O (= Z_O^{hl} / \sqrt{Z_V^{hh}\, Z_V^{ll}} \approx 1)$ perturbatively.
  $O(\alpha_s) \approx 1-5\%$ (Harada *et.al.*; Nobes *et.al.*)
  $\Longrightarrow \;\; O(\alpha_s^2) \leq 1\%$.