



Twisted
mass lattice
QCD

A. Shindler

Cutoff
effects

Recovery of
symmetry

$N_f = 2$

Summary
and
outlooks

Twisted mass lattice QCD: recent developments and results

Andrea Shindler

NIC/DESY-Zeuthen



Lattice 2005

Outline



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- 1 Cutoff effects
 - Lattice QCD action
 - $O(a)$ improvement and small pion masses
- 2 Recovery of symmetry
 - Flavour symmetry
- 2 $N_f = 2$
 - Phase diagram of Wilson fermions
 - Minimal pion mass and all that
- 3 Summary and outlooks

The perfect world



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- Lattice QCD with $N_f = 2$ dynamical light quarks and 1(+1) heavier quarks;
- Efficient algorithm;
- Reach a pion mass ($m_\pi < 300$ MeV) where it is possible to match χ_{pt} ;
- The volume must be large enough $L \geq 2\text{fm}$
- The lattice action should have good scaling properties and simplified renormalization patterns;
- Is twisted mass QCD a possible way to reach this goal?

$N_f = 2$ twisted mass QCD



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This lattice action has a long history [S. Aoki: 1988];
It was proved to be an alternative lattice QCD action in
[R. Frezzotti, P. A. Grassi, S. Sint, P. Weisz: hep-lat/0101001]

$$S = S_g[U] + S_F[U, \psi, \bar{\psi}]$$

$$S_F = a^4 \sum_{\mathbf{x}} \{ \bar{\psi}(\mathbf{x}) [D[U] + m_0 + i\mu\gamma_5\tau^3] \psi(\mathbf{x}) \}$$

- $D[U] = \frac{1}{2}[\gamma_\mu(\nabla_\mu + \nabla_\mu^*) - a\nabla_\mu^* \nabla_\mu]$: massless Wilson Dirac operator.
- m_0 : untwisted (bare) quark mass parameter.
- μ : (bare) twisted quark mass parameter.
- τ_3 : third Pauli matrix acting in flavour space.

Parity even correlators

- Using spurionic symmetries it was proved in [R. Frezzotti, G. C. Rossi:2003] that parity even correlator do not have $O(a)$ effects;
- In the appendix A of [R. Frezzotti, G. Martinelli, M. Papinutto, G.C. Rossi:2005] the proof does not make use of spurionic symmetries;

$$S_{\text{eff}} = S_0 + aS_1 + \dots$$

$$S_0 = \int d^4x \bar{\psi}(x) [\gamma_\mu D_\mu + i\mu_R \gamma_5 \tau^3] \psi(x)$$

$$S_1 = \int d^4y \mathcal{L}_1(y) \quad \mathcal{L}_1(y) = \sum_i c_i \mathcal{O}_i(y)$$

$$\mathcal{O}_1 = \bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi \quad \mathcal{O}_2 = \mu^2 \bar{\psi} \psi \quad \mathcal{O}_3 = \Lambda^2 \bar{\psi} \psi$$

$$\psi(x) \longrightarrow \gamma_0 (i\gamma_5 \tau^3) \psi(x_0, -\mathbf{x})$$

$$\bar{\psi}(x) \longrightarrow \bar{\psi}(x_0, -\mathbf{x}) (i\gamma_5 \tau^3) \gamma_0$$

$$\langle \Phi \rangle = \langle \Phi \rangle_0 + a \int d^4y \langle \Phi \mathcal{L}_1(y) \rangle_0 + a \langle \Phi_1 \rangle_0 + \dots$$

Contact terms amount to a redefinition of Φ_1

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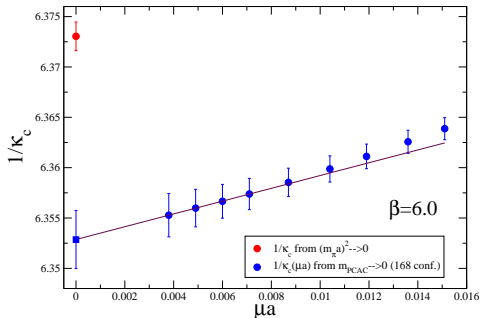
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Determination of the critical mass

[K. Jansen, M. Papinutto, A.S., C. Urbach, I. Wetzorke: 2005]
[Talk of M. Papinutto]

$$m_0 = m_c \rightarrow \text{maximal twist}$$

$$m_{\text{PCAC}} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(\mathbf{x}) P^a(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^a(\mathbf{x}) P^a(0) \rangle} \quad a = 1, 2$$



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Pseudoscalar decay constant



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As with overlap fermions to compute the pseudoscalar decay constant one does not need any improvement coefficient or any renormalization constant!

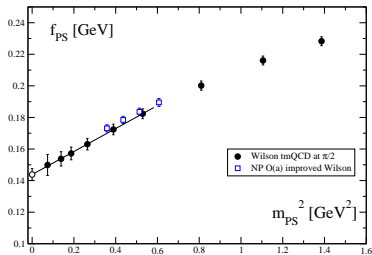
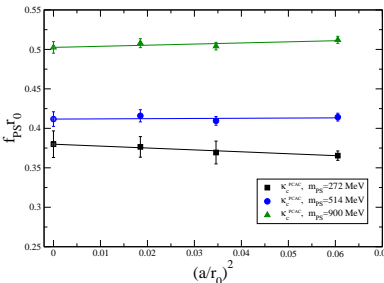
$$2\mu \langle 0 | P^1 | PS \rangle = \partial_\mu^* \langle 0 | \tilde{V}_\mu^2 | PS \rangle = f_{PS} m_{PS}^2 \quad Z_P = Z_\mu^{-1}$$

$$f_{PS} = \frac{2\mu}{m_{PS}^2} \langle 0 | P^1 | PS \rangle$$

[R. Frezzotti, S. Sint; M. Della Morte, R. Frezzotti, J. Heitger: 2002]

[K. Jansen, A.S., C. Urbach, I. Wetzorke:2003]

Pseudoscalar decay constant



The minimal pion mass reached is $m_{\pi} = 272$ MeV

[K. Jansen, M. Papinutto, A.S., C. Urbach, I. Wetzorke: 2005]

The NP O(a) improved data are from:

[J. Garden, J. Heitger, R. Sommer, H. Wittig:2000]

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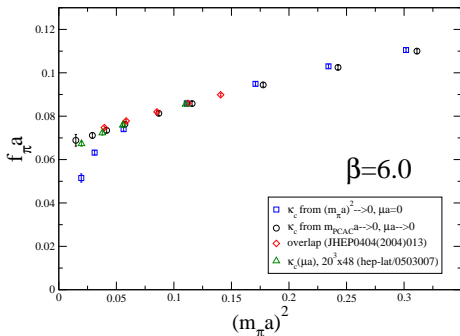
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The infamous “bending phenomenon”

What happens if the critical mass is chosen from the pion mass in the pure Wilson theory?



[K. Jansen, M. Papinutto, A.S., C. Urbach, I. Wetzorke: 2005]

Overlap results: [Giusti et al.: 2004]

$m_c(a\mu)$: [A. Abdel-Rehim, R. Lewis, R. M. Woloshyn: 2005]

The bending phenomenon in W_χ PT

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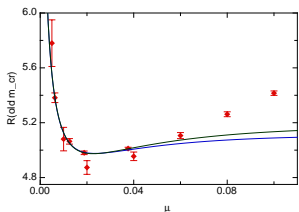
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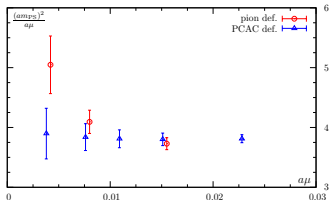
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$$R = \frac{m_\pi^2}{2\mu}$$



◆ R(old m_cr)
— $\mu_{\max} = 0.04$
— $\mu_{\max} = 0.06$



[Talk of O. Bär]

Data from :

[χ LF coll.: Bietenholz et al.: 2004]

$m_c(a\mu)$: [A. Abdel-Rehim, R. Lewis, R. M. Woloshyn: 2005]

- Power counting: $\mu \sim a^2$. Try $\mu \sim a$.
- It would be difficult to describe cutoff effects at rather heavy masses using W_χ PT.

Happy ending...



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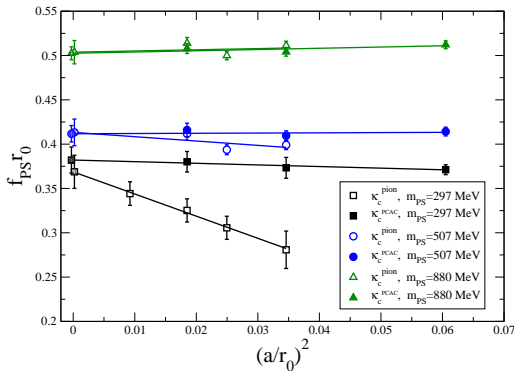
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[K. Jansen, M. Papinutto, A.S., C. Urbach, I. Wetzorke: 2005]

Universality of the continuum limit

NO bending phenomenon in the continuum

Choice of the critical mass



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- The problem is raised in [Aoki,Bär:2004]: No “ $O(a)$ improvement” at $\mu \simeq a^2 \Lambda^3$; use the PCAC relation to define the critical point;
- This is a problem that appears also at bigger masses $a\Lambda^2 > \mu > a^2 \Lambda^3$; use PCAC, parity conservation [Sharpe,Wu:2004];
- Infrared divergent $O(a^2)$ effects [Frezzotti, Martinelli, Papinutto, Rossi:2005] [See talk of G. C. Rossi];
 - Parity conservation tunes the critical mass in order to cancel the leading infrared divergent effects;
 - Identical result could be obtained using a non-pertubatively determined C_{SW} ;

Comparison Clover-Wilson



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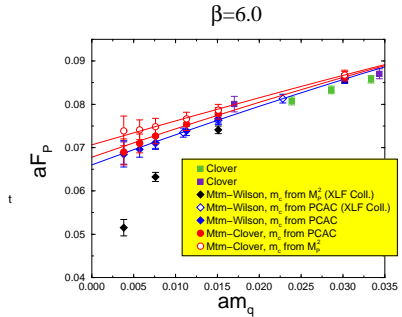
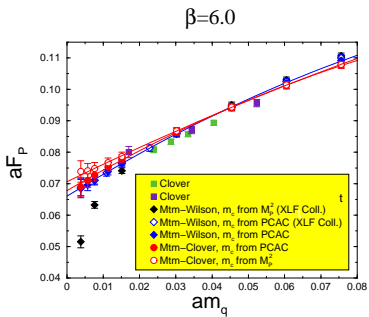
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[Talk of V. Lubicz]

The Mtm-clover results independently on the choice of the critical mass, are consistent with Mtm without clover term.

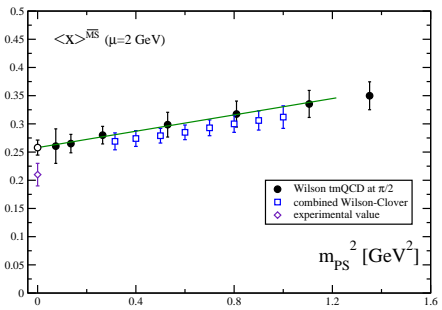


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Example of the potential of twisted mass QCD



[S. Capitani, K. Jansen, M. Papinutto, A.S., C. Urbach, I. Wetzorke:2005]
[Talk by K. Jansen]

Renormalization



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- In the talk of [S. Sint] it has been presented a construction of a Schrödinger functional with twisted boundary conditions that preserves the nice properties of O(a) improvement without bulk improvement coefficients; see [R. Frezzotti, G. C. Rossi: 2005] for and alternative SF.
- Based on the consideration that in a finite volume (with p.b.c) the Wilson theory is O(a) improved.
- The construction makes use of the orbifolding technique (see [Y. Taniguchi: 2004])
- The new boundary projectors are $Q_{\pm} = \frac{1}{2}(1 + i\gamma_0\gamma_5\tau^3)$
- It is interesting to notice that these projectors commute with the previous parity transformation and can be obtained by a chiral rotation of the original projectors
- As a consequence the running of the coupling constant should be identical to the running computed with the “old” SF
- The O(a) uncertainty in the critical mass does not harm the O(a) improvement

Symmetries breaking



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- When tm is used to define the standard QCD correlation functions some of the physical symmetries are restored only in the continuum limit \rightarrow flavour and parity symmetries;
- No flavour symmetry \rightarrow splitting between charged and neutral pion;
- No parity symmetry \rightarrow states of opposite parity can appear in the spectral decomposition of usual correlators;
- These phenomena are expected to vanish, at maximal twist, with a rate $O(a^2)$

Flavour breaking



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$$P^\pm(\mathbf{x}) = \bar{\psi}(\mathbf{x})\gamma_5\frac{\tau^\pm}{2}\psi(\mathbf{x}) \quad \tau^\pm = \frac{\tau^1 \pm i\tau^2}{2}$$

$$S^0(\mathbf{x}) = \bar{\psi}(\mathbf{x})\psi(\mathbf{x})$$

$$C_{\pi^+}(x_0) = a^3 \sum_{\mathbf{x}} \langle P^+(\mathbf{x})P^-(0) \rangle \quad C_{\pi^0}(x_0) = a^3 \sum_{\mathbf{x}} \langle S^0(\mathbf{x})S^0(0) \rangle$$

$$C_{\pi^0}(x_0) = a^3 \sum_{\mathbf{x}} \{ \langle -\text{Tr} [G(0, \mathbf{x})G(\mathbf{x}, 0)] + \text{Tr} [G(\mathbf{x}, \mathbf{x})] \text{Tr} [G(0, 0)] \rangle \}$$

In [K. Jansen, C. McNeile, C. Michael, K. Nagai, M. Papinutto, J. Pickavance, A. S., C. Urbach, I. Wetzorke: 2005] quenched pilot study;

- The vacuum contribution has been subtracted out
- For the technical details see [poster of J. Pickavance];

Flavour breaking



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$$S_{\text{OS}} = a^4 \sum_x \{ \bar{\psi}(x) [D[U] + m_0 + i\mu\gamma_5] \psi(x) \}$$

- Using the OS action it is possible to interpret the connected piece in terms of local operators: the disconnected piece is zero if the action for the “up” and “down” quarks are the same; see [R. Frezzotti; G. C. Rossi: 2004]
- Even if this is not the π^0 with the tm action, it is interesting to study with precise data in view of a possible use of mixed actions;

Flavour breaking



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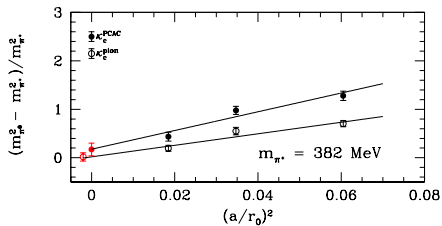
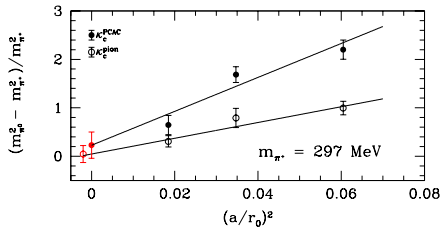
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The choice of the critical mass from the PCAC (parity restoration) “maximise” the flavour breaking effects.

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- The pion mass splitting has been studied in the framework of χ PT by [L. Scorzato; S. R. Sharpe, J. M. S. Wu: 2004]
- The results are in agreement and can be summarized by

$$m_{\pi^0}^2 - m_{\pi^\pm}^2 = -\frac{32}{f^2} \hat{a}^2 W' s^2$$

- W' is the LEC that describes the term $\langle \hat{A}^\dagger \Sigma + \Sigma^\dagger \hat{A} \rangle^2$
- Natural explanation for the increased flavour breaking when the PCAC mass is tuned to zero

Flavour breaking



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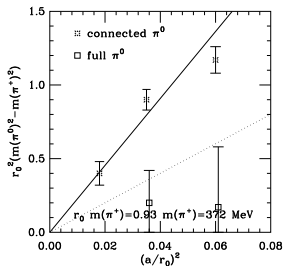
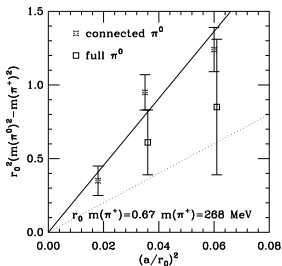
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[Poster by J. Pickavance]

Flavour breaking effects are reduced with the inclusion of the disconnected piece.

Flavour breaking



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- The pion mass splitting with both the full and only the connected (OS) correlator goes to zero with a rate of $O(a^2)$;
- The flavour breaking are substantial $r_0^2(m^2(\pi^0) - m^2(\pi^+)) = c(a/r_0)^2$ with $c \approx 10$;
- For naive staggered fermions with quenched Wilson gauge action $c \approx 40$ [N. Ishizuka, M. Fukugita, H. Mino, M. Okawa, A. Ukawa: 1994];
- The only available unquenched results are dynamical improved staggered fermions have $c \approx 10$ [C. Aubin et al.: 2004];
- The inclusion of disconnected pieces is crucial to reduce the flavour breaking effects (a factor of 2 smaller);

Flavour breaking

$$\Delta_i^{++} = \epsilon_{abc} [u_a^T C \gamma_i u_b] u_c \quad \Delta_i^+ = \frac{2}{\sqrt{3}} \epsilon_{abc} [u_a^T C \gamma_i d_b] u_c + \frac{1}{\sqrt{3}} \epsilon_{abc} [u_a^T C \gamma_i u_b] d_c$$

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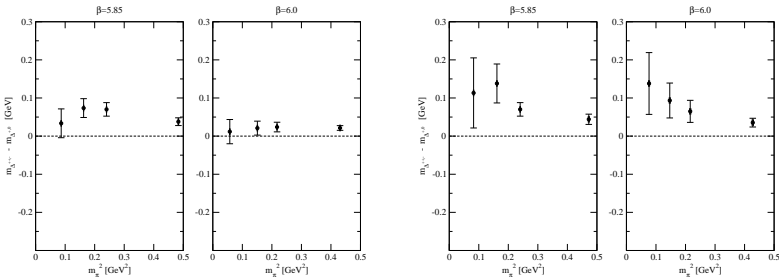
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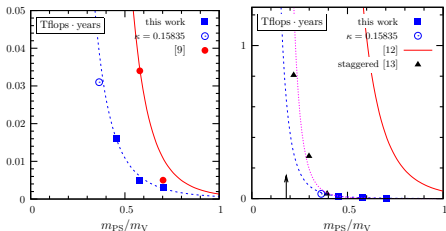
[A. M. Abdel-Rehim, R. Lewis, R. M. Woloshyn: 2005]

[Talk of A. M. Abdel-Rehim]

See the [Poster of R. Lewis] for flavour breaking in the kaon system.

Algorithmic improvements

- In the [talk of C. Urbach] a very efficient HMC algorithm will be presented based on mass preconditioner with a multiple time scale integrator [C. Urbach, K. Jansen, A.S., U. Wenger: 2005]



- In [Plenary talk of M. Lüscher] it was shown that with a domain decomposition preconditioning for the HMC light quark masses ($m_\pi = 294$ MeV) are reachable with Wilson fermions; see [M. Lüscher: 2003,2003,2004]

$$\nu = 10^{-3}(2N + 3)\tau_{\text{int}}(P)$$

Action	Algo	r_0/a	m_π [MeV]	ν	τ_{int}
W+W	(mt)(DD)HMC	6.40(15)	294	0.74(18)	21(5)
tlSym+Wtm	(mt)(H)HMC	5.05(24)	274	0.345 – 1.035	15 – 45

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- “Let me describe a typical computer simulation: the first thing to do is to look for phase transitions” [G. Parisi: 1988]
- Exploration of the phase structure of Wilson fermions in general, at zero temperature
- Important to have at least 2 algorithms in order to check if it is an algorithmic issue;
- The exploration should be carried out in the 3-d space (β, m_0, μ)

Thermal cycles



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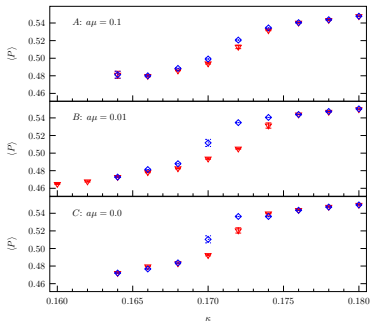
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[F. Farchioni et al.:2004]

$\beta = 5.2$ $a \approx 0.16 fm$

$8^3 \times 16$ lattice

Use of several algorithms:

TSMB [I. Montvay: 1996];

GHMC [M. Hasenbusch: 2001;

M. Hasenbusch, K. Jansen: 2002];

Thermal cycles



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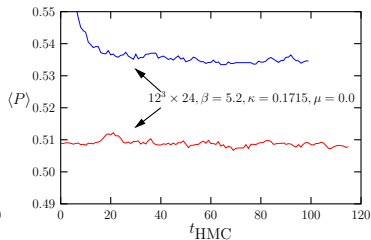
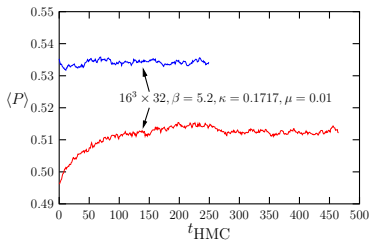
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Are we surprised by these results?

No surprises



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Not really!!

- Results from [S. Aoki, A. Ukawa: 1994] at finite temperature indicate difficulties of observing the Aoki phase at $\beta > 4.8$;
- In 1994 the MILC collaboration [T. Blum et al.] finds a surprising bulk first order phase transition for Wilson fermions at $\beta \simeq 4.8$;
- In a series of paper [1996] M. Creutz makes an ansatz on a possible first order phase transition scenario for Wilson fermions;
- In 1998 a very important paper (according to my opinion);

An important paper



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[S. Sharpe, R. Singleton Jr.:1998]

- For the first time the concept of chiral lagrangian at finite lattice spacing is given;
- “We now include the Pauli term. This is straightforward since it transforms under chiral rotation exactly as does the mass term.”
- It shows the existence of 2 scenarios for the phase diagram of Wilson fermions:
 - The Aoki phase;
 - The existence of a 1st order phase transition;
- “Removing the quenched approximation could, in principle, lead to a change of scenarios.”
- The analysis is **identical** with non-perturbatively improved clover fermions;

The famous c_2



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- Generalize the continuum chiral lagrangian to include the effects of the clover term;
- Neglect the derivative interaction, since we are interested in the vacuum state;

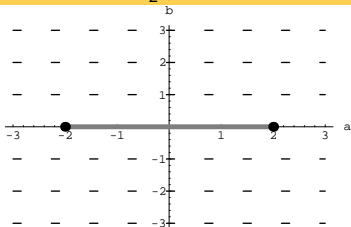
$$\mathcal{V} = -\frac{c_1}{4} \langle \Sigma + \Sigma^\dagger \rangle + \frac{c_2}{16} \langle \Sigma + \Sigma^\dagger \rangle^2$$

$$c_1 \sim m' \Lambda^3 \quad c_2 \sim m'^2 \Lambda^2 + m' a \Lambda^4 + a^2 \Lambda^6 \quad m' = m - a \Lambda^2$$

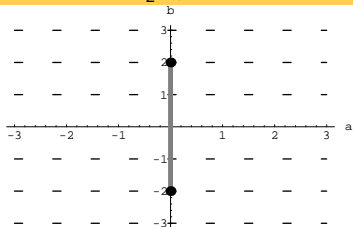
- For $m' \sim a^2 \Lambda^3$ the two terms in the potential become comparable;

Phase structure of Wilson fermions

$c_2 > 0$



$c_2 < 0$



- [G. Münster; L. Scorzato; S. Sharpe, J. Wu:2004]
- The above phase diagram only when $m' \sim \mu \sim a^2$
- The unbroken $U(1)$ symmetry ensures that the charged pions are degenerate
- $m_{\pi 3}^2 - m_{\pi \pm}^2 \propto c_2$
- The extension of the first order line is given exactly by the splitting of the pion masses, i.e. $O(a^2)$
- For $c_2 < 0$ **The metastabilities at low quark masses at fixed lattice spacing are a generic phenomenon for Wilson fermions**



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The Gross-Neveu model

Twisted mass lattice QCD

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Cutoff effects

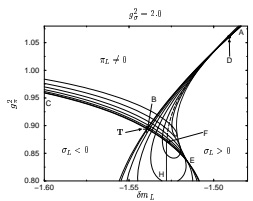
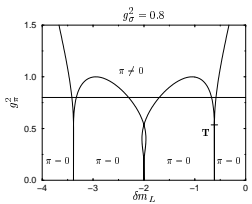
Recovery of symmetry

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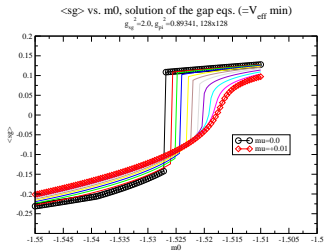
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[T. Izubuchi, J. Noaki, A. Ukawa: 1998]



K. Jansen, K. Nagai *work in progress*

Phase diagram of Wilson lattice QCD



Twisted mass lattice QCD

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Cutoff effects

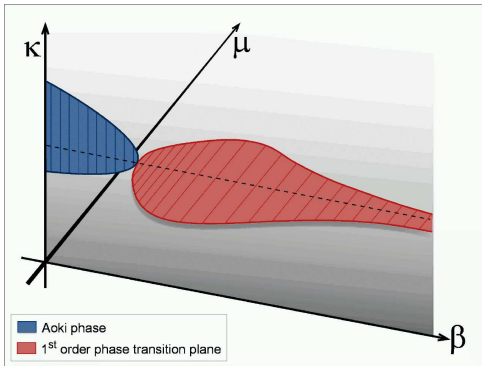
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[F. Farchioni, K. Jansen, I. Montvay, E. Scholz, L. Scorzato, A.S., N. Ukita, C. Urbach, I. Wetzorke : 2004]

How to reduce $|c_2|$



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- The value of c_2 depends on the gauge action (and also on the value for c_{sw})
- It might be speculated that at the microscopic level the occurrence of a 1st order phase transition is accompanied by a massive rearrangement of small eigenvalues of the Wilson-Dirac operator
- Results from JLQCD [S. Aoki et al.: 2004] indicate that a metastability seen in the average plaquette can be suppressed by replacing the Wilson with the Iwasaki gauge action.

TISym gauge action



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The gauge action is given by

$$S_G = \beta \left[b_0 \sum_{x; \mu < \nu} \left(1 - \frac{1}{3} P^{1 \times 1}(x; \mu, \nu) \right) + b_1 \left(1 - \frac{1}{3} P^{1 \times 2}(x; \mu, \nu) \right) \right]$$

$$b_0 = 1 - 8b_1 \quad b_1 = -\frac{1}{12}$$

$\beta = 3.65,$	$\beta = 3.75,$	$\beta = 3.90,$
$a\mu = 0.01$	$a\mu = 0.0094 - 0.005$	$a\mu = 0.0075 - 0.004$
$a \approx 0.13fm$	$a \approx 0.12fm$	$a \approx 0.09fm$
$L \approx 1.56fm$	$L \approx 2fm$	$L \approx 1.5fm$
$m_\pi \approx 390MeV$	$m_\pi \approx 250MeV$	$m_\pi \approx 280MeV$

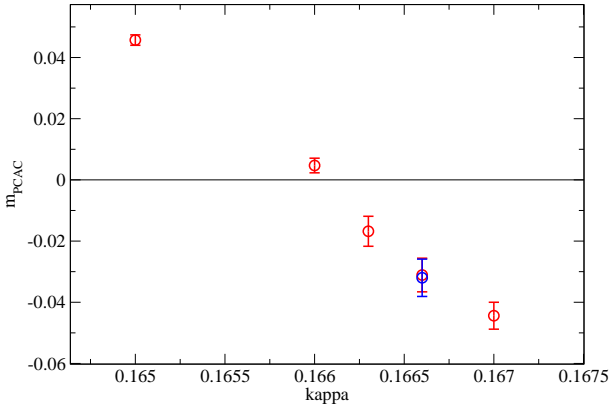
[F. Farchioni, K. Jansen, I. Montvay, M. Papinutto, E. Scholz, L. Scorzato, A.S., N. Ukita, C. Urbach, U. Wenger, I. Wetzorke, *work in progress*]

[Talk of U. Wenger]



TISym gauge action

$N_f=2$ TM tSym @ $\beta=3.75, 12^{3 \times 24}, m=0.005$



$\beta = 3.75, a \approx 0.12 \text{ fm}$

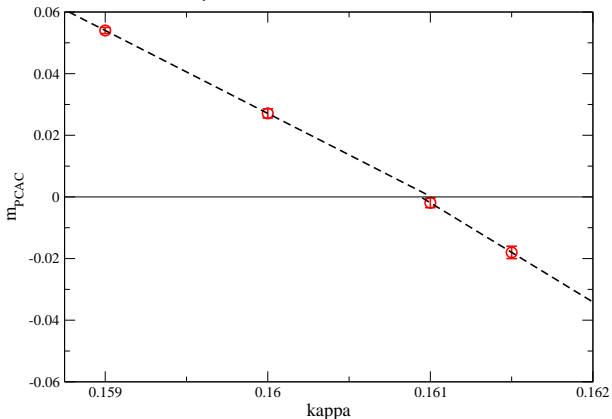
In a big volume $L \simeq 2 \text{ fm}$ we are at $m_{\text{PCAC}} \simeq 0$.

We have $m_\pi = 402(19)$ and $m_\pi = 250(50)$ (**Very preliminary!**);

- Twisted mass lattice QCD
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TISym gauge action

$N_f=2$ TM tISym @ $b=3.90, 16^3 \times 32, m=0.0075$



$\beta = 3.9, a \approx 0.09 \text{ fm}$

In a small volume $L \simeq 1.5 \text{ fm}$ we are at $m_{\text{PCAC}} \simeq 0$.

We have $m_\pi = 453(32)$ and $m_\pi = 274(25)$;

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Wilson gauge action ($b_1 = 0$)

Twisted mass lattice QCD
A. Shindler

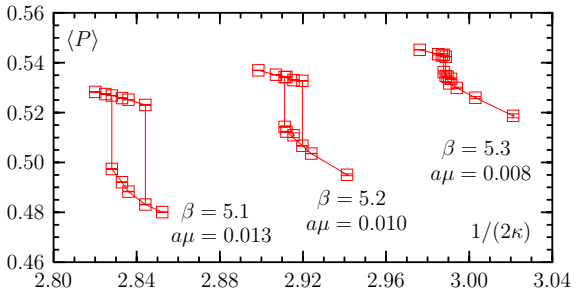
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β	$m_{PS}^{\min} [\text{MeV}]$	ΔP
5.1	$> \sim 600$	0.0399(1)
5.2	$> \sim 630$	0.0261(1)
5.3	$> \sim 470$	0.0077(4)



[F. Farchioni, K. Jansen, I. Montvay, E. Scholz, L. Scorzato, A.S., N. Ukita, C. Urbach, U. Wenger, I. Wetzorke: 2005]
[Talk of I. Wetzorke]

DBW2 gauge action



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$\beta = 0.67, \mu a = 0.01$	$\beta = 0.74, \mu a = 0.0075$
$a \approx 0.19 fm$	$a \approx 0.12 fm$
$L \approx 2.3 fm$	$L \approx 2 fm$
$[M_\pi L]_{min} \approx 3.35$	$[M_\pi L]_{min} \approx 3.13$
$[M_\pi]_{min} \approx 300 MeV$	$[M_\pi]_{min} \approx 320 MeV$

[F. Farchioni, K. Jansen, I. Montvay, E. Scholz, L. Scorzato, A.S., N. Ukita, C. Urbach, U. Wenger, I. Wetzorke: 2004 and *work in progress*]
[Talk of F. Farchioni and Poster of N. Ukita]

At $\beta = 0.74$ the pion mass splitting is consistent with zero indicating a small metastability region. [Poster of J. Pickavance]

DBW2 gauge action



Twisted mass lattice QCD

A. Shindler

Cutoff effects

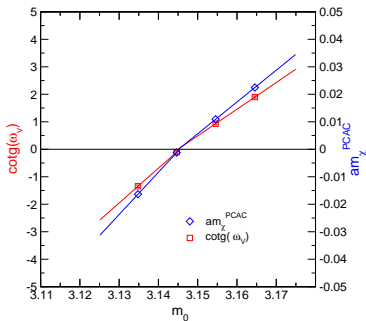
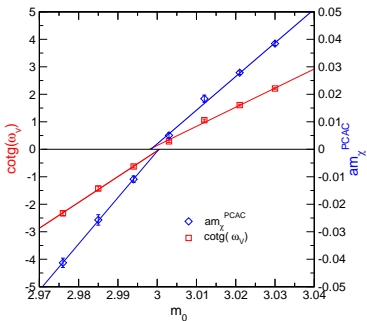
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$$\cotg\omega_V = \frac{\sum_{\mathbf{x}} \langle A_0^a(\mathbf{x}) P^b(\mathbf{y}) \rangle}{\sum_{\mathbf{x}} \langle V_0^a(\mathbf{x}) P^b(\mathbf{y}) \rangle}$$

$$m_{PCAC} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(\mathbf{x}) P^a(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^a(\mathbf{x}) P^a(0) \rangle}$$

Dependence on the gauge action



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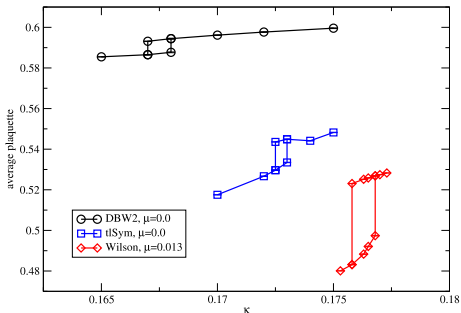
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$$b_1 = -1.4088 \quad b_1 = -\frac{1}{12} \quad b_1 = 0$$

Non-zero b_1 reduces the gap considerably. The choice on tISym is because:

- It has good scaling properties
- Well behaved in perturbation theory


$$N_f = 2 + 1 + 1$$

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- Using off diagonal splitting (degenerate and non degenerate doublet have different flavour orientation) the (1+1) determinant is real and positive [R. Frezzotti, G. C. Rossi:2004]; see [C. Pena, S. Sint, A. Vladikas: 2004] for an alternative;

$$m_c^R \simeq 1.5\text{GeV} \quad m_s^R \simeq 0.1\text{GeV} \rightarrow \frac{Z_P}{Z_S} > 0.875$$

$$m_c^R \simeq 0.9\text{GeV} \quad m_s^R \simeq 0.1\text{GeV} \rightarrow \frac{Z_P}{Z_S} > 0.8$$

- First simulations are starting [poster of N. Ukita];
- Several algorithms are under investigation:
 - TSMB [I. Montvay: 1996]
 - PHMC [R. Frezzotti, K. Jansen: 1997]
 - Stochastic PHMC [I. Montvay, E. Scholz:2005]
 - (P)HMC [poster of T. Chiarappa]

Short but not less interesting



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- In the talk of [R. Frezzotti] a strategy using a mixed actions approach it has been proposed to obtain $O(a)$ improved B_K and matrix elements related to the $\Delta I = 1/2$ rule without mixing with operators with wrong chiralities; for an alternative see [C. Pena, S. Sint, A. Vladikas:2005]
- A strategy to compute B_B without mixings it has been discussed in the [talk of F. Palombi]
- The effect of a twisted mass term in the low lying modes of the Wilson-Dirac operator and a remnant of index theorem for twisted mass fermions has been discussed in the [talk of C.Gattringer]

The perfect world



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- Lattice QCD with $N_f = 2$ dynamical light quarks and 1(+1) heavier quarks;
- Efficient algorithm;
- Reach a pion mass where it is possible to match χ_{pt} ;
- The volume must be large enough $L \geq 2\text{fm}$
- The lattice action should have good scaling properties and simplified renormalization patterns;
- Is twisted mass QCD a possible way to reach this goal?

Summary

- Is twisted mass QCD a possible way to reach this goal? **I think YES!**
- Important lessons from quenched studies:
 - Cutoff effects are $O(a^2)$ and small (NO bending phenomenon)
 - It is possible to reach small pion masses ($m_\pi = 272$ MeV)
 - The flavour breaking is an issue and it has to be investigated with dynamical simulation (promise to C. Michael: no more quenched!)
 - We have indication of an Aoki phase at $a \simeq 0.1$ fm.
- Algorithmic improvement are crucial to go dynamical. Efficient simulations performant as staggered fermions;
- Better understanding of the phase structure.
- The metastabilities are a general phenomenon of Wilson fermions
- Theoretically well founded action (tlSym+Wtm) allows to perform dynamical simulation at $N_f = 2$ at pion masses of $m_\pi < 299$ MeV. Matching with χ PT.
- Effects of different gauge actions has been studied.
- Simulations with $2 + 1 + 1$ flavour are starting.
- Operator mixing as in the continuum;
- It is important to have a number (>1) of fermion actions to better control the continuum limit;



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