High Precision Fundamental Constants using Lattice PT

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HPQCD Collaboration

Lattice
2005

Strong Coupling and Light Quark Masses

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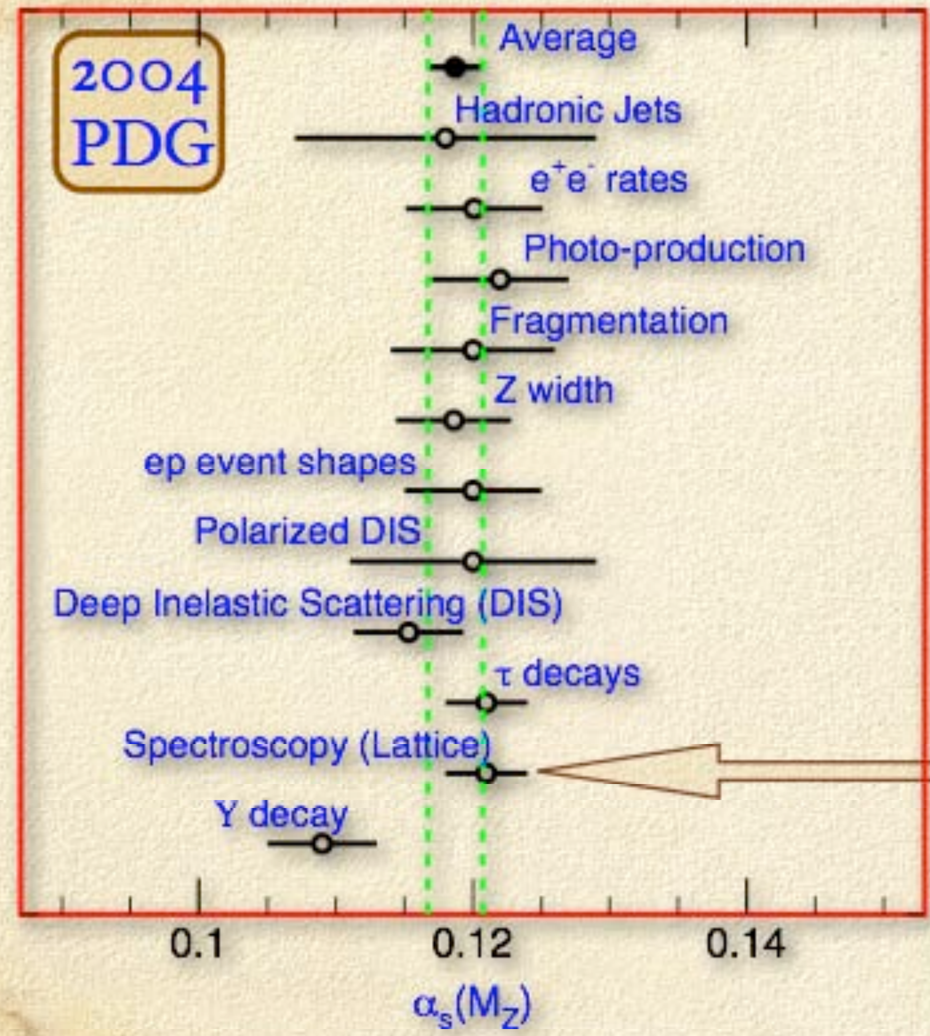
nf = 2 + 1
Asqtad

using MILC: fine, coarse, super-coarse

HPQCD

Determination of $\alpha_{\overline{\mathrm{MS}}}(M_Z)$



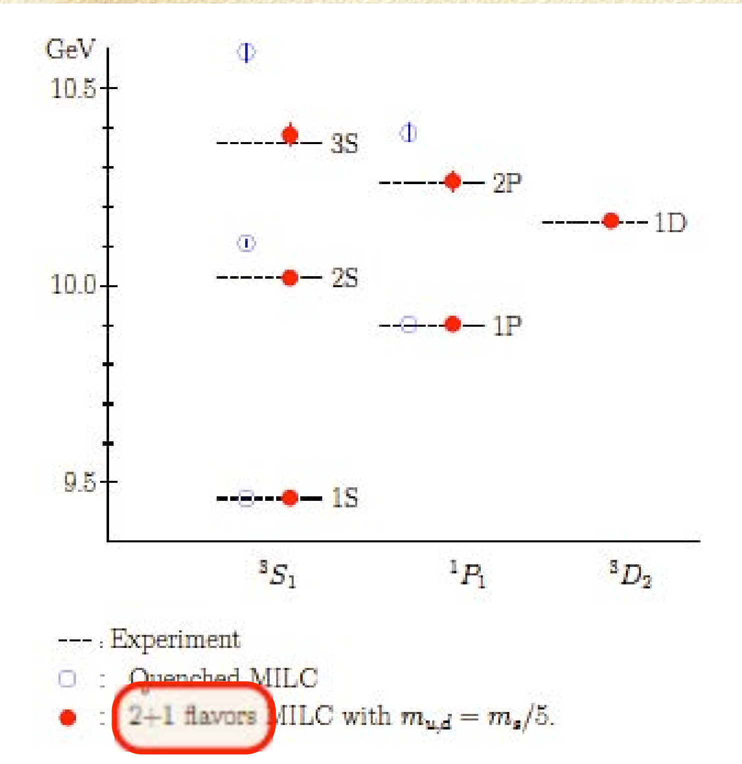
HPQCD Collab'n

Previous lattice NLO analysis

NNLO: reduce uncertainty factor ~ 2

“Novel” use of *both* long- & short-distance QCD

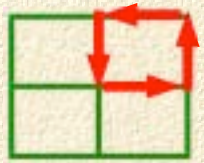
(NRQCD Collab’n 1997)



What’s new?

(i) NPT input e.g. $\Upsilon' - \Upsilon \Rightarrow a$

(ii) Measure short-distance quantity



Wilson loop

▶ Characteristic scale $q^* \propto 1/a$

(iii) Use perturbation theory:

$$\langle \mathcal{O} \rangle = c_1 \alpha(q^*) + c_2 \alpha^2(q^*) + \boxed{c_3 \alpha^3(q^*)} + \ldots$$

(iv) Evolve $\alpha(q^*)$ to $\alpha_{\overline{\mathrm{MS}}}(M_Z)$

Lattice PT essential, but hard

$$= \int [dU_\mu(x)][d\bar\psi d\psi] \quad e^{-\beta(S_{\rm gluon}+S_{\rm quark}^{\rm stagg})}$$

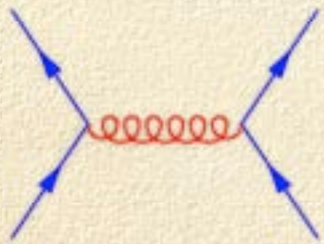
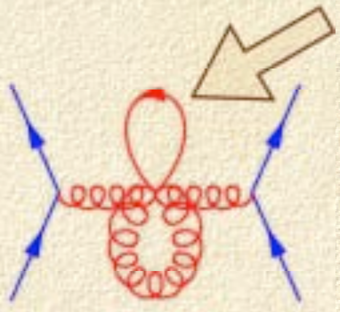
$$\text{e.g.} - \ln \mathrm{W}_{1\times 1} = 3.0684\,\alpha_{\rm lat}\left[1 + 2.421\alpha_{\rm lat} + 8.436(5)\alpha_{\rm lat}^2\right] + \ldots$$

- Input is lattice-regularized bare coupling $\alpha_{\rm lat}$

$$= 3.0684\,\alpha_V(3.33/a)\left[1 - 1.068\alpha_V + 1.69(4)\alpha_V^2\right] + \ldots$$

- We need to reorganize series $\alpha_{\overline{\rm MS}}$

- Peculiarities lattice regulator

$$-4\pi C_F \frac{\alpha_V(q^2)}{q^2} =$$  $+$  $+ \ldots$

Lattice PT is more difficult

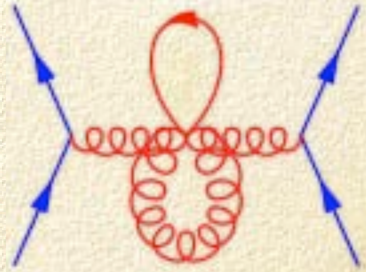
- Same as regular QFT perturbation theory BUT:
  - lattice Feynman Rules much larger
  - many more lattice Feynman Diagrams
  - lattice integrals non-analytic

Lattice PT is more difficult

- Same as regular QFT perturbation theory BUT:
  - **automated**
  - **automated**
  - **numerical brute force**

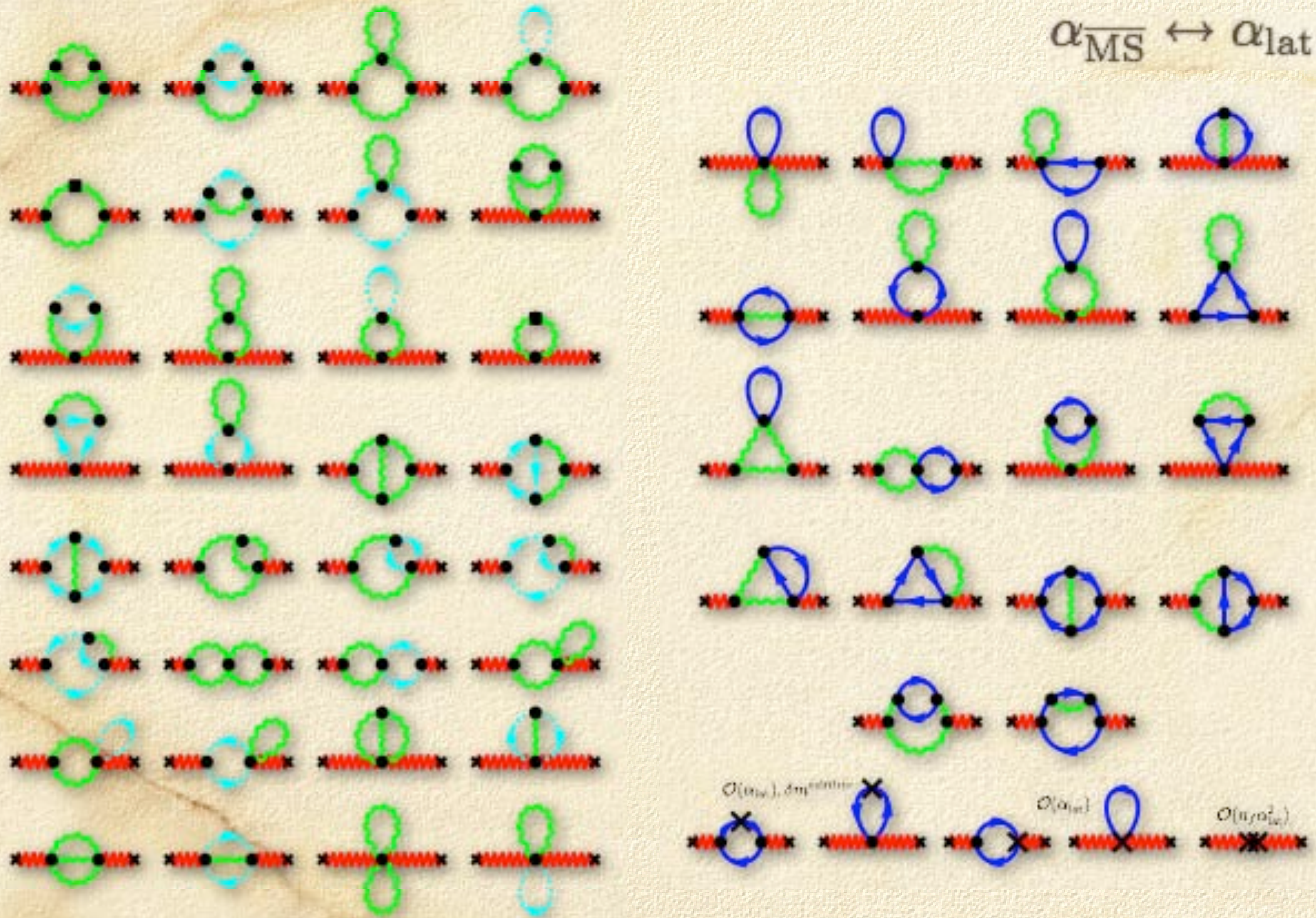
Flexible, action and process agnostic approach. Highly automated construction, adaptive Monte-Carlo integration.

Lattice Regulator Unhappiness

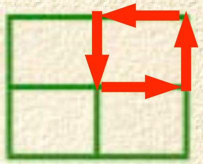


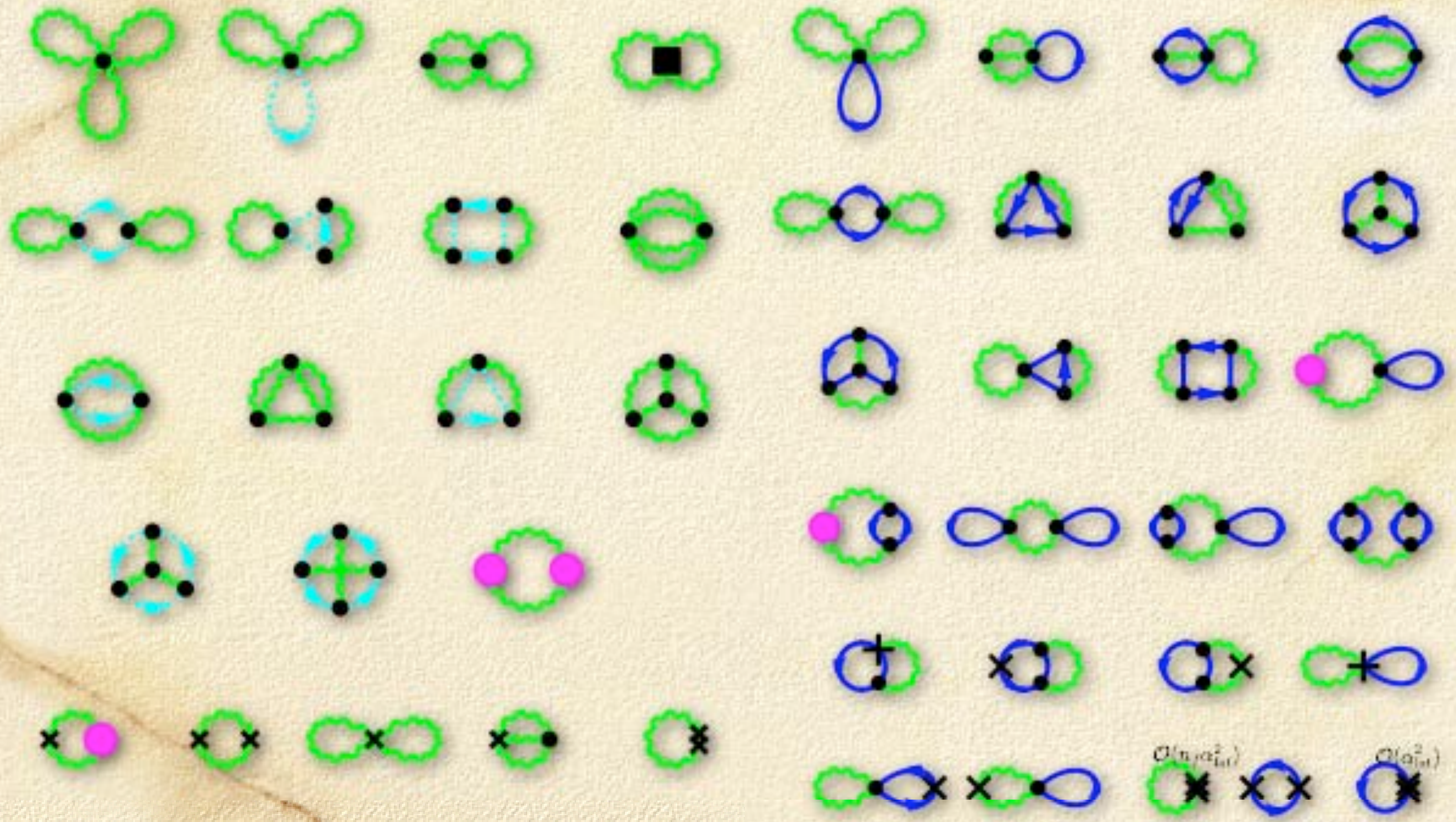
Fortunately, PT algebra can be automated, loop integrals done numerically

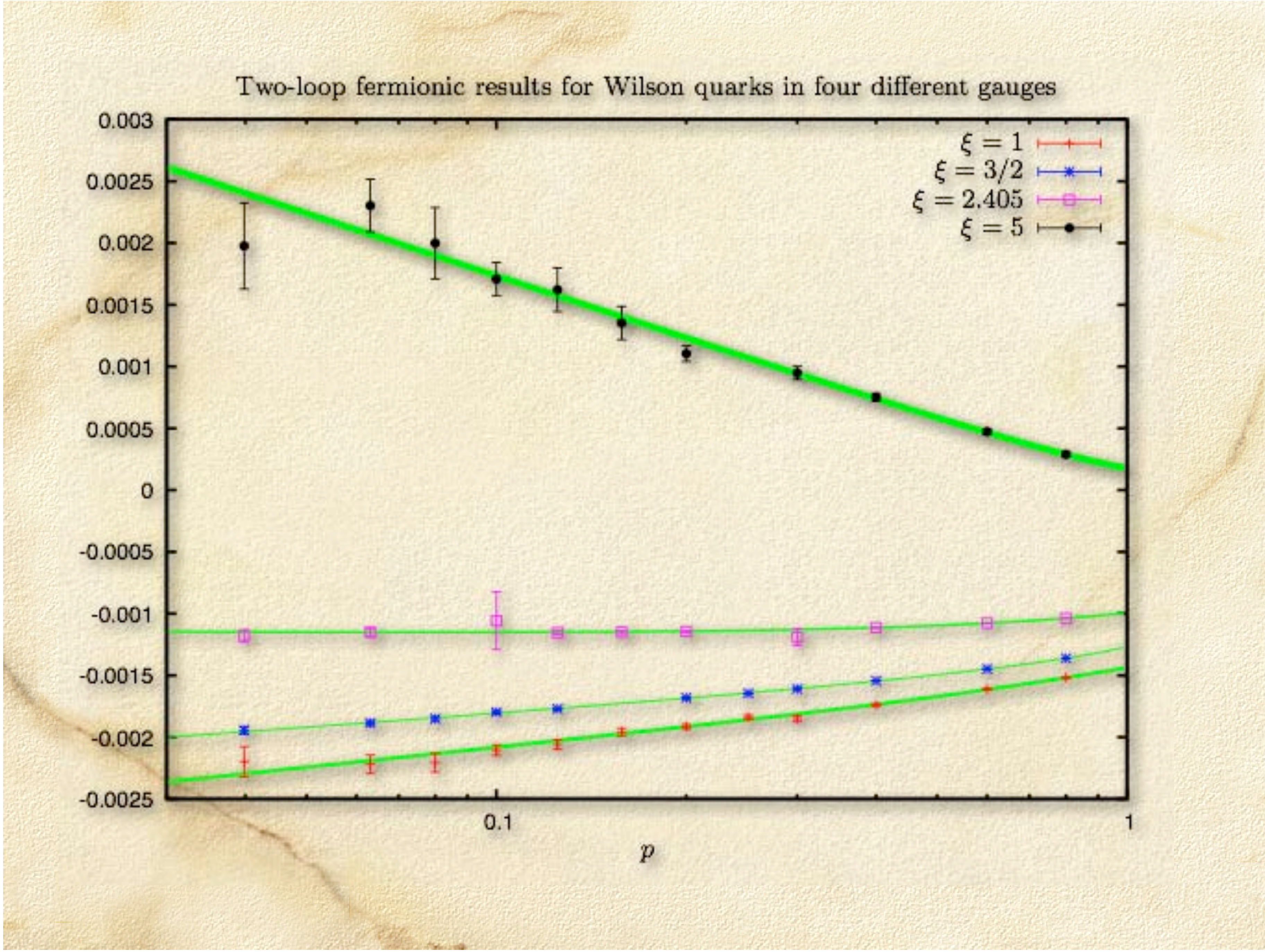
Two-loop Diagrams



Vacuum Bubble Diagrams





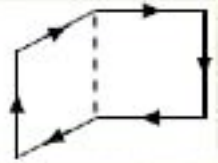


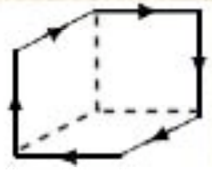
Two-loop fermionic results for Wilson quarks in four different gauges
$\xi = 1$
$\xi = 3/2$
$\xi = 2.405$
$\xi = 5$
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
-0.0005
-0.001
-0.0015
-0.002
-0.0025
0.1
1
p

$\alpha_{\overline{\rm MS}}(M_Z)$ **analysis**

- NNLO perturbation theory
  - 8 different Wilson loops:

$$W_{1\times1}, W_{1\times2}, \ldots, W_{2\times3},$$

$W_{\rm BR} =$ 

$W_{\rm CC} =$ 

  - static potential @ 6 R's

$$V(R) = -C_F \frac{\alpha_V(0.5614/R)}{R}\left[1 + \frac{\beta_0^2}{48}\alpha_V^2 + \ldots\right]$$

continuum

remove perturbative discretizations

- Simulations at 3 different lattice spacings:

$$a^{-1} = 1.239(49),\ 1.596(30),\ 2.258(32)\ \text{GeV}$$

  - m_s brackets physical value, $m_{u/d} \downarrow \frac{1}{5} m_s$

} MILC

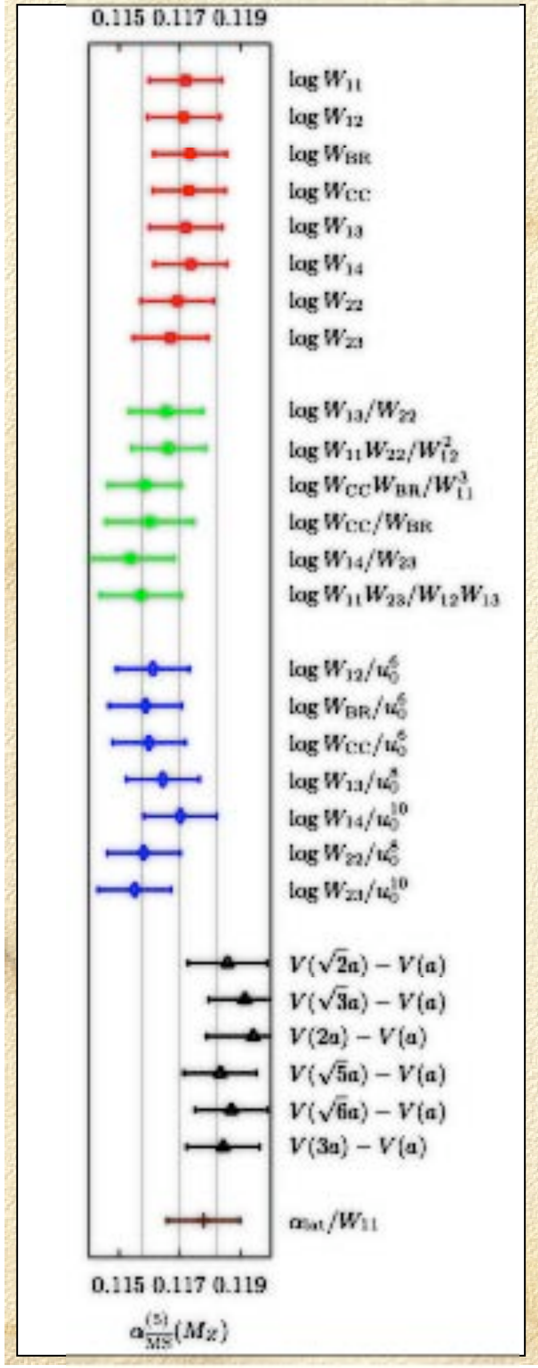
- Estimate systematic uncertainties, incl.
  - higher-order perturbative corrections
  - non-perturbative condensates

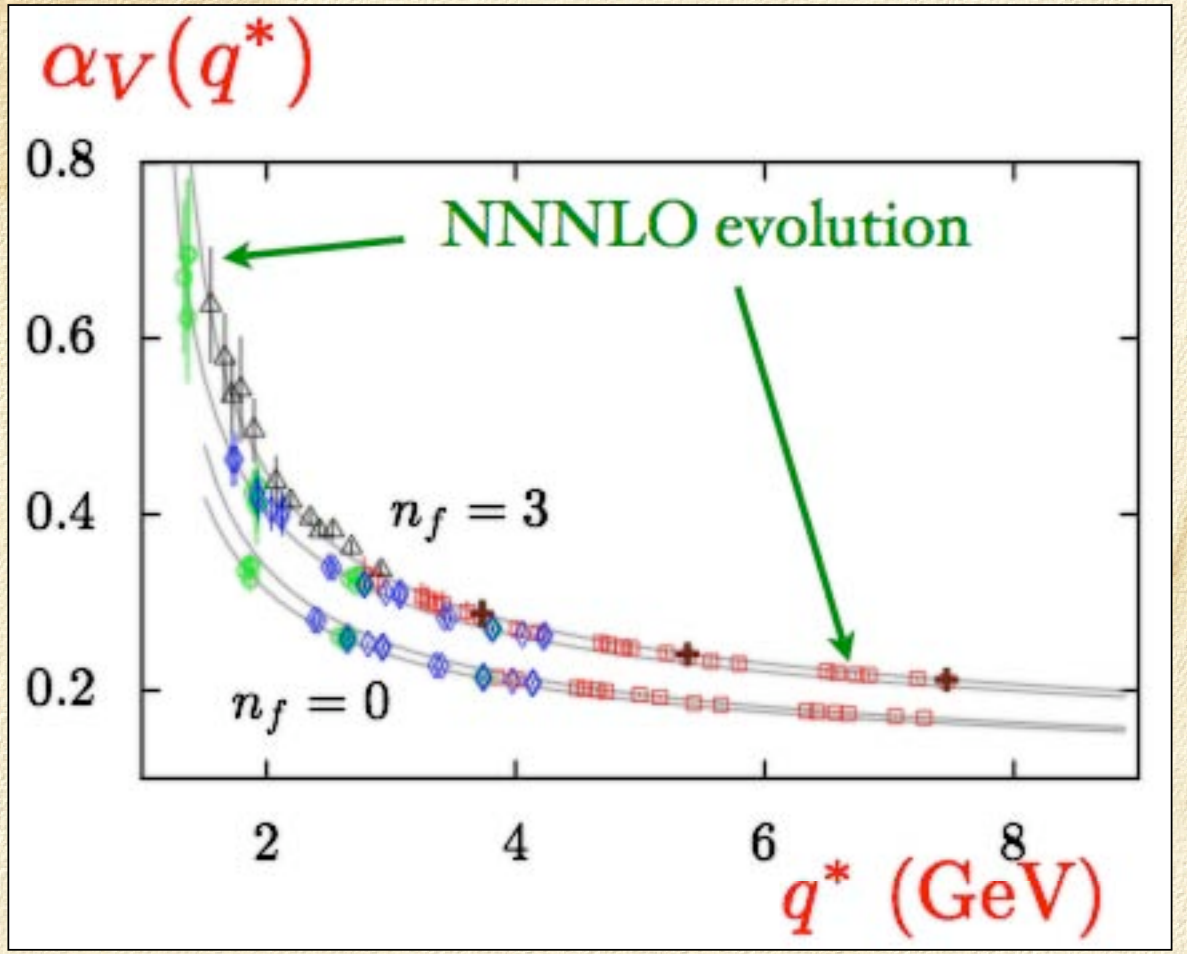
Thanks to three a's

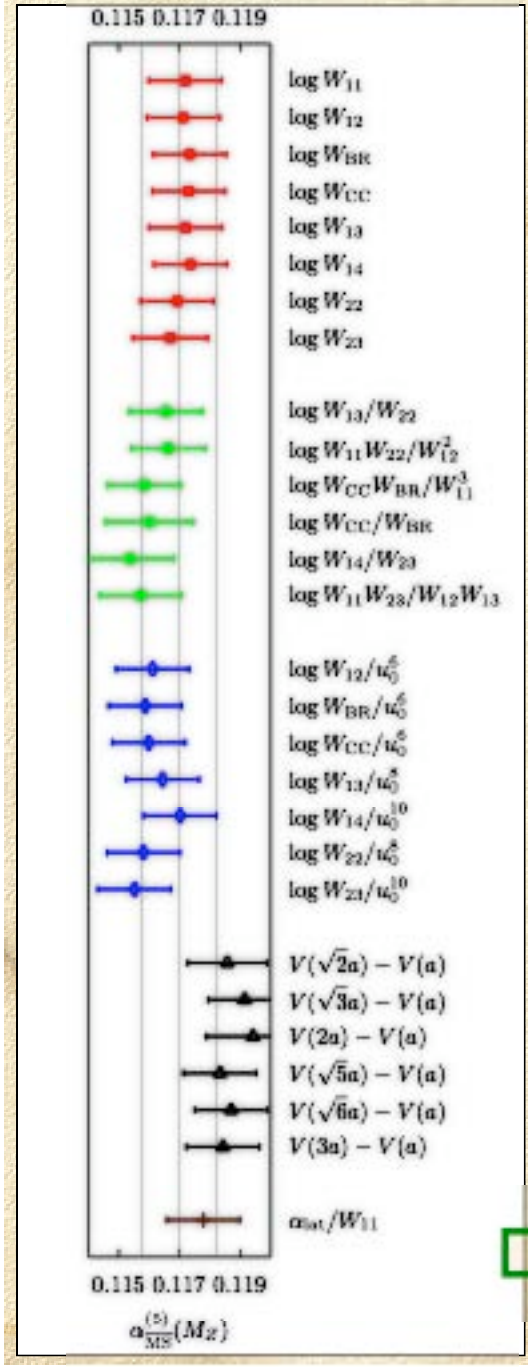
- $\log W_{RT} = \sum_{n=1}^{3+\ldots} c_n \alpha_V^2(d/a) - \frac{\pi}{36} a^4 (RT)^2 \langle \alpha_s F^2 \rangle + \ldots$
  - use NNNLO $\beta_{\text{“}V\text{”}}$ evolve from one input: $\alpha_V(7.5\ \mathrm{GeV})$ $=$ $0.208(4)$
  - use constrained curve-fitting:

$c_{n\geq 4} = \mathcal{O}(1)$

$\langle \frac{\alpha_s}{\pi} F^2 \rangle = (0.009 \pm 0.007)\ \mathrm{GeV}^4$ (Ioffe & Zyablyuk)


0.115 0.117 0.119
log W_{11}
log W_{12}
log W_{BR}
log W_{CC}
log W_{13}
log W_{14}
log W_{22}
log W_{23}
log W_{13}/W_{22}
log $W_{11}W_{22}/W_{12}^2$
log $W_{CC}W_{BR}/W_{11}^3$
log W_{CC}/W_{BR}
log W_{14}/W_{23}
log $W_{11}W_{23}/W_{12}W_{13}$
log W_{12}/u_0^6
log W_{BR}/u_0^6
log W_{CC}/u_0^6
log W_{13}/u_0^8
log W_{14}/u_0^{10}
log W_{22}/u_0^8
log W_{23}/u_0^{10}
$V(\sqrt{2}a) - V(a)$
$V(\sqrt{3}a) - V(a)$
$V(2a) - V(a)$
$V(\sqrt{5}a) - V(a)$
$V(\sqrt{6}a) - V(a)$
$V(3a) - V(a)$
α_{lat}/W_{11}
0.115 0.117 0.119
$\alpha_{\overline{MS}}^{(5)}(M_Z)$


$\alpha_V(q^*)$
0.8
0.6
0.4
0.2
NNNLO evolution
$n_f = 3$
$n_f = 0$
2
4
6
8
q^* (GeV)



Error Budget

	$\log W_{11}$	$\log W_{13}/W_{22}$	$V(\sqrt{2}a) - V(a)$
a^{-1}	0.0008	0.0010	0.0008
$c_1 \ldots c_3$	0.0001	0.0004	0.0006
c_n for $n \geq 4$	0.0008	0.0005	0.0006
$V \to \overline{\mathrm{MS}} \to M_Z$	0.0001	0.0001	0.0001
condensate	0.0002	0.0001	0.0001
m_u, m_d, m_s	0.0003	0.0001	0.0001
m_c, m_b	0.0002	0.0002	0.0002
simulation errors	0.0000	0.0000	0.0001
total uncertainty	0.0012	0.0012	0.0012

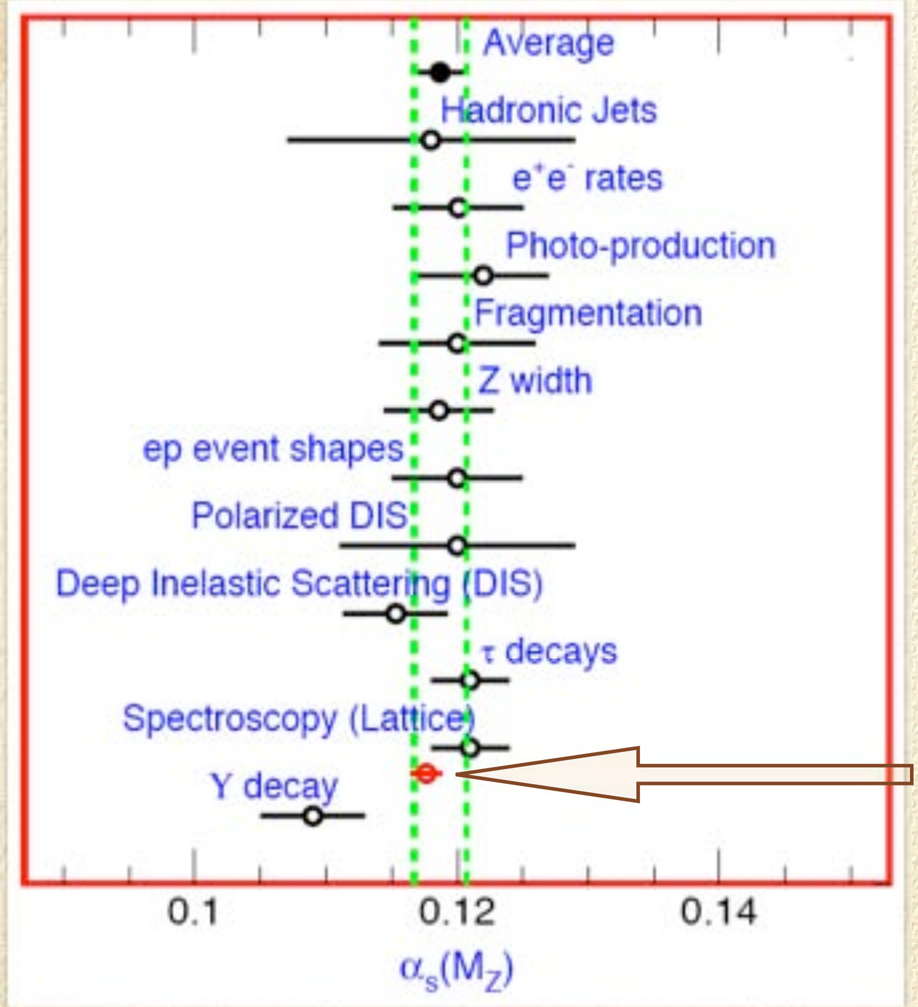
Weighted average

$$\alpha_{\overline{\mathrm{MS}}}(M_Z) = 0.1171(13)$$

Result of new NNLO analysis

QJM,
Trottier,
Davies,
Foley,
Gray,
Lepage,
Nobes,
Shigemitsu

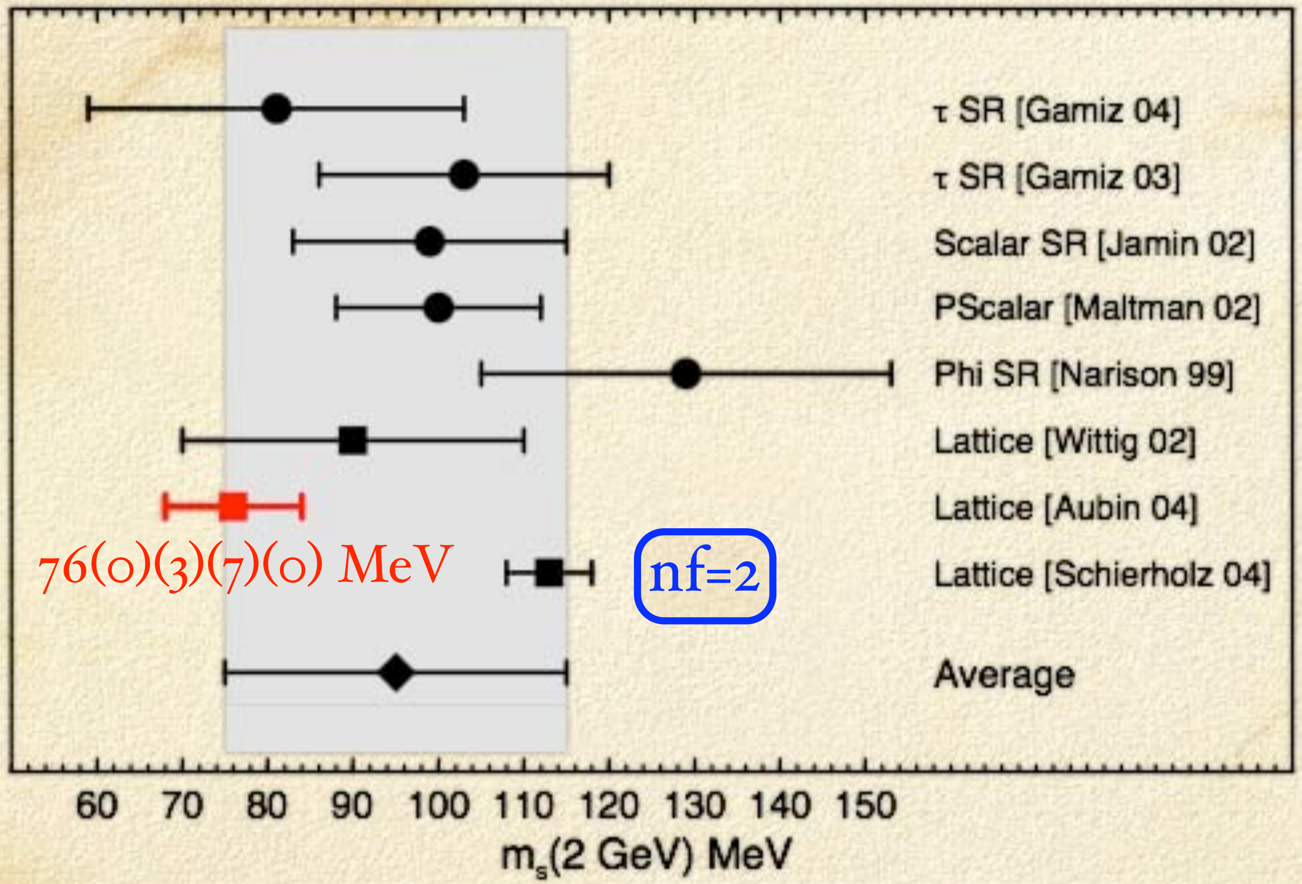
PRL,
to appear



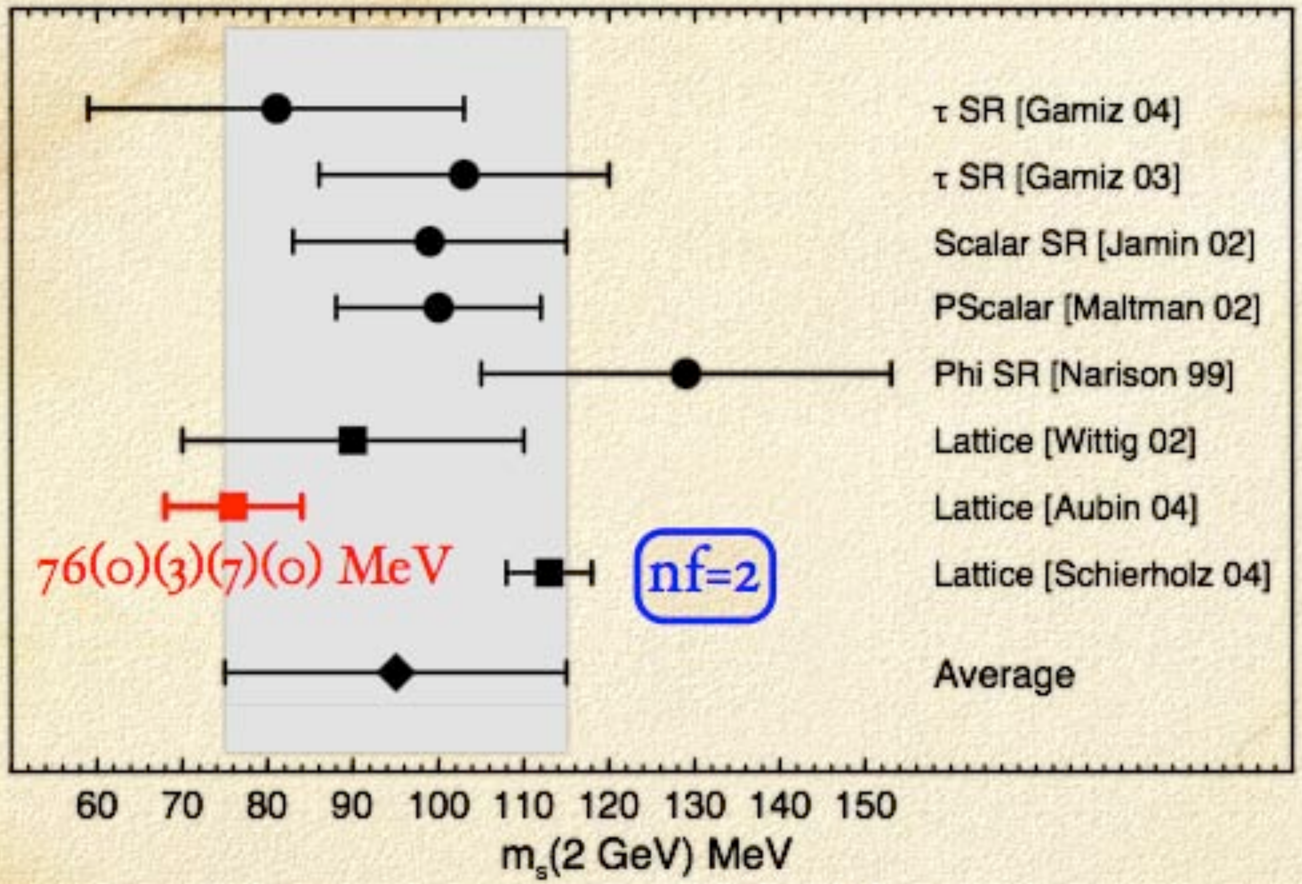
$\alpha_{\overline{\rm MS}}(M_Z)$
=0.1171(13)

PDG world avg = 0.1187(20)

Strange Quark Mass



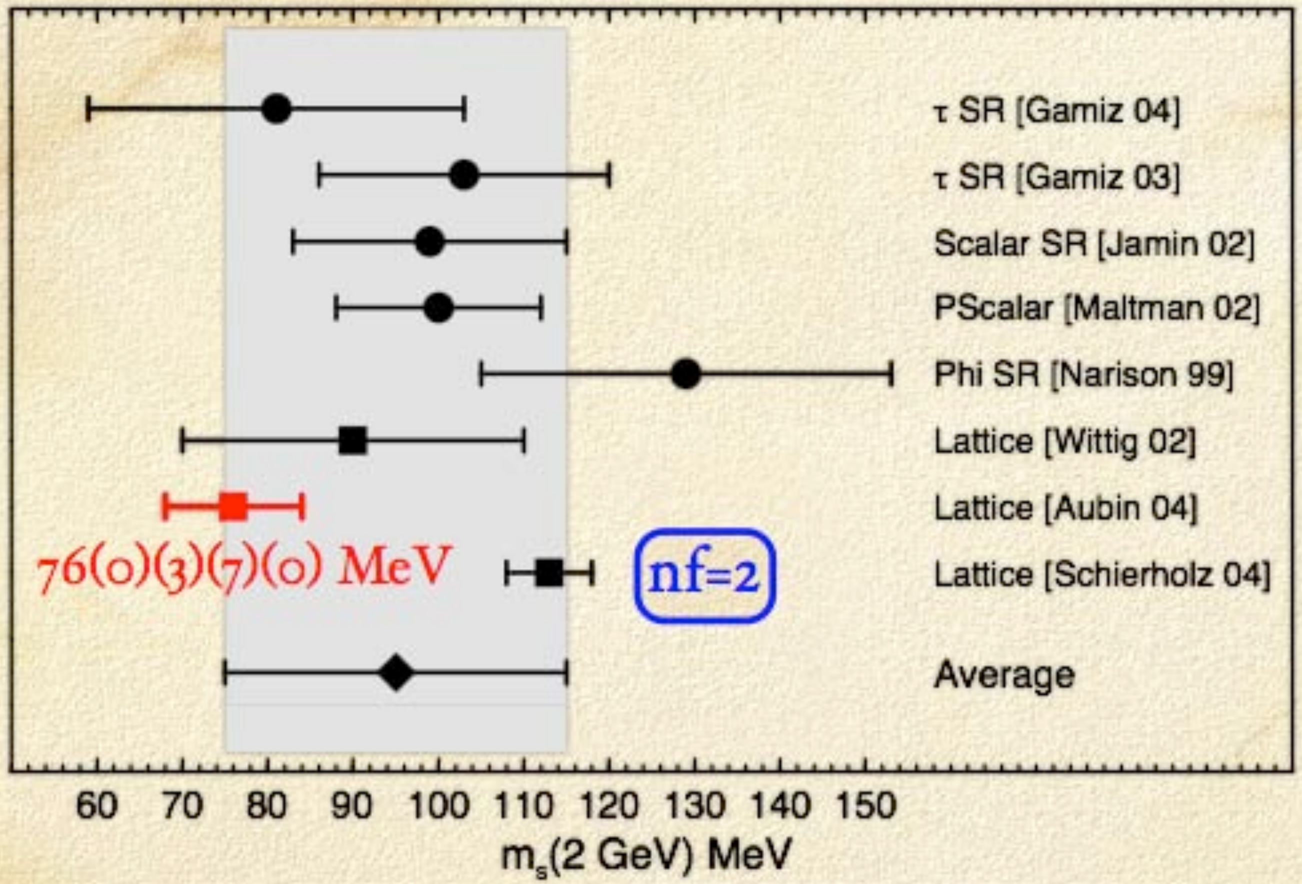
Strange Quark Mass



$M_{\text{pole}} = m_{\overline{\text{MS}}}(\mu) + \ldots$ (Tarrach, Broadhurst, Melnikov)

$$M_{\text{pole}} = m_{\text{lat}}\left(1 + \mathcal{O}(\alpha) + \mathcal{O}(\alpha^2) + \ldots\right)$$

Strange Quark Mass



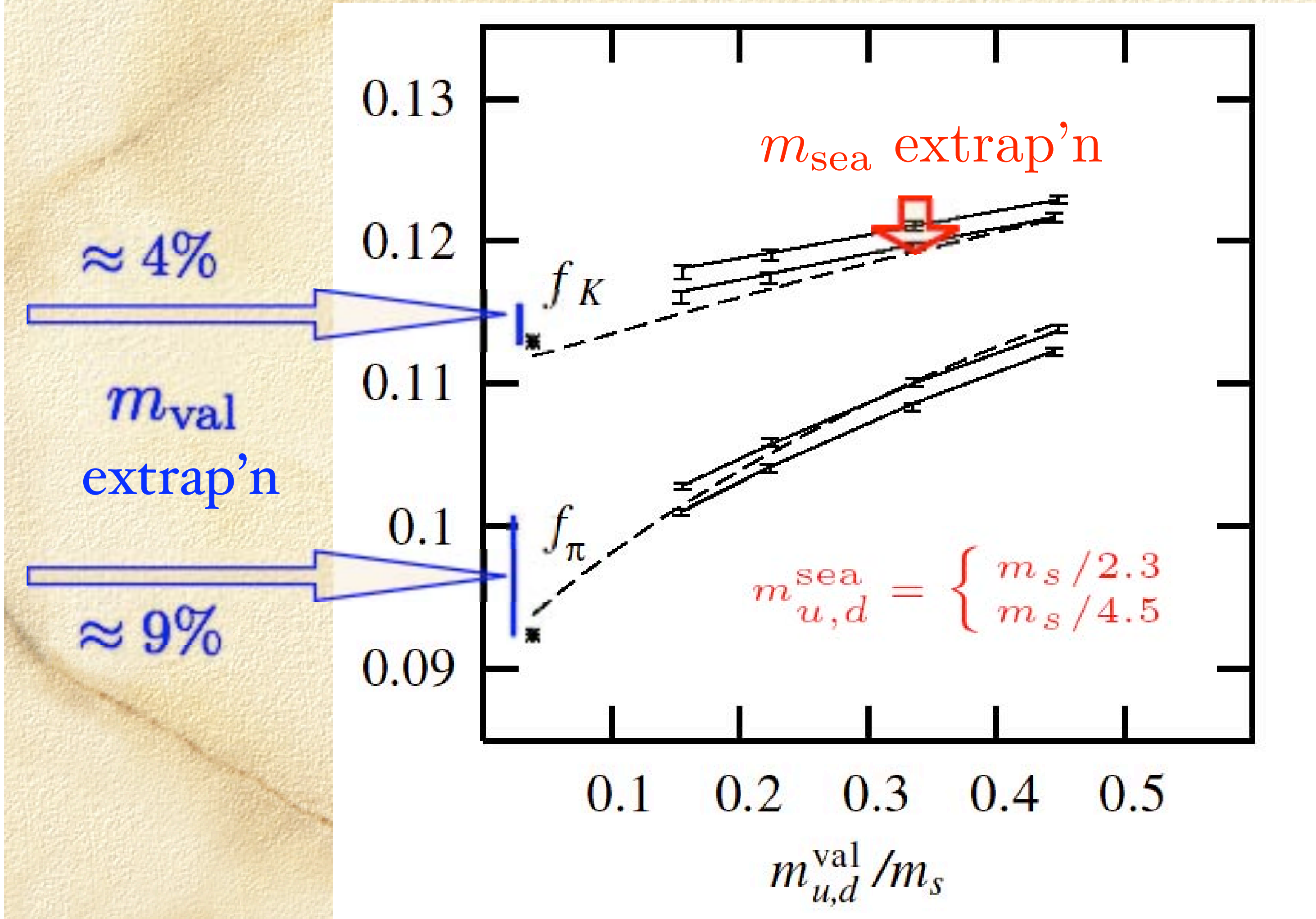
$$m_{\overline{\rm MS}}(\mu a) = \frac{m_0 a}{a}\left(1 + Z_m^{(2)}(ma, \mu a)\,\alpha_V(q^*) \boxed{+ Z_m^{(4)}(\mu a)\,\alpha_V^2} + \ldots\right)$$

New

Partially Quenched χ PT

Lee & Sharpe
Bernard & Aubin

Run at several $m_{u/d,s}^{\text{valence}}$ not necessarily equal to m_{sea}

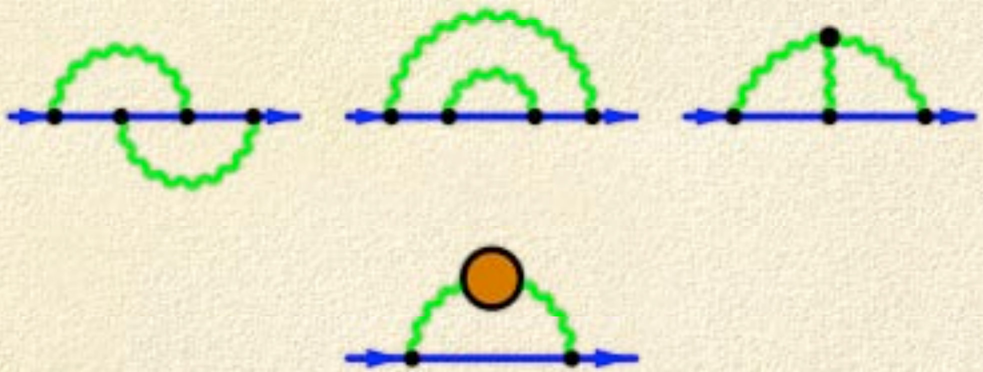


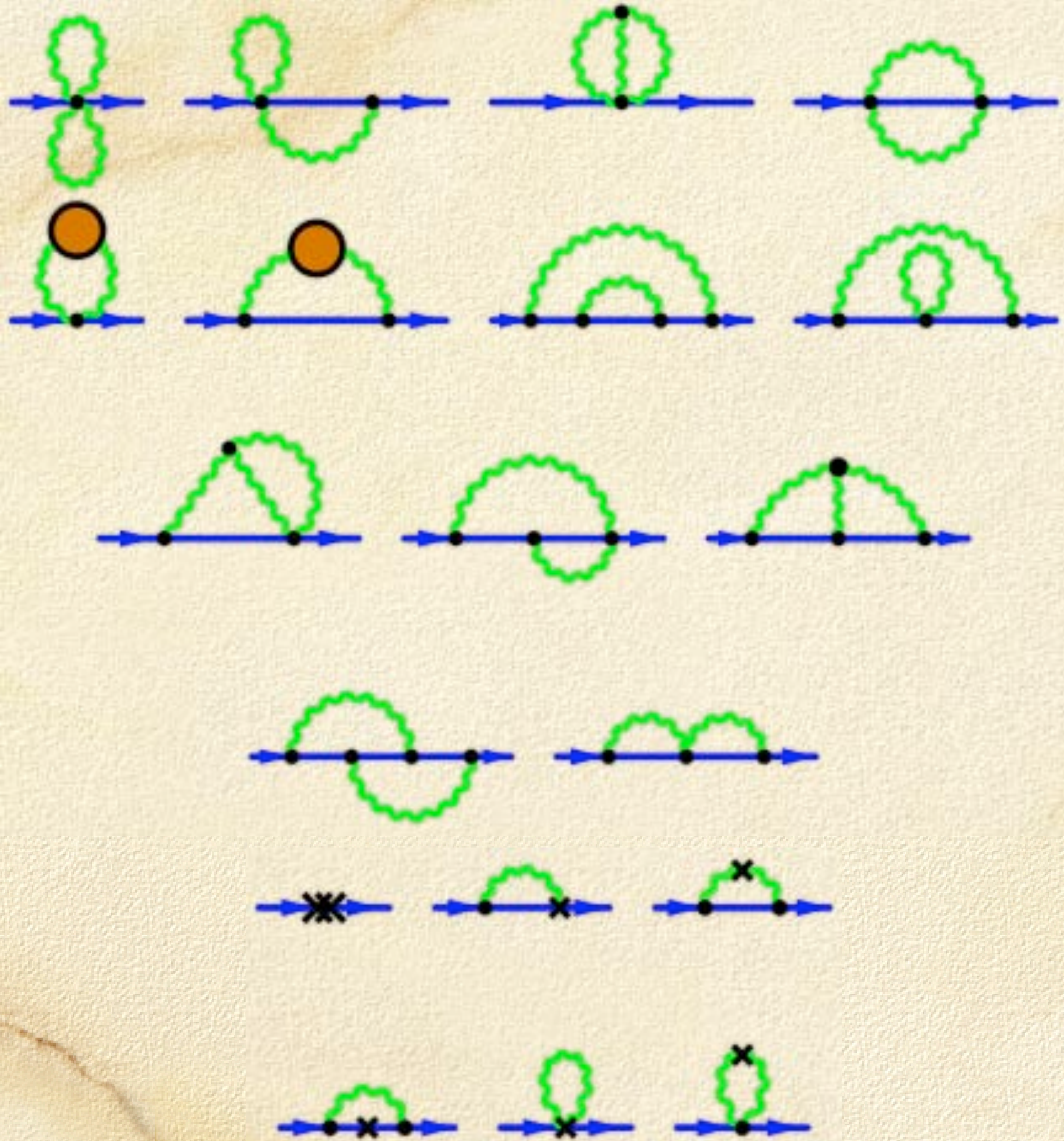
Sea quark masses ~3-5 times smaller than in previous unquenched

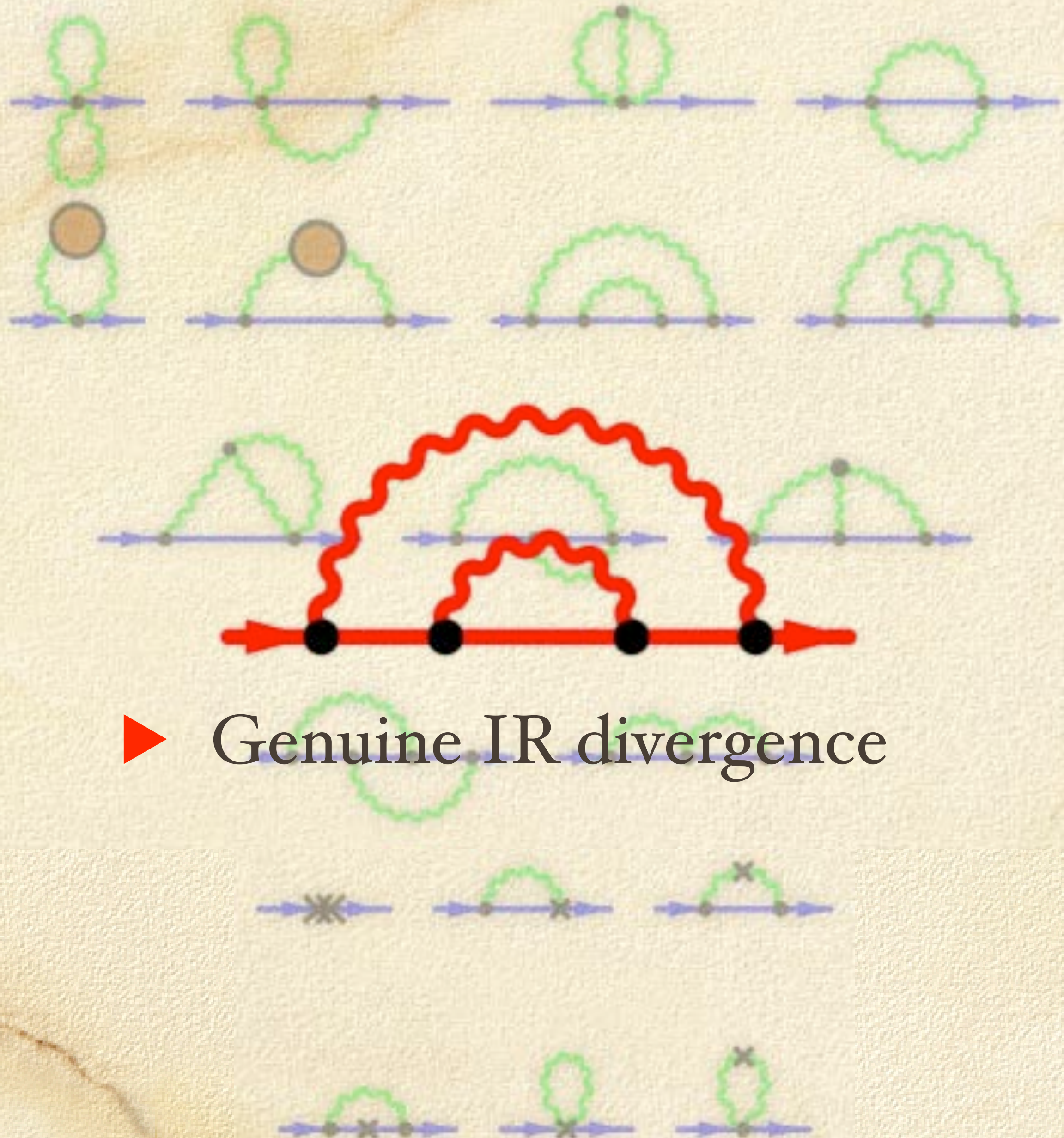
2-loop pole mass

$$\mathrm{Tr}\left[\frac{1+\gamma_0}{4}\right.$$

$$p_{\text{ext}} = (iM, \vec{0})$$

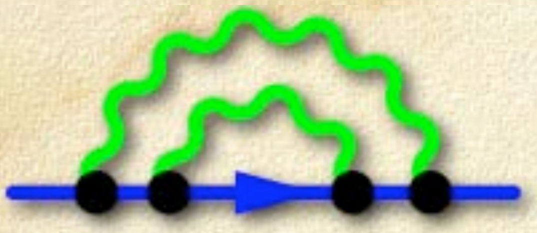


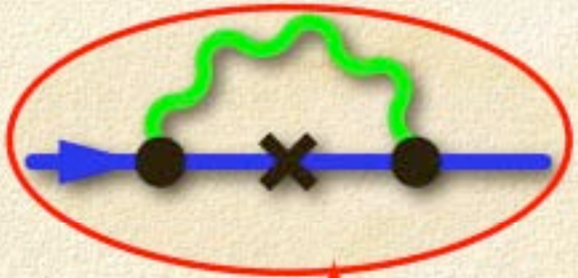




- Genuine IR divergence

IR divergence (continuum)





Single 2-loop IR divergence

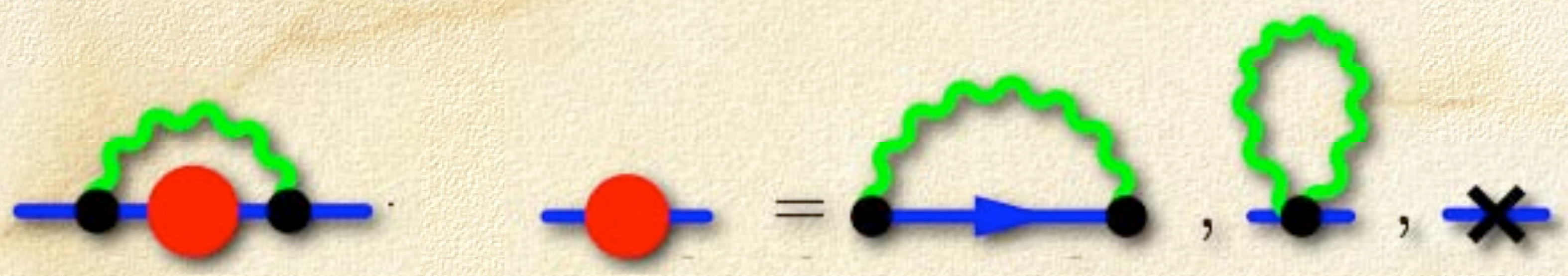
Given:

$$\Sigma_1(p, m_0) \sim m_0 f(p^2/m_0^2)$$

Comes from using:

$$\frac{\partial \Sigma_1}{\partial p} = -\frac{\partial \Sigma_1}{\partial m_0} + \frac{\Sigma_1}{m_0}$$

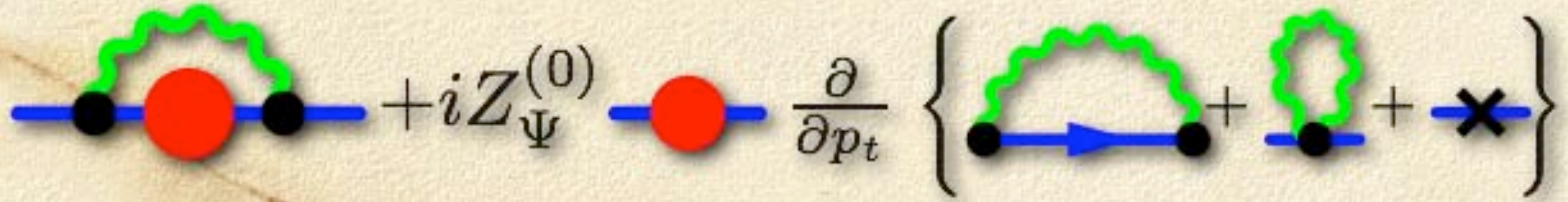
IR divergence (lattice)



- Lattice has 3 IR divergent diagrams at 2-loops!
- Furthermore the one-loop diagram has new scales:

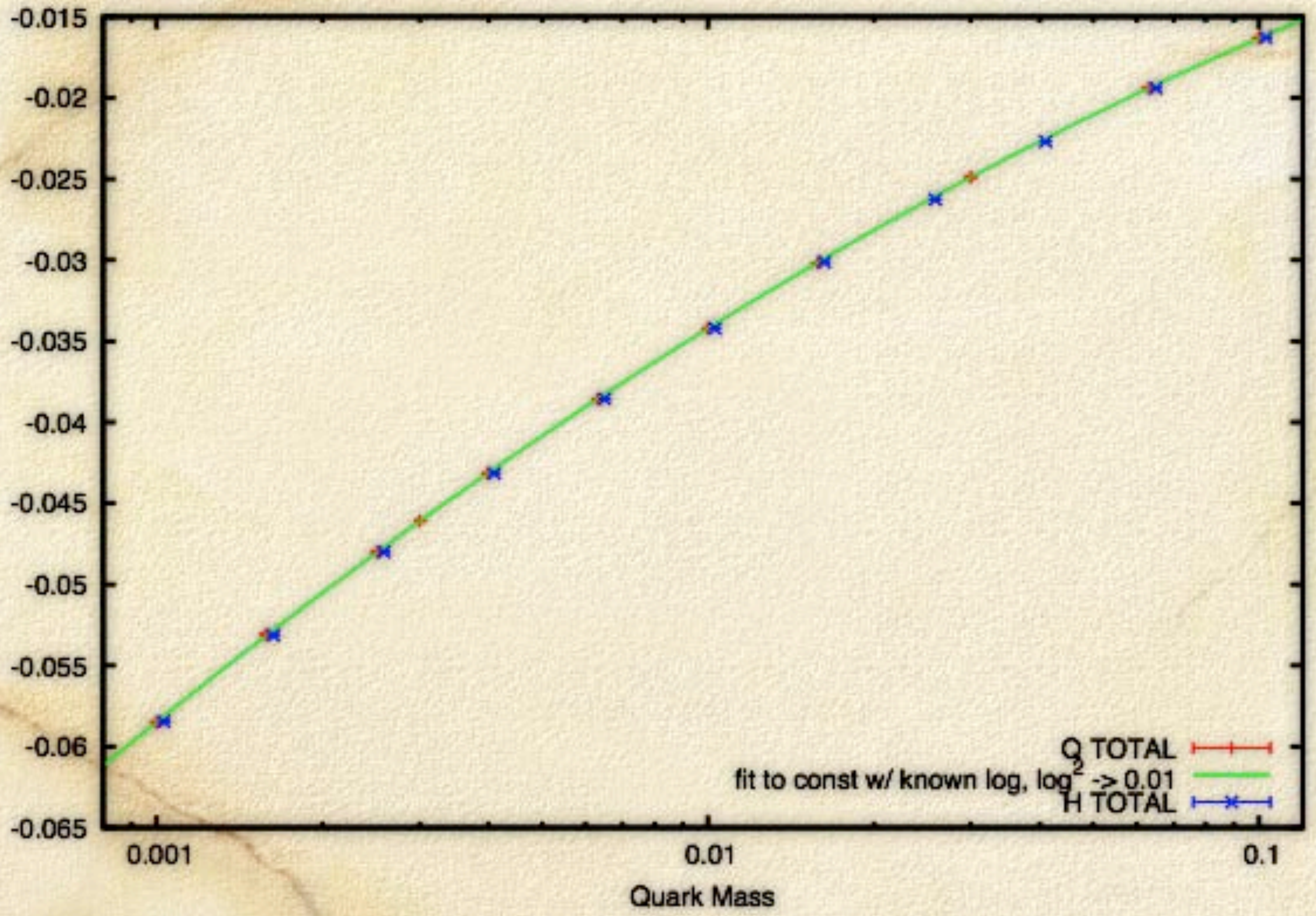
$$\Sigma_1(p, m_0) \sim m_0\, g\Big(p^2/m_0^2, (ap), (am_0)\Big)$$

so trick of transforming d/dp into d/dm does NOT work.



- So keep d/dp and do three separate IR finite combinations

Gluonic Total



Results

$$M = m_0\Big[1+\alpha_L(A\log m_0 a + D) + \alpha_L^2(B\log^2 m_0 a + C\log m_0 a + E)\Big],$$

Take logs and differente wrt log(a), use the fact that the pole mass, M, is RG invariant, the beta-function and the anomalous dimension equation:

$$\frac{d\log m_0}{d\log a} \equiv \gamma_0\alpha_L(a) + \gamma_1^L\alpha_L(a),$$

$$A = -\gamma_0, \qquad A = -\frac{3}{2\pi}C_2(R) = -\frac{2}{\pi}$$

$$B = \frac{1}{2}A^2 - A\frac{\beta_0}{4\pi}$$

$$C = -\gamma_1^L + A^2 - D\left(\frac{\beta_0}{2\pi} - A\right)$$

Deriving γ_1^{lat}

- Can get this from the MS version:

$$\frac{\partial m_{\text{lat}}}{\partial a} = \gamma_0 \alpha_{\text{lat}} + \boxed{\left(\gamma_1^{\overline{\text{MS}}} + C_\alpha \gamma_0 - C_m \frac{\beta_0}{2\pi}\right)} \alpha_{\text{lat}}^2 + \dots$$

$$\mu = a^{-1}$$

$$\alpha_{\overline{\text{MS}}}(a^{-1}) = \alpha_{\text{lat}} \left(1 + C_\alpha \alpha_{\text{lat}} + \mathcal{O}(\alpha^2)\right)$$

$$m_{\overline{\text{MS}}}(a^{-1}) = m_{\text{lat}} \left(1 + C_m \alpha + \mathcal{O}(\alpha^2)\right)$$

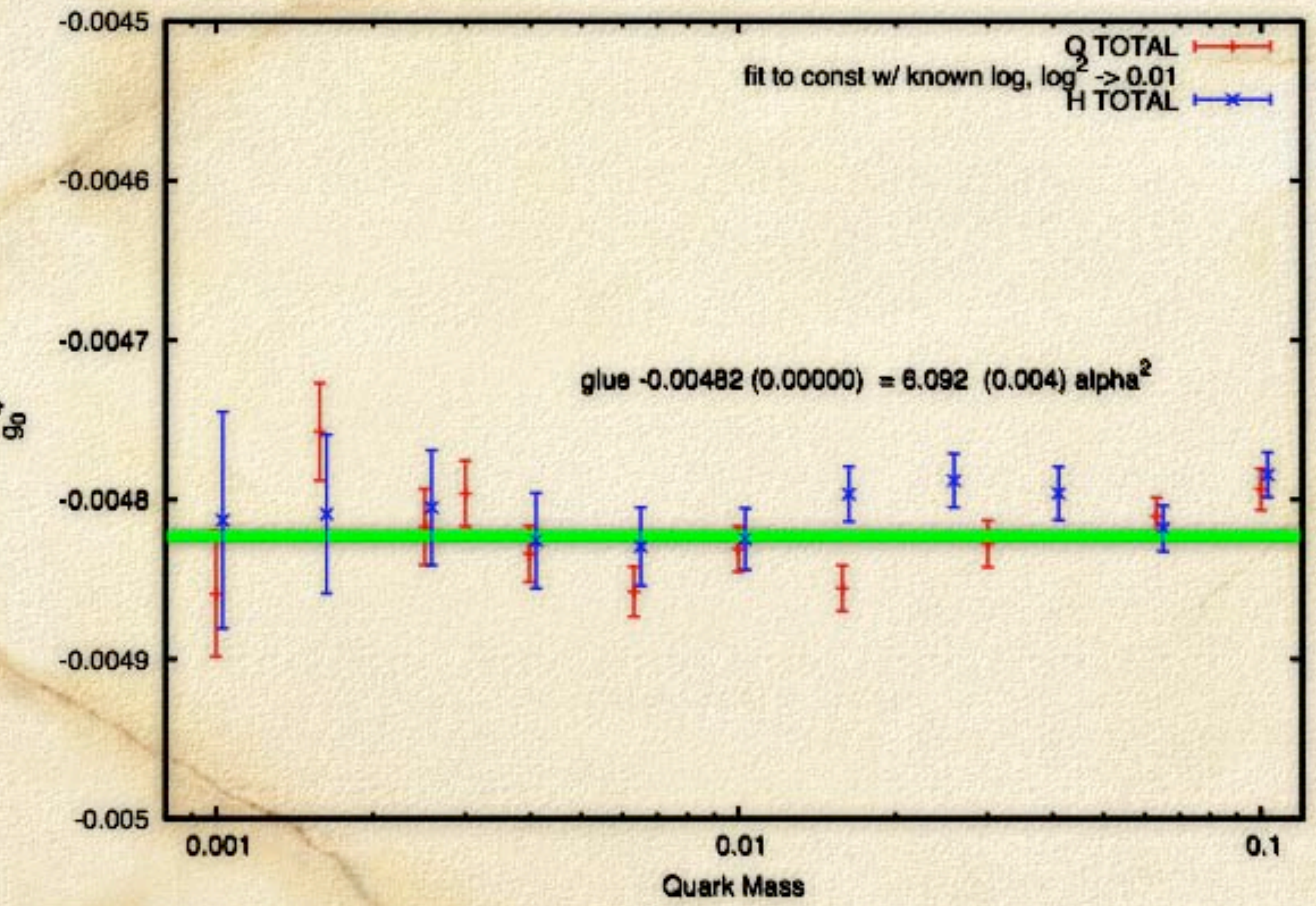
$$\frac{\partial \alpha}{\partial \log a} = \frac{2\beta_0}{4\pi} \alpha^2 + \mathcal{O}(\alpha^3)$$

β-function

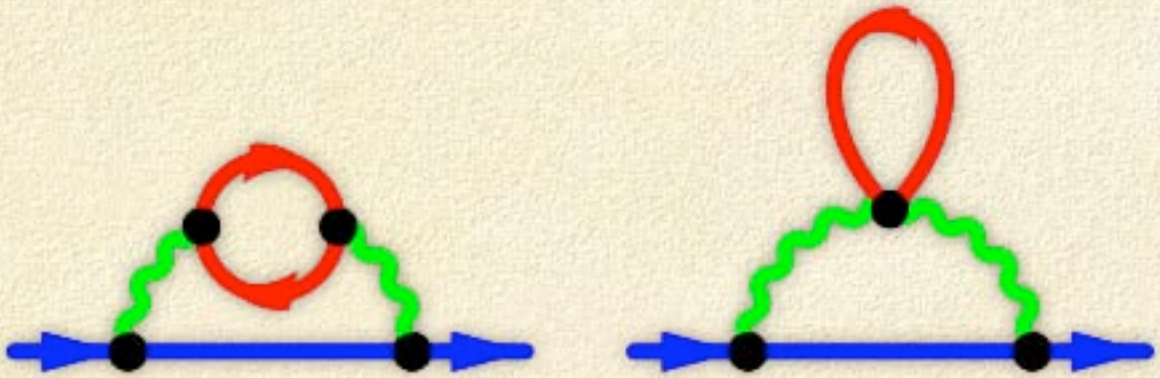
- Assumes that lattice is mass-independent scheme

- Can now remove both log and double-log, and look for artifacts

Subtracting known logs



- Important effect for $\frac{m_s}{\frac{1}{2}(m_u + m_d)}$

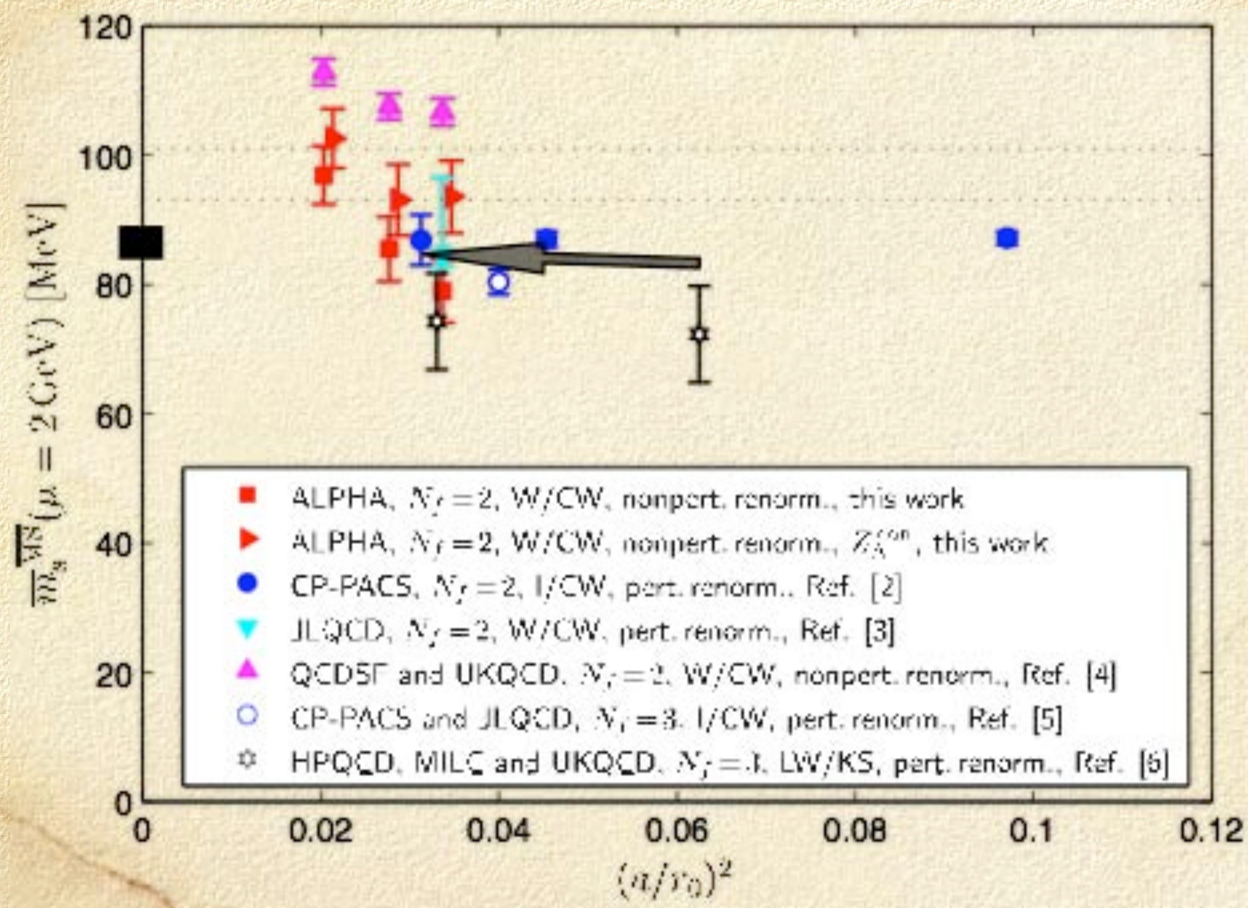


m_{sea} different from m_{valence}

$$\mathcal{O}(1) \cdot \left(\frac{m_{\text{sea}}}{m_{\text{valence}}} \right) \cdot N_f \cdot \alpha^2$$

- Continuum mass dependence calculated by Broadhurst (1990)

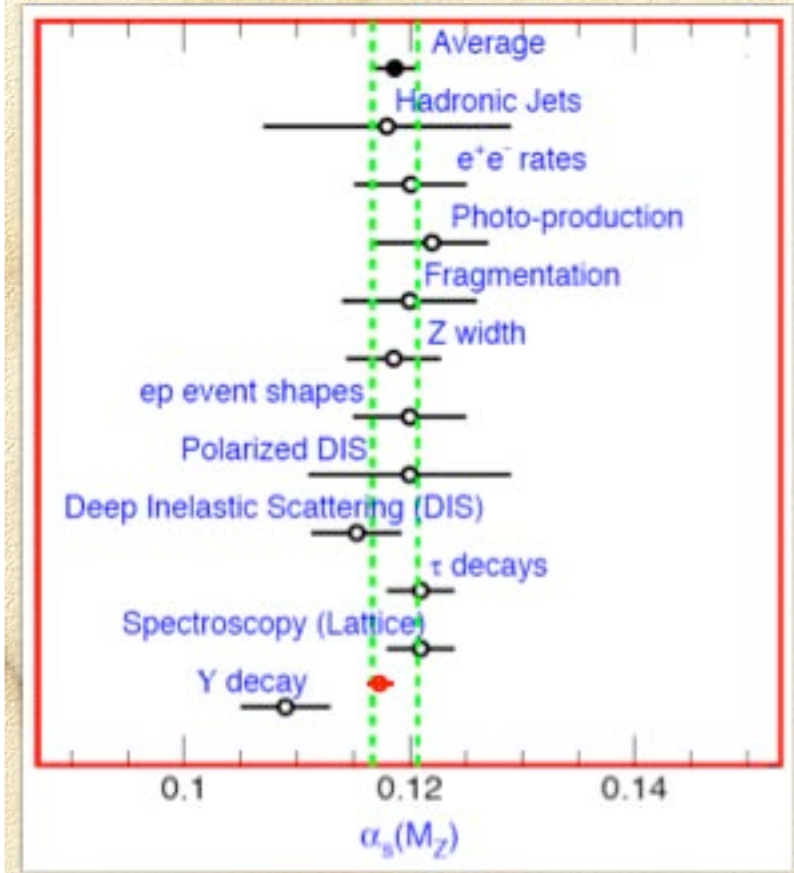
Lattice "comparison" for strange quark mass



HPQCD (prelim): 86(0)(4)(3)(0) MeV

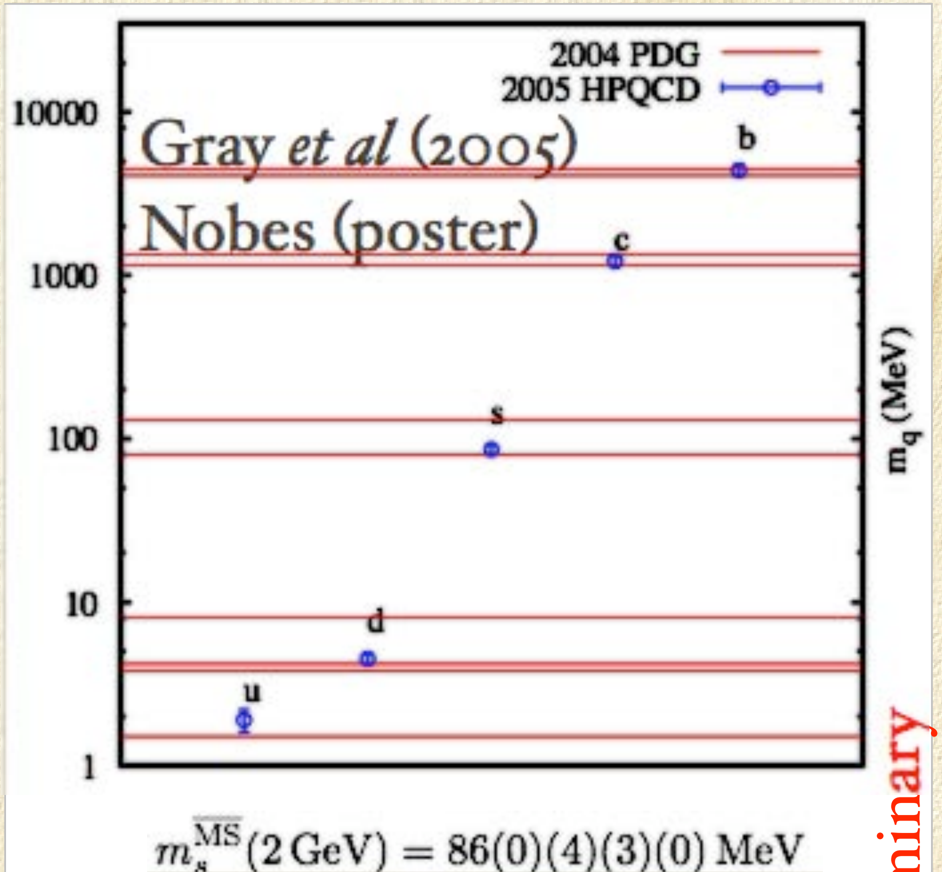
HPQCD (2005) vs PDG (2004)

Strong Coupling Constant



$$\alpha_{\overline{\mathrm{MS}}}(M_Z) = 0.1171(13)$$

Quark Masses



$$m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 86(0)(4)(3)(0)\ \mathrm{MeV}$$

$$m_d^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 4.5(0)(2)(1)(3)\ \mathrm{MeV}$$

$$m_u^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 1.9(0)(1)(1)(3)\ \mathrm{MeV}$$

Preliminary

Summary

Results:

- First with 2+1 flavours, chiral + continuum extrapolations
- Masses and coupling agree with PDG, smaller errors
- Multi-loop PT with highly improved actions

PT:

- Light quark masses to 2nd order
- Coupling constant value to 3rd order
- $\alpha_{\mathrm{lat}} \leftrightarrow \alpha_{\overline{\mathrm{MS}}}(q^*)$ to 2nd order
  - IR divergences at 2-loops by numerical brute force

Future:

- Next: assault on theory errors in CKM
  2-loop leptonic & semileptonic D-decays
  - precision measurements @ CLEO-c