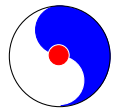


Recent results
from
unquenched light quark simulations

Taku Izubuchi



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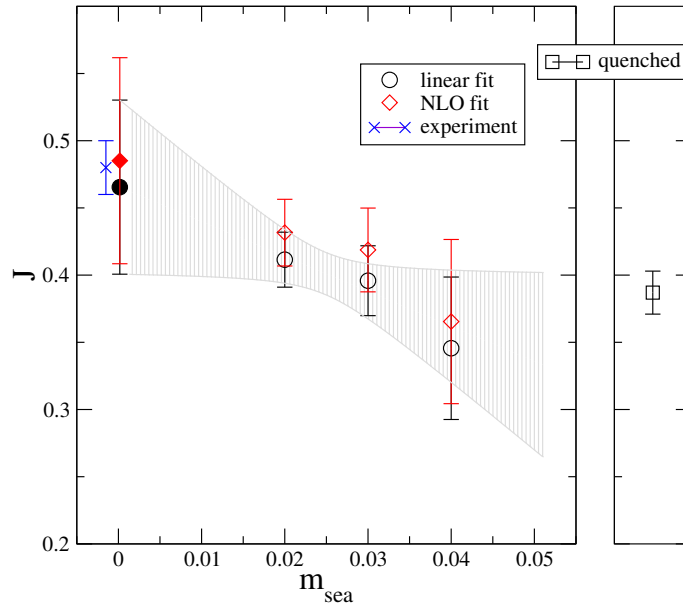
Kanazawa University

Contents

- Introduction
- Performance
- $N_f=2, 3$ Dynamical simulations
- Quark mass
- Conclusions

Introduction

- In quenched simulations ($N_F = 0$, no sea quark in QCD vacuume), Hadron spectrum found to be 5-10% different from experiments (CP-PACS, JLQCD, UKQCD)



- (RBC) $N_F = 0, 2$ DWF

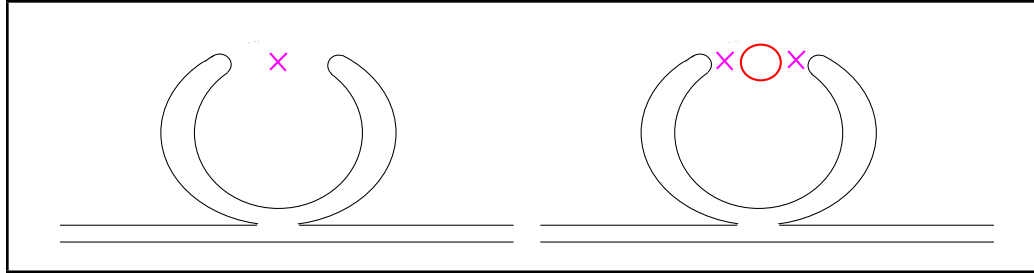
$$J = m_V \left. \frac{dm_{ps}^2}{dm_V} \right|_{m_V/m_{ps}=1.8}$$

- Is it only 5-10% ?

- More obvious quenched pathology was found in NS scalar meson

$$a_0 \quad I^G(J)^{PC} = 1^+(0^{++})$$

(Bardeen *et.al.*)



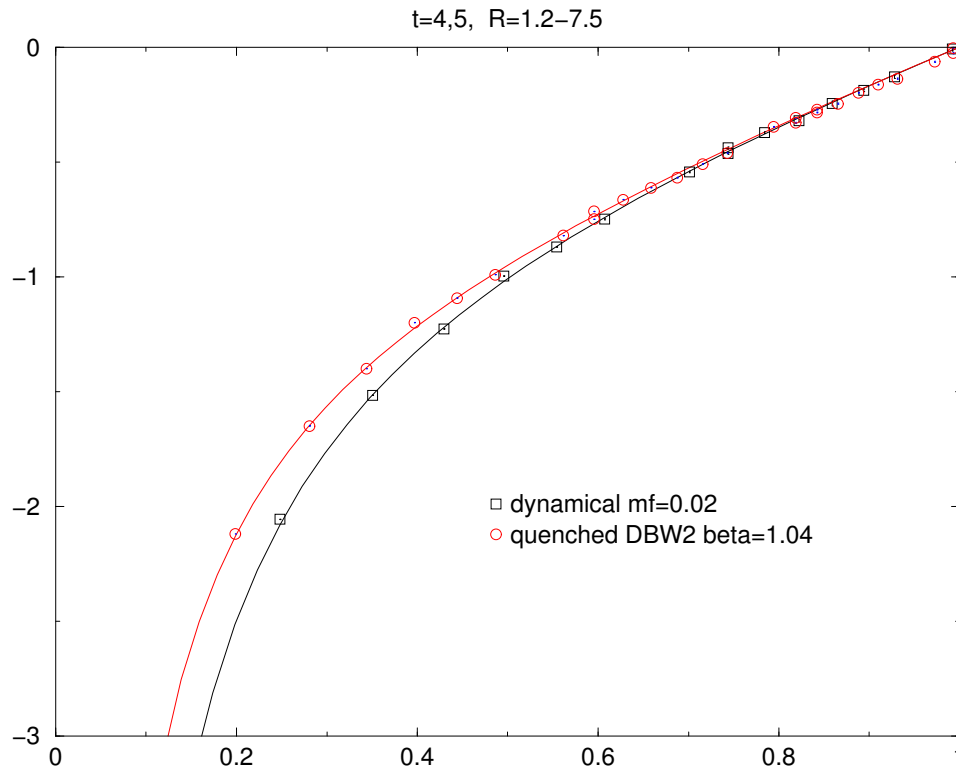
$$a_0 \rightarrow \eta'(\text{quenched}) + \pi \rightarrow a_0$$

As $\eta'(\text{quenched})$ failed to get heavy having **double pole**, this contribution was argued to make a_0 propagator to be **negative** using QChPT in finite volume.

- quenched theory is not unitary nor local field theory.

Introduction...

Dynamical simulation is **difficult** not only it's computationally demanding, but also gauge field tend to be **more fractuating** at short distance than quenched simulation (fixing scale at Hadronic scale).
 \iff milder running $\alpha_S(\mu)$ for $N_F > 0$ (asymptotic freedom).



$r_0 \times [V(r) - V(r_0)]$ vs r/r_0 for
DFW $N_F = 0, 2$.

This makes dynamical simulation even more difficult.

Dynamical simulation for DWF (GW)

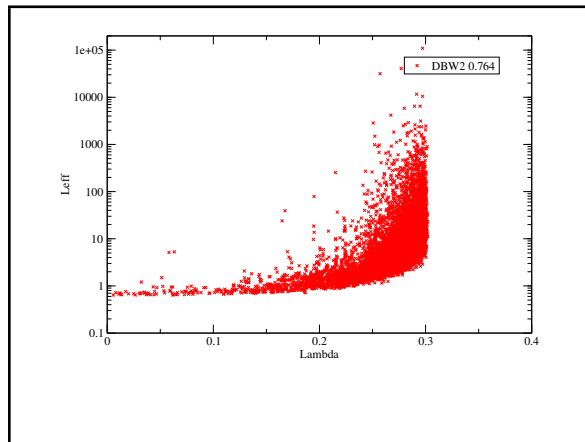
- Such **short distance fluctuation** has an impact on DWF (GW) fermion simulations.
- Axial Ward-Takahashi identity,

$$\begin{aligned} \partial_\mu \mathcal{A}_\mu^a(x) &= 2m_f J_5^a(x) + 2J_{5q}^a(x) \\ &\approx 2 (m_f + m_{res}) J_5^a(x) \end{aligned}$$

$J_5^a(x)$: non-singlet pseudoscalar,
 $J_{5q}^a(x)$: explicit breaking term
 consists of field at $s = L_s/2 - 1, L_s/2$.

A measure of the residual chiral symmetry breaking,

$$m_{res} = \frac{\sum_{x,y} \langle J_{5q}^a(y,t) J_5^a(x,0) \rangle}{\sum_{x,y} \langle J_5^a(y,t) J_5^a(x,0) \rangle} \sim e^{-\lambda L_s}, \lambda \sim H_W = \gamma_5 D_W$$



- Sudden growth of eigenmode size at $\lambda = \lambda_c$.
 Roughly consistent with $m_{res}(L_s)$ behaviors.
 DWF $N_F = 3$ (P.Boyle, N.Christ @ chiral)
 Mobility edge (M.Golterman, Y.Shamir)
 (S.Aoki, Y.Taniguchi)

m_{res} in dynamical simulations

- In practice $L_s \lesssim$ a few 10 is preferable. At the same time am_{res} must be small, less than a few MeV, to realize the advantages of DWF.
- In $N_F = 0$ DWF QCD (RBC) tuning RG action, the negative coefficients to the rectangular plaquette, suppresses small dislocations drastically, but the parity broken phase, still exists for small enough β (S. Aoki) .
- In $N_F = 2$, $m_{res} \sim \mathcal{O}(1)$ MeV, for $L_s = 12$, $a^{-1} = 1.7$ GeV using DBW2 gauge action.
- In $N_F = 3$, $L_s = 8$, an order of magnitude larger m_{res} than $N_F = 2$, $L_s = 12$. Tuning of rectangular actions (R.Mawhiney @ spectrum 11) .
- Fructuations at short distance might cause bad things (taste breaking, exceptional configuration) for other fermions as well.

Sincere apologies

I apologise sincerely to those whom I won't cite. There are also many interesting and important works and talks I should have covered, but it was totally beyond my capability.

Let me try to understand and mention in the proceedings.

If you could drop an email to taku@bnl.gov to call my attention , I will highly appreciate that.

1. Performance

Simulation for Dynamical Fermion

It is important to improve the performance of dynamical simulation to **reduce the statistical error** on physical output.

$$\text{statistical error} \propto \sqrt{\frac{1}{N_{conf}}}$$

Hybrid Monte Carlo (Exact algorithm)

$$\text{Prob}(U_\mu(x)) \propto e^{-S(U_\mu)}[dU_\mu] \implies e^{-\mathcal{H}}[dU_\mu][d\Pi_\mu], \quad \mathcal{H} = \frac{1}{2}\Pi^2 + S(U)$$

Conjugate momentum $\Pi_\mu(x)$

- 1. **Refresh** momentum Π , (Φ, Φ_{PV}) .
- 2. Approximately solve Hamilton's equation (\mathcal{H} preserved, reversible, area-preserving) : **1 trajectory**.
- 3. correct the approximation by a Metropolis reject/accept test : **acceptance**.

Factors of Simulation Performance

Three factors that have impact on the performance of dynamical simulations

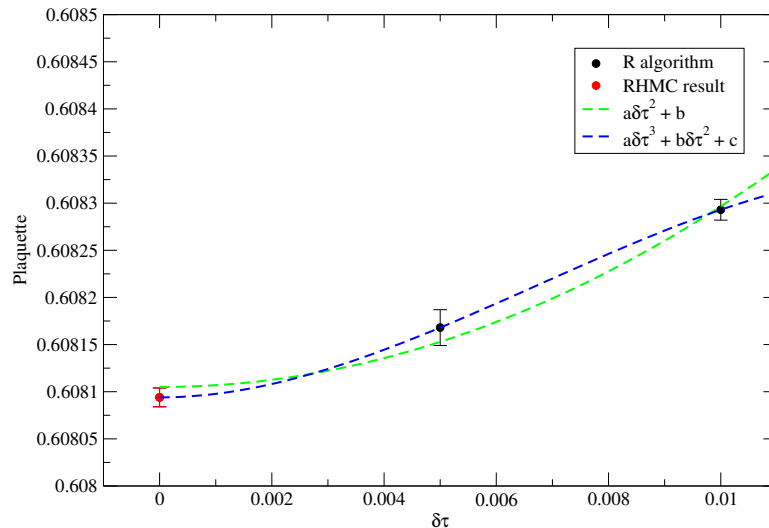
1. **Speed of integrator** for Hamilton's equation (many Matrix inversions)
2. **Acceptance**
3. **Autocorrelation**, τ_{int} , between consecutive trajectories

In this conference:

- Schwartz-preconditioned HMC (domain decomposition) (M.Lüscher @ plenary)
- mass preconditioning (Hasenbusch trick) & multiple time scale integration (M.Hasenbusch, C.Urbach @ algorithm 2)
- Twisted mass (A.Shindler @ plenary, and therein)
- Ginsparg-Wilson fermions (M.Clark, P.Hasenfratz @ algorithm 2)
(B.Joo, R. Edwards @ chiral 4) (A.Borici, S.Krieg @ algorithm poster)

DWF (R)HMC experiences

- started by Columbia Univ. (G. Fleming, P. Vranas, *et.al.*)
- Force term modification (P.Vranas, C.Dawson)
- Chronological inverter (Brower, Ivanenko, Levi, Orginos)
- Acceptance, autocorrelation, τ_{int} , was insensitive to quark mass.
- On $N_f = 2 + 1$ DWF RHMC (double precision), acceptance is almost flat in light quark mass with multiple gauge steps (~ 4 per a fermion step) in the integrator.



- $N_F = 3$ DWF, Plaquette
- R algorithm (inexact) vs RHMC (exact) (M.Clark @ algorithm 2)
 \implies Use exact algorithm, unless the performance is away worse.

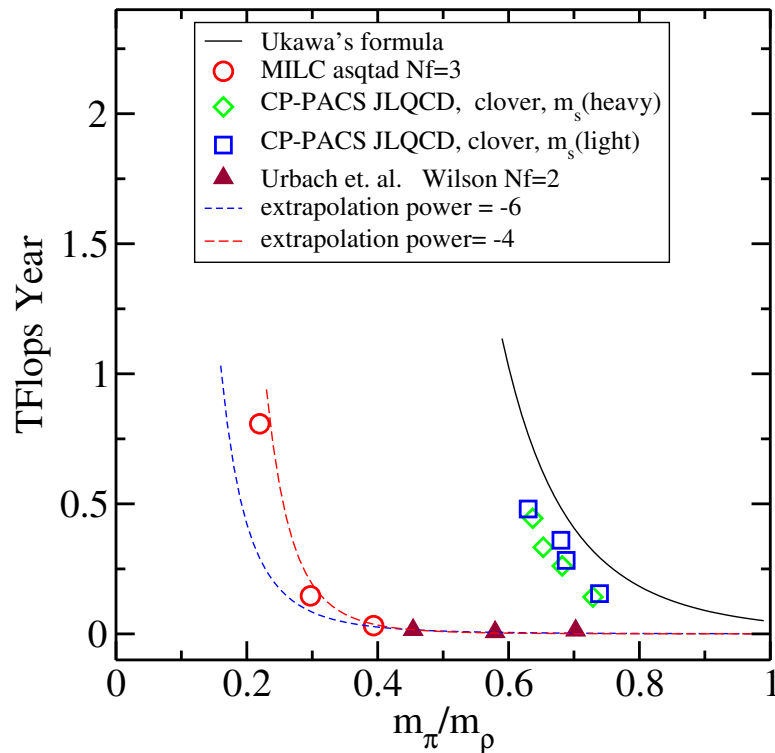
Cost estimation

- Panel discussion @ Berlin Lattice conference.
- Assuming $((m_\pi/m_\rho)^2 \propto m_q)$
 - $\tau_{int} \propto 1/m_q$,
 - (inversion cost) $\propto 1/m_q$,
 - $\Delta t \propto m_q$
- TFLOPS \times Year to generate 1,000 independent configuration
(Thanks to K.Jansen, A.Ukawa, T.Yoshie) :

$$\text{TflopsY} = C \left[\frac{\#conf}{1,000} \right] \left[\frac{(m_\pi/m_\rho)}{0.6} + S \right]^{-6} \left[\frac{L}{2.12\text{fm}} \right]^5 \left[\frac{a^{-1}}{2.60\text{GeV}} \right]^7$$

S : cost for strange quark. $C \approx 0.312$ (Ukawa) .

1k configs, $a=0.08$ fm, $\text{Vol}=24^3 40$



- MILC points are from Urbach *et.al*
- Scaled for $a = 0.08\text{fm}$, $24^3 \times 40$ lattice using the formula.
- CP-PACS JLQCD $N_F = 3$ clover on Earth simulator $\tau_{int} = 0.6/(am_q)$
- Urbach *et.al* multiple time scale $\tau_{int}(\text{plaq})$ (Sexton, Weingarten) + Hasenbusch accel.
- Note the formula's assumptions are not totally confirmed: try lighter quark mass to see if $\tau_{int}, C_{\Delta H}$ change.
- fixing a^{-1} has no absolute meaning.
c.f. $\mathcal{O}(a)$ vs $\mathcal{O}(a^2)$ discretization.

2. Dynamical Simulations

Dynamical simulations

Dynamical Wilson fermions (M.Lüscher @ plenary)

Dynamical twisted mass Wilson (R. Frezzotti and G.C. Rossi) (A.Shindler @ plenary)

parameters of dynamical simulations

- Gauge action
- Fermion action
- N_F : number of dynamical light quarks
- a^{-1}
- m_{sea} (m_π/m_ρ)
- renormalization

MILC collaboration

RG(Symanzik) + Improved staggered(Asqtad) $N_F = 2 + 1$
R algorithm

300-700 configurations, $(2.4 \text{ fm})^3$

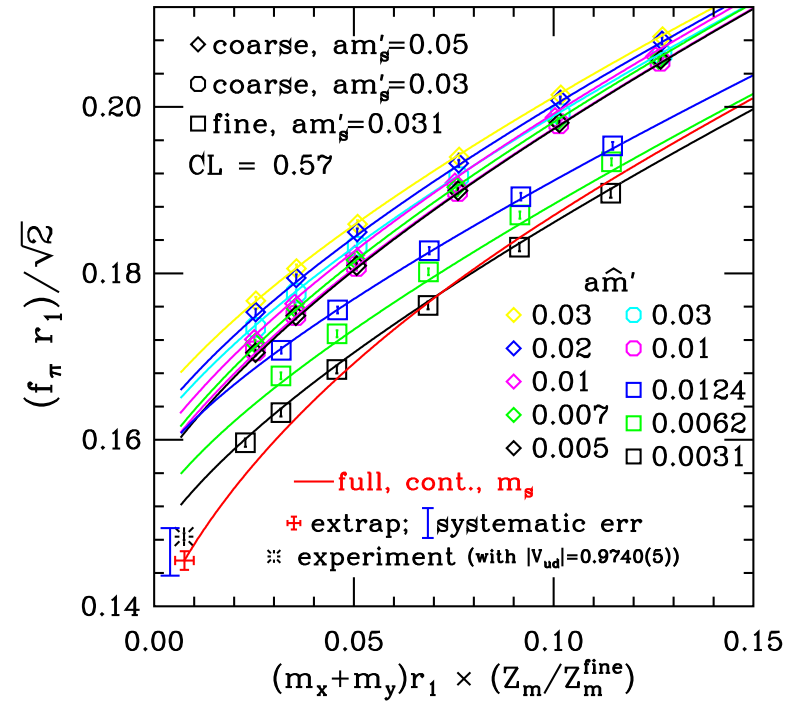
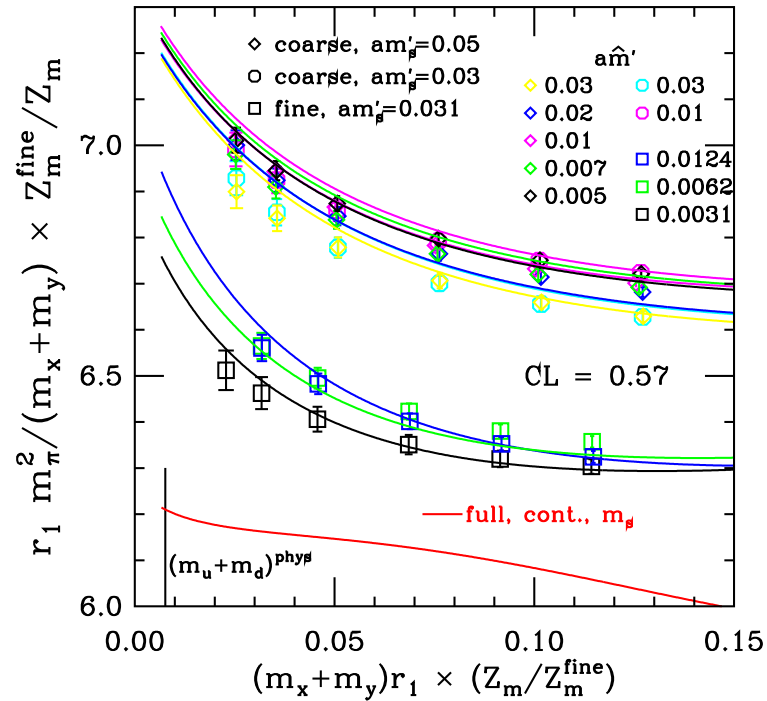
($(2.9\text{fm})^3$ / $(3.4\text{fm})^3$ for coarse/fine lightest mass)

- coarse lattice $a^{-1} = 1.6 \text{ GeV}$ $m_q = 10 - 50 \text{ MeV}$ 5 points ($m_\pi/m_\rho = 0.3 - 0.6$)
Added second (40% lighter) strange quark points (2 mass points).
- fine lattice $a^{-1} = 2.3 \text{ GeV}$ $m_q = 10 - 30 \text{ MeV}$ 3 points ($m_\pi/m_\rho = 0.3 - 0.5$)
Added lightest quark mass points (now 170 confs)
- Also quenched simulations on similar lattice spacing/size to study quenched staggered chiral perturbation theory.

perturbative renormalization

(C.Bernard @ spectrum poster, also thanks to D. Toussaint)

MILC collaboration



- The new lightest quark points doesn't change mass results much, so doesn't quark masses.
- Including new data, decay constants shifted by a significant amount.
- new (preliminary) results of decay constants.

MILC collaboration

- only analytic terms in NNLO, (and NNNLO) are included in the fits.
- R algorithm.
- electromagnetic effects is the biggest error on m_u/m_d .
(N.Yamada, Y.Namekawa @ spectrum 10)
- Quenched f_π is larger than experimental value by 28%.
(Setting scale by r_0, σ , 21%, 14% larger respectively).
consistent with other quenched simulations ?

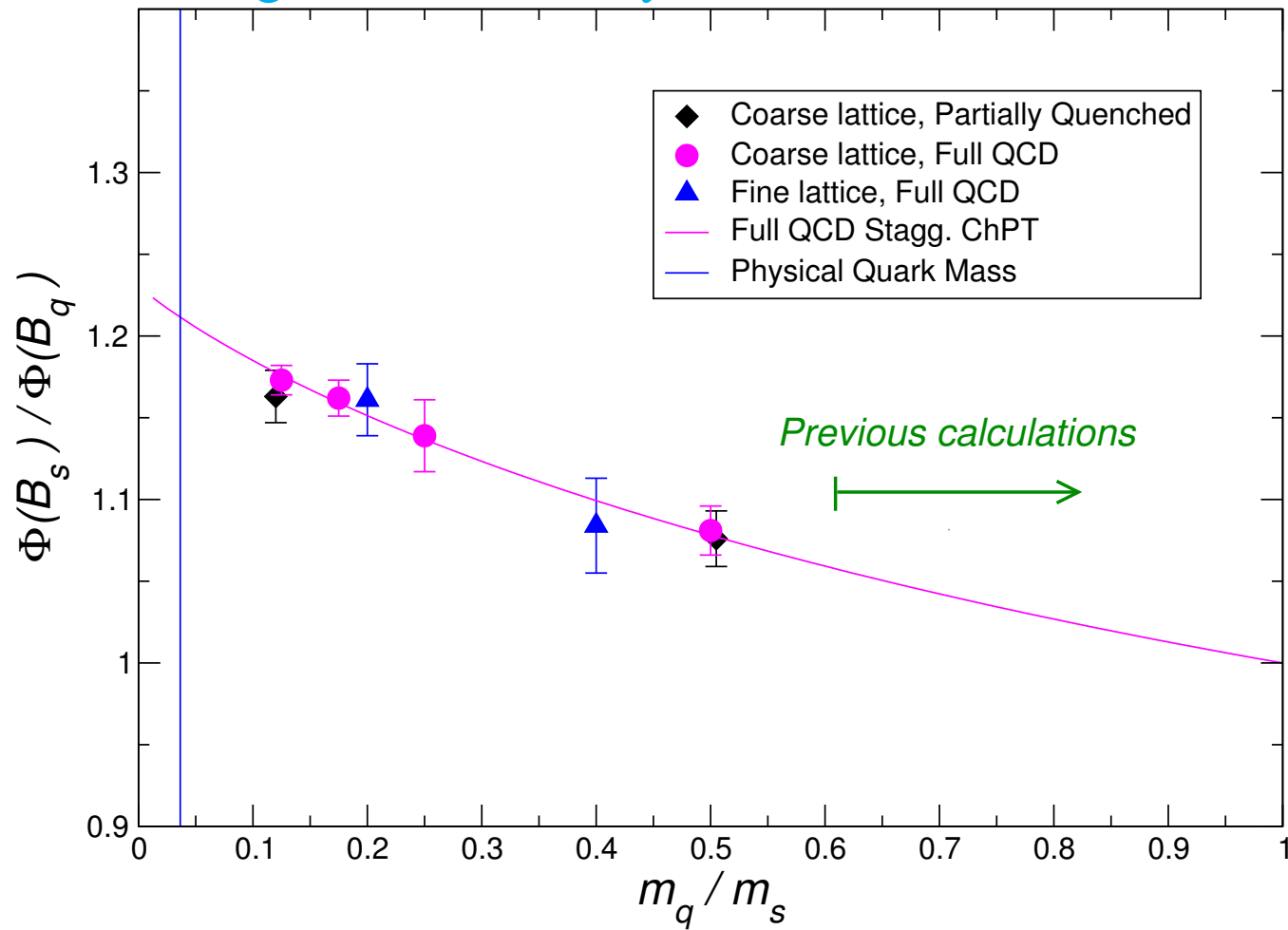
(A.Mason @ plenary)

(C. McNeile @ spectrum 1)

(J. Bailey @ spectrum 2)

(C.Aubin @ spectrum 10)

(J.Shigemitsu @ heavy 2)

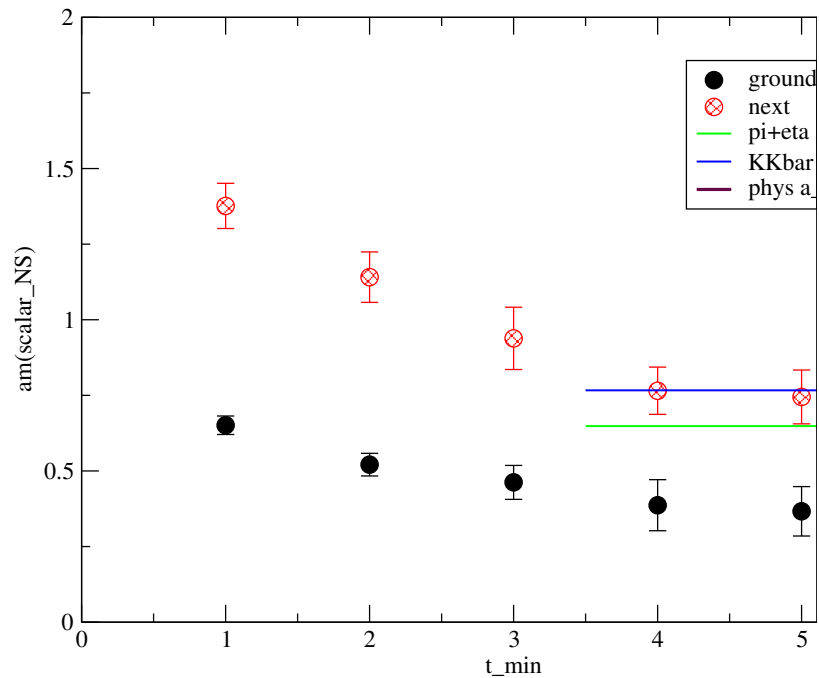


- B_q : b (NRQCD) + light quark (staggered) meson

- $\Phi_q = f_{B_q} \sqrt{M_{B_q}}$

staggered a_0 on staggered sea

(A.Irving @ spectrum poster)



- flavour nonsinglet scalar
- Spin \times Taste = $1 \otimes 1$
- **excited state** is compatible to $a_0(980)$
- ground state : **partially quenched effect** ?
- (S.Dürr @ plenary)

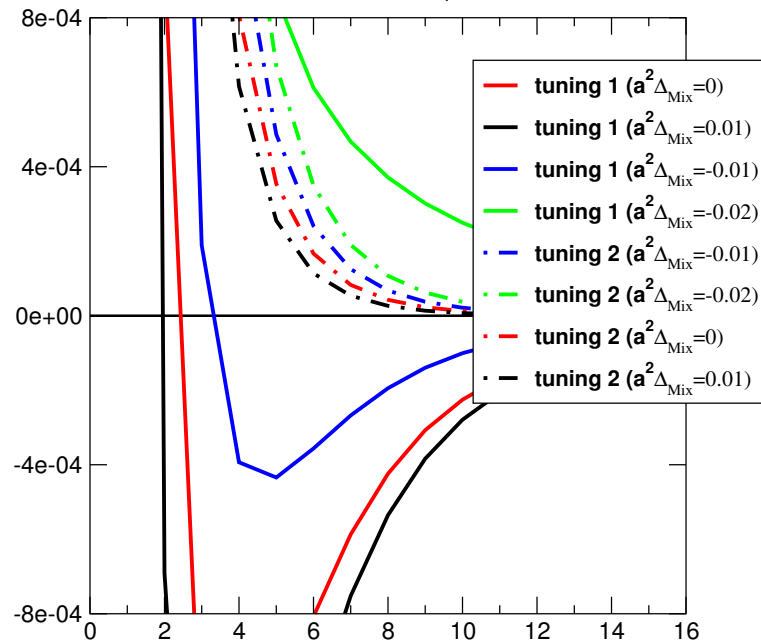
$$C(t) = \sum_n A^{(n)} e^{-m^{(n)}t} + A_*^{(n)} e^{+m_*^{(n)}t}$$

m_* : $\gamma_4 \gamma_5 \otimes \gamma_4 \gamma_5$ (taste-split pion)

DWF a_0 on staggered sea

(S.Prelovsek @ spectrum 8)

tuning 1: $M_\pi^{\text{DWF}} = M_{\pi_5}$ (LHPC); tuning 2: $M_\pi^{\text{DWF}} = M_{\pi I}$
 a_0 correlator for $m_q = 0.01/0.05$



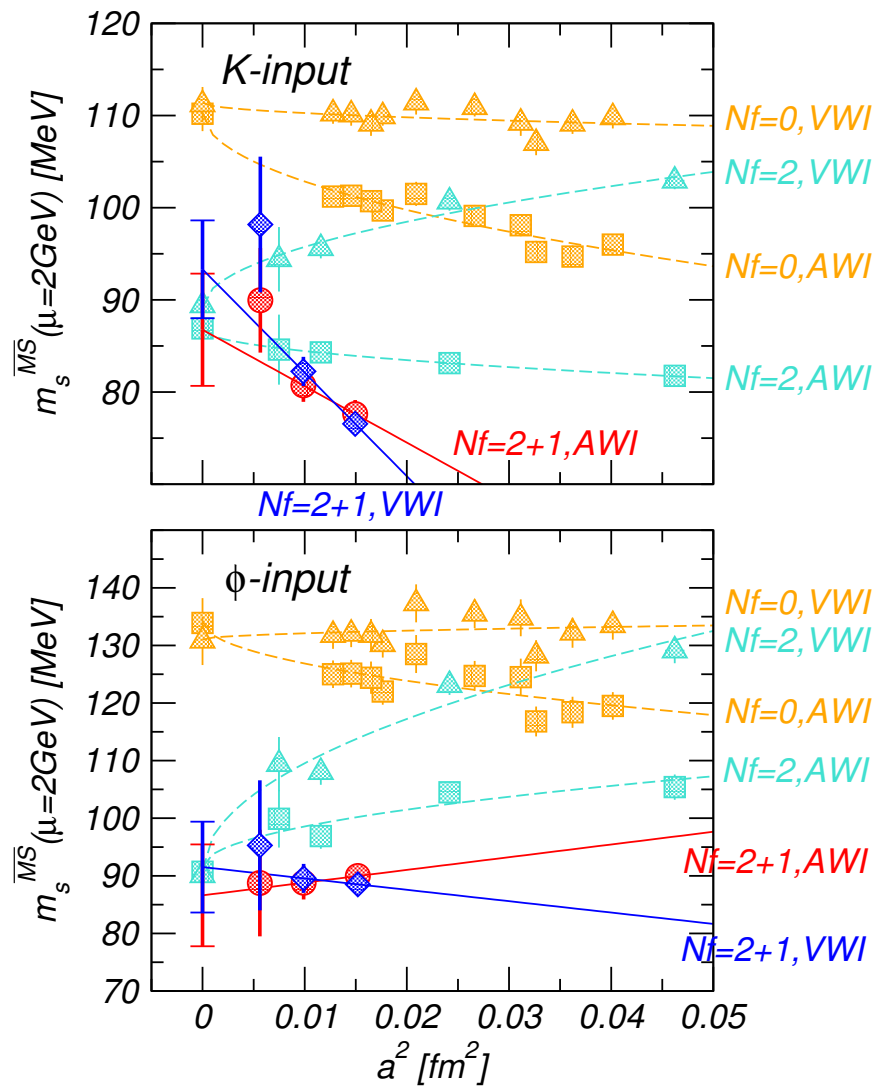
- Partially quenched ChPT.
- For DWF-valence + DWF-sea, $C(t) < 0$ for $m_\pi(\text{sea}) > m_\pi(\text{val})$, vice versa. (S. Prelovsek, *et.al.*)
- DWF a_0 could be a “detector” of $m_\pi(\text{sea})$.

- For GW-valence pion on staggered sea: $m_\pi(\text{val}) = m_{\pi, I}(\text{sea})$ leads “continuum like” NLO pion mass formula (O.Baer *et.al.*) .
- For DWF-valence + staggered-sea mixed action , a_0 's $C(t)$ is still **negative** for $m_{\pi, 5}(\text{sea}) = m_\pi(\text{val})$ for a range of unknown parameter Δ_{mix} .
- DWF a_0 *feels* staggered $m_\pi(\text{sea})$ is **heavier** than NG pion $m_{\pi, 5}$?

CP-PACS and JLQCD collaboration

- RG improved gauge action + nonperturbatively $O(a)$ improved Wilson $N_F = 2 + 1$,
- 3k-10k trajectories, $(2\text{fm})^3$ box.
- $a^{-1} = 1.6, 2.0, 2.6$ GeV ($m_\pi/m_\rho = 0.60 - 0.78$)
- measure on dynamical (unitary, $m_{sea} = m_{val}$) points.
- **Exact** $N_F=2+1$, PHMC (K.Ishikawa)
- perturbative renormalization
- AWI quark masses
$$m_q^{AWI} = \frac{\Delta_4 A_4(t) J_5(0)}{2(J_5(t) J_5(0))}$$
- $r_0 = 0.5\text{fm}$ is consistent with m_ρ input.

(T. Ishikawa's talk @ spectrum 3, S.Takeda @ spectrum 2)



- polynomial chiral extrapolations.
- perturbative renormalization.

$N_F = 2$ Dynamical Wilson simulations

- ALPHA

Wilson + nonperturbatively $O(a)$ improved Wilson $N_F = 2$,
 $a^{-1} = 2.1, 2.4, 2.8$ GeV ($m_{ps} = 495$ MeV)

nonperturbative renormalization (Schrodinger functional)

(M.D.Morte's talk @ improvement 1, also thanks to F.Knechtli)

- QCDSF-UKQCD

Wilson + nonperturbatively $O(a)$ improved Wilson $N_F = 2$,
 $a^{-1} = 2.1, 2.4, 2.8$ GeV ($m_\pi/m_\rho \geq 0.6$)

NLO

nonperturbative renormalization (RI-MOM)

(R.Horsley's talk @ spectrum 8, D.Pleiter @ spectrum 4,
also thanks to G.Schierholz)

- SPQcdR

Wilson + Wilson $N_F = 2$,

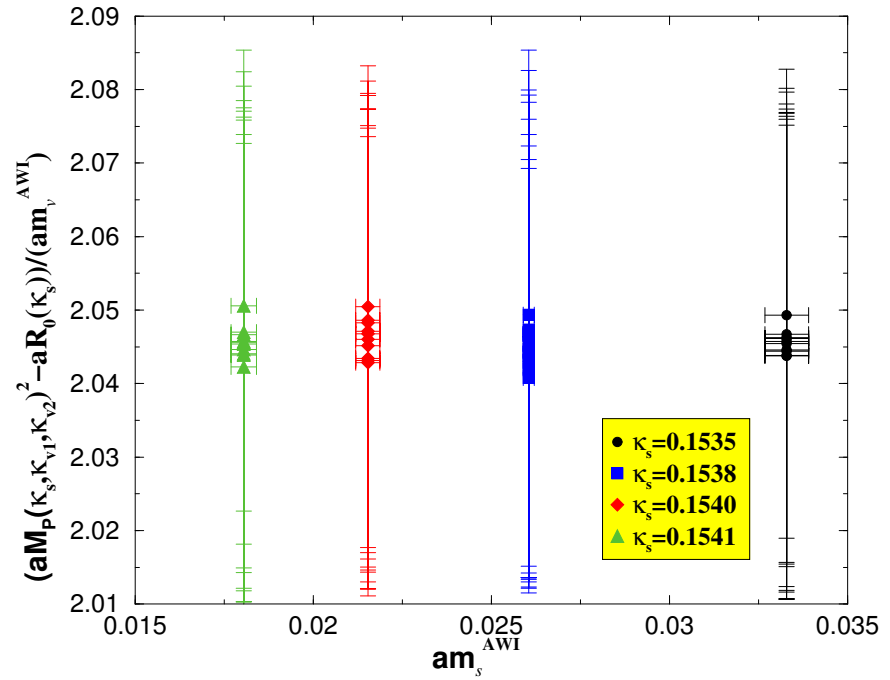
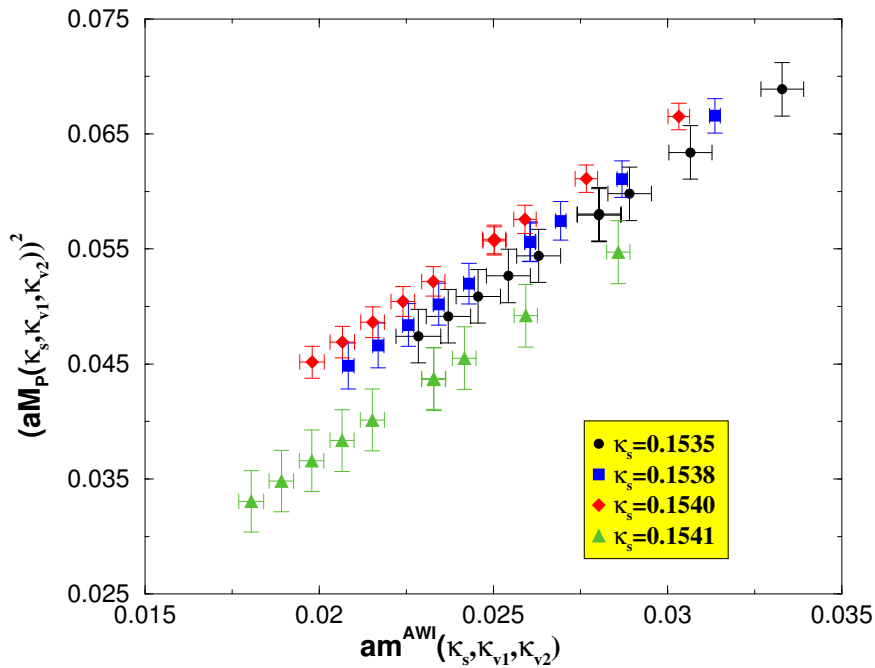
$a^{-1} = 3.2$ GeV ($m_\pi/m_\rho = 0.63 - 0.75$)

nonperturbative renormalization (RI-MOM)

Also quenched simulation at similar at a

(C.Tarantino's talk @ spectrum 8)

SPQcdR



- $m_\pi/m_\rho \geq 0.6$, too heavy ?
- Chiral extrapolation, Wilson ChPTs (O.Baer *et.al.*, S. Aoki) .
- Perturbative renormalizations tend to underestimate quark mass ?

RBC collaboration

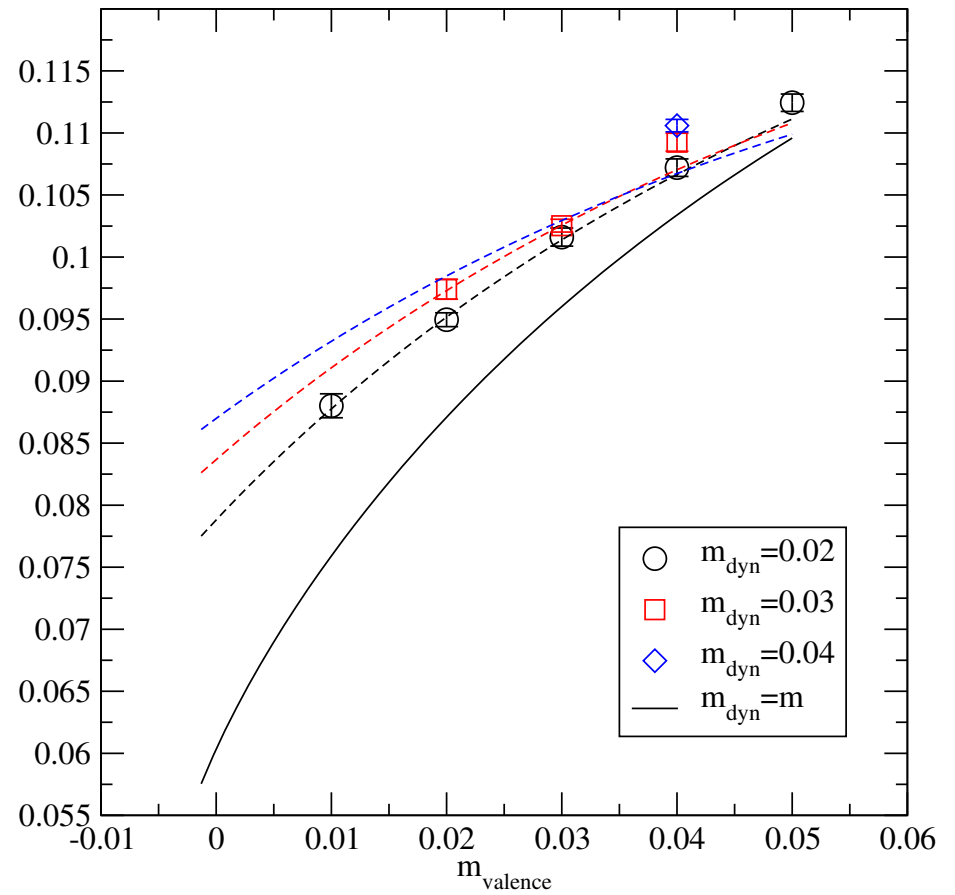
- RG(DBW2) + Domain Wall Fermions, $N_F = 2$
- $a^{-1} = 1.7$ GeV ($m_\pi/m_\rho = 0.54 - 0.65$), $(1.9\text{fm})^3$ box.
6k trajectories, 94 confs.
- non-perturbative renormalization (RI-MOM)
- LO and **NLO** ChPT fit.
- $L_s = 12$, $m_{res} = 0.001372(44) \sim 0.1m_{sea} \sim$ a few MeV.

(C.Dawson, T.Blum's talk @ plenary)

(S.Prelovsek @ spectrum 8, N.Yamada @ spectrum 10)

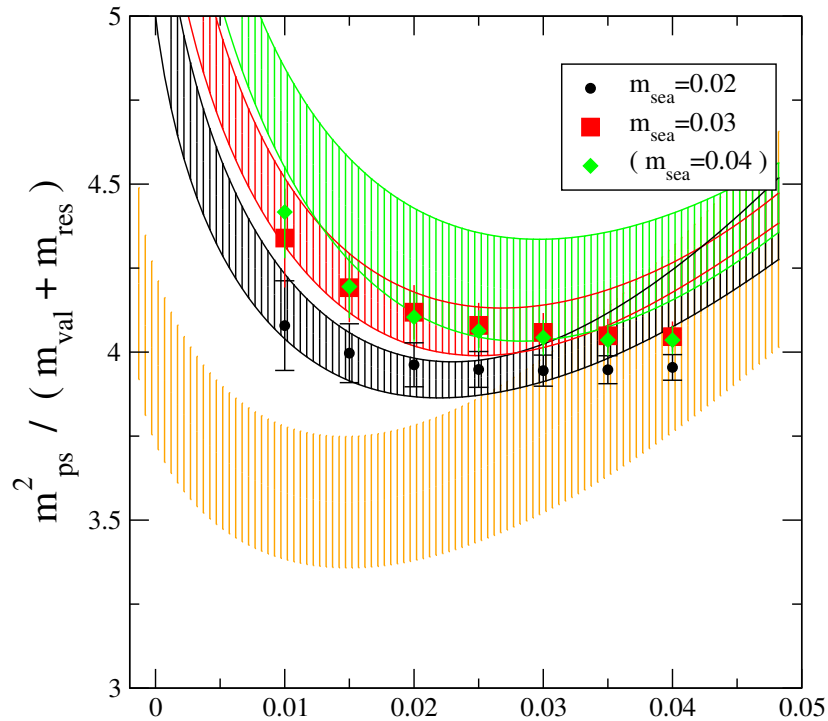
Pseudoscalar decay constant

- NLO fits are also examined.
- $m_{val}, m_{sea} \in [0.01, 0.03]$
- **30% smaller f** than linear fit.
- Larger mass points are missed badly.
 \implies LO (linear) extrapolation.



Pseudoscalar Meson mass

Fit using $m_{\text{sea, val}} \leq 0.03$ only



- NLO fit using $m_{\text{sea, val}} \leq 0.03$ not inconsistent.
- constraints:
 - $m_{ps}^2 = 0$ at $m_{val, sea} = -m_{res}$,
 - $f = 0.0781$ from linear fit of f_{ps} .

Physical Results

- NLO fits results using m_{ps}^2 at $m_f = m_{sea, val} \leq m_f^{(max)}$. Pseudo-scalar wall-point (upper two column), and axial-vector wall point. uncorrelated χ^2 . Gasser-Leutwyler low energy constants L_i multiplied by 10^4 at $\Lambda_\chi = 1$ GeV.

$m_f^{(max)}$	χ^2/dof	$2 B_0$	$L_4 - 2L_6$	$L_5 - 2L_8$
0.03	0.1(1)	4.0(3)	-1.5(7)	-2(1)
0.04	2(1)	4.2(1)	-0.2(4)	-1.1(4)
0.03	0.3(2)	4.0(3)	-1.9(8)	-1(1)
0.04	1.9(9)	4.2(1)	-0.4(4)	-0.8(3)

- By linear extrapolations/interpolations for f_{ps} to \bar{m} and m_s ,

	$N_F = 2$	experiment	$N_F = 0$
f_π	134(4)	130.7	129.0(50)
f_K	157(4)	160	149.7(36)
f_K/f_π	1.18(1)	1.224	1.118(25)

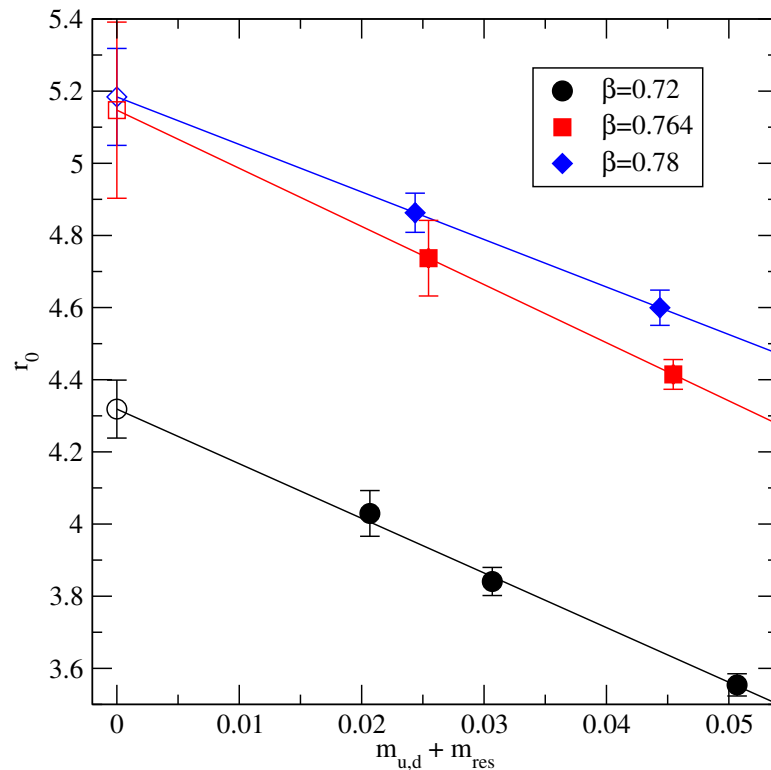
better agreement with experiment than quenched DWF simulations.

RBC and UKQCD collaboration

- RG(Iwasaki, DBW2) + Domain Wall Fermions $N_F = 2 + 1$
- $L_s = 8$ parameter search runs. $am_{res} \lesssim \mathcal{O}(0.01)$
- $a^{-1} = 1.7\text{-}2.0$ GeV
- QCDOC, QCDOC collaboration (T.Wettig @ plenary)
- R algorithm and Exact RHMC algorithm (M. Clark @ algorithm 2)
in CPS++ (maintainer: C.Jung)
- 1.5-6 k trajectories.
- $24^3 \times 64$, $L_s = 16$ started.

preliminary investigations

Nf=3, DWF (DBW2)



- Static quark potential (preliminary)

- $r_0 = 0.5$ fm

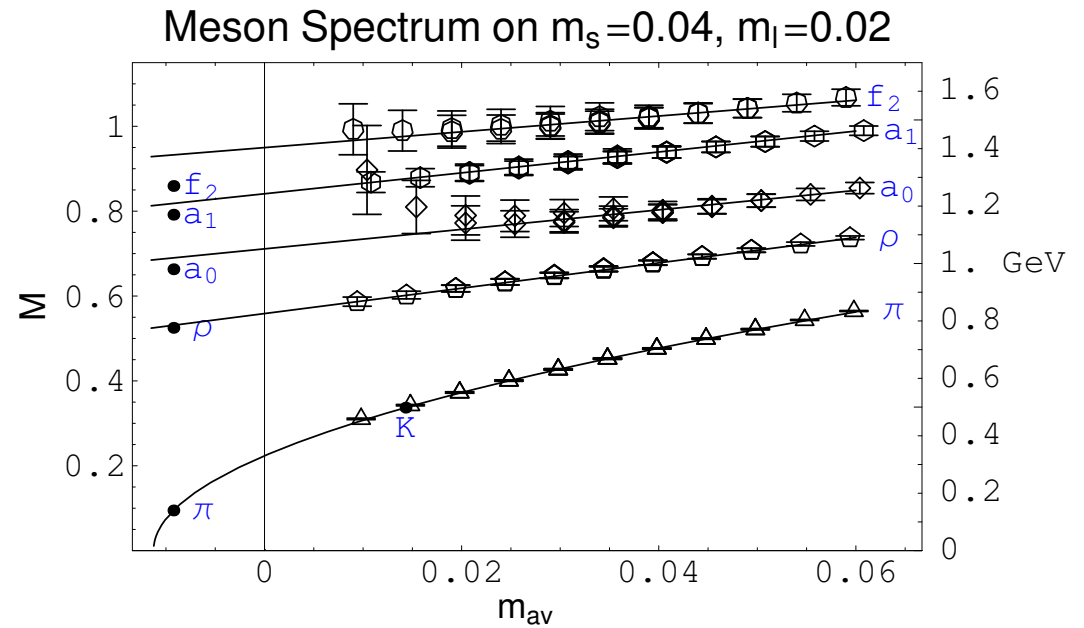
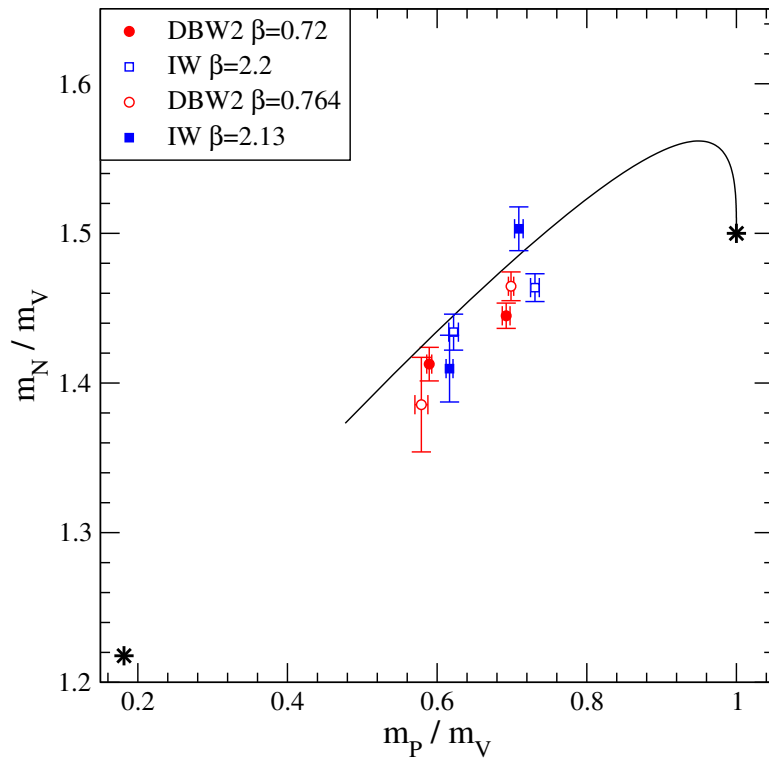
	β	a^{-1}
DBW2	0.764	2.0 GeV
	0.72	1.7 GeV
Iwasaki	2.2	2.1 GeV
	2.13	1.8 GeV

- consistent with f_π, m_ρ inputs within $\sim 10\%$.

Spectrum

(C. Maynard @ spectrum 11)

(S. Cohen @ weak 2)



roughly consistent with experimental value

(A.Yamaguchi @ chiral 2)

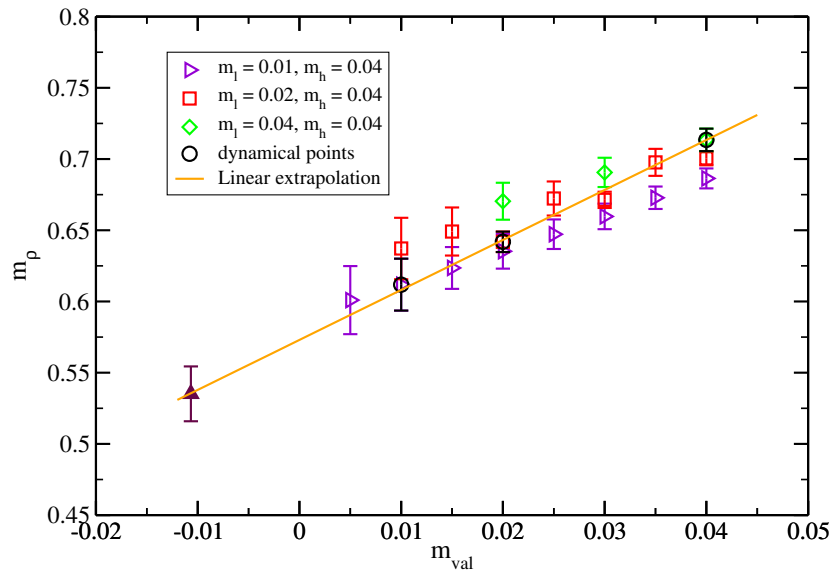
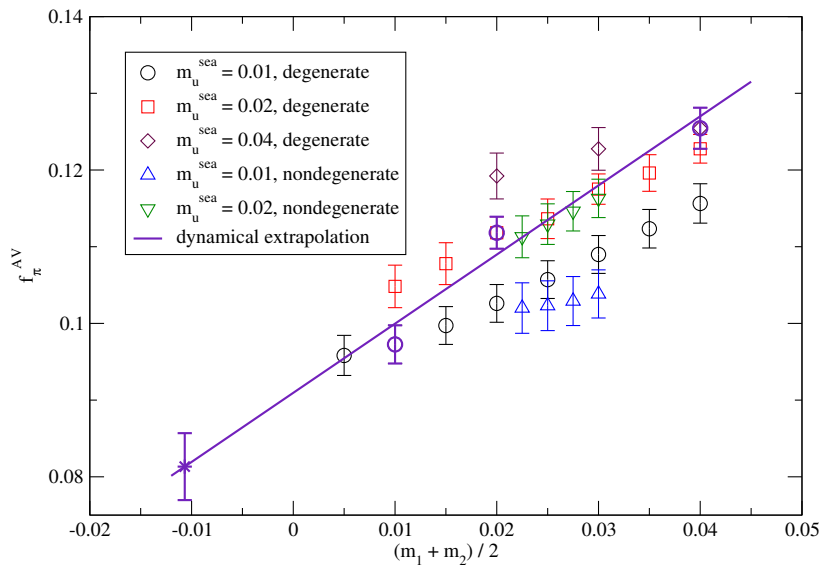
topology, $\langle \bar{q}q \rangle$, ...

(R. Tweedie @ spectrum 8)

Scaling study of decay constants,

(M.Lin @ spectrum 11)

evolution details, decay constants....



GW/overlap/chirally improved dynamical simulations

(P.Majumdar @ chiral 2) (W.Ortner @ chiral poster)

(D.Kadoh @ chiral 2) (Y.Kikukawa @ chiral poster)

(T.DeGrand @chiral 3) (S.Schaefer @chiral 3)

(B.Joo @chiral 4) (R.Edwards @chiral 4)

(W. Kamleh @ algorithms poster) (S. Kreig @ algorithms poster)

3. quark masses

Light quark masses

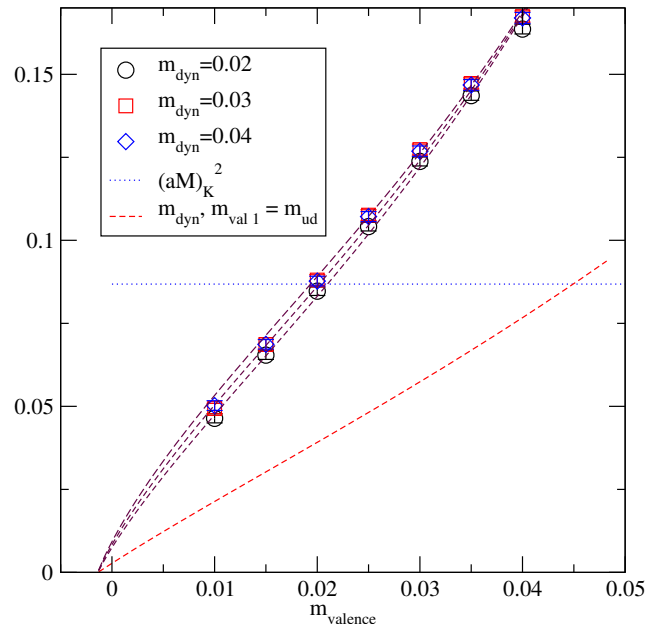
Quark mass is a **fundamental parameter** of the standard model Lagrangian, which is not directly accessible from experiments due to confinement.

- Lattice QCD : **map** between hadronic observables (hadron mass, decay constant) and quark mass,

$$M_{had}(m_q) = M_{had}^{(exp)}$$

- fix **lattice scale**, a^{-1} (Sommer scale r_0 , m_ρ , f_π)
- **Extrapolate to chiral** regime ($m_{u,d} \sim \mathcal{O}(1)$ MeV).
- mass renormalization, **Z factors**, for non-lattice community.
- Extrapolate to continuum ($a \rightarrow 0$).

map between hadronic mass and quark mass on lattice



- Set lattice scale, a^{-1} , from m_ρ , r_0 , or f_π .
- quark mass at physical Kaon mass (horizontal line)
- By using non-degenerate ChPT formula (red dots), $am_{strange} = 0.0446(29)$ is extracted.
- If one uses dynamical, $m_{sea} = m_{val}$, points instead of $N_F = 2$ sea quarks, one finds $am_{strange} = 0.04177(64)$, 7% smaller than partially quenched analysis.

Operator Renormalization on lattice

Lattice perturbative calculation (improved)

RI-MOM (Rome/Southampton)

Schrodinger Functional (ALPHA)

Real-space NPR (Giménez *et.al.*) (V.Porretti @ spectrum 8)

NPR(RI-MOM) on dynamical lattice

- measure quark propagator, $S_F(q)$, on Landau gauge fixed gauge configuration.
- calculate amputated green function of bilinear operators, $\Gamma = 1, \gamma_5, \gamma_5\gamma_\mu, \dots$

$$\Pi_\Gamma = \langle u(-p)[\bar{u}\Gamma d]\bar{d}(q) \rangle_{AMP}$$

$$\Lambda_\Gamma = \frac{\text{Tr}(\Gamma\Pi_\Gamma)}{\text{Tr}(\Gamma\Gamma)} \Big|_{p^2, q^2 = \mu^2}$$

on lattice ensemble.

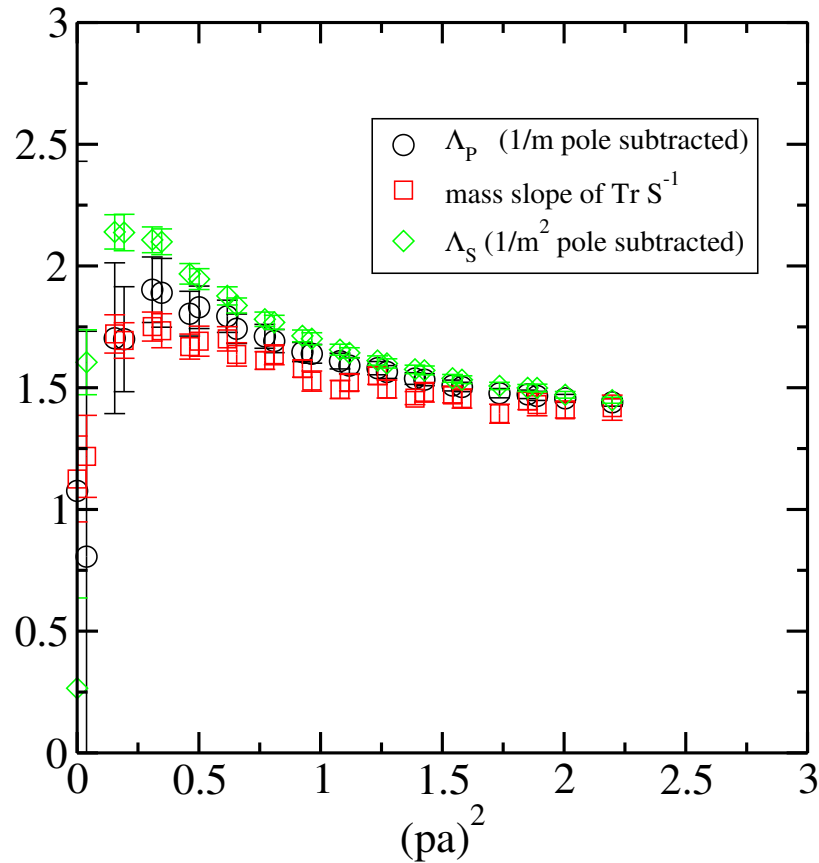
- Subtract mass pole to avoid non-perturbative effects ($\langle \bar{q}q \rangle$) by fitting

$$\Lambda_{\gamma_5} = \frac{c_1}{m_q} + \frac{Z_q}{Z_P} + c_3 m_q + \dots$$

at $\Lambda_{QCD} \ll |p| \ll a^{-1}$ on each sea quark ensemble.

(Z_q quark field normalization, $Z_{P,S,A}$ pseudoscalar, scalar, axial current)

NPR(RI-MOM) on dynamical lattice...

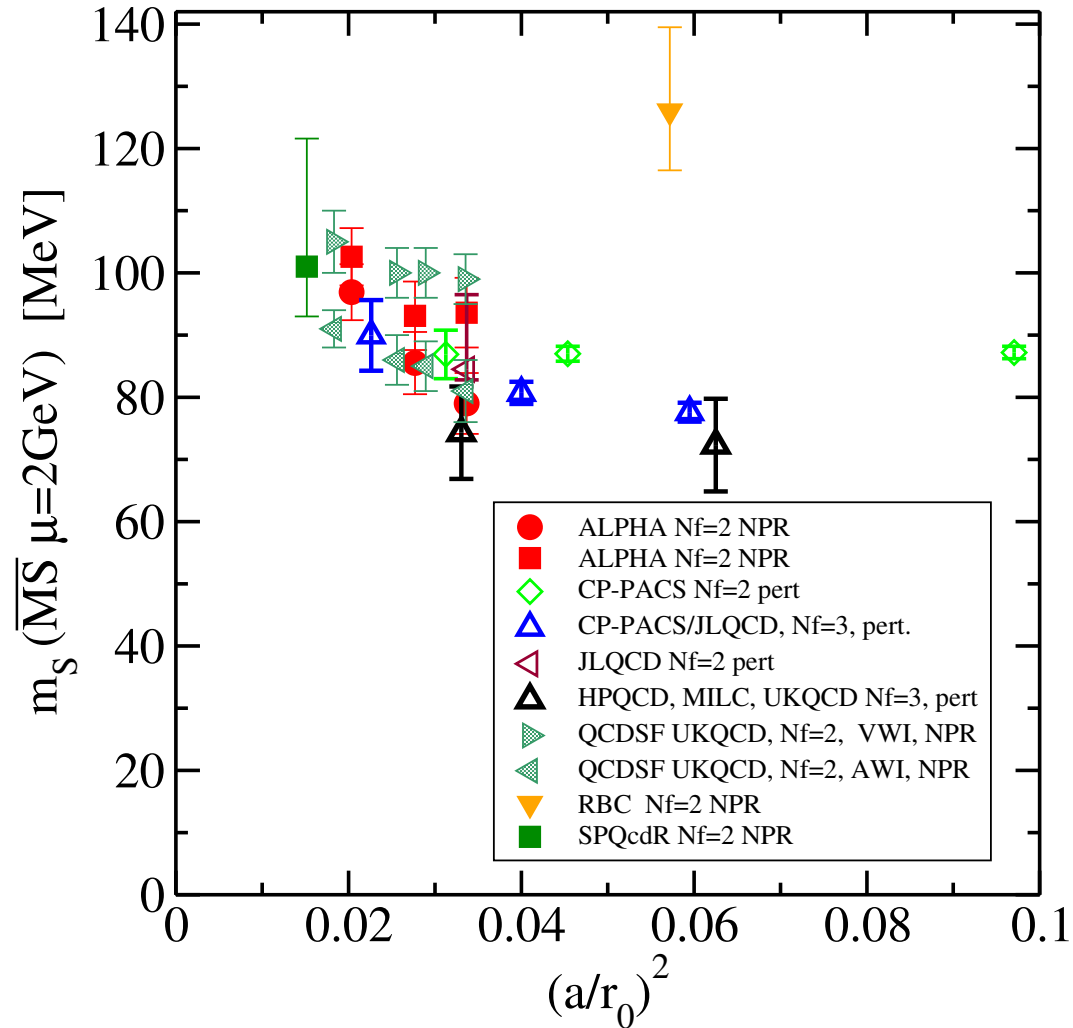


- $\Lambda_P \approx \Lambda_S \approx \partial S^{-1} / \partial m \rightarrow \frac{Z_q}{Z_P}$
- convert from RI to \overline{MS} with $c(pa)^2$ subtraction.
- constant fit all m_{sea} (**mild** dependency).
- $Z_P = 0.62(5)$
(250 MeV $\leq \Lambda_{QCD} \leq$ 300 MeV)

recent dynamical strange quark masses

based on ALPHA's compilation (Thanks to F.Knechtli) + new results.

Preliminary



$N_F = 2, 3$ (difference is small in CP-PACS/JLQCD)

scale is $r_0 = 0.5$ fm. (MILC corresponds to $r_0 = 0.467$ fm, not corrected)

chiral extrapolation ?

perturbation tends to give smaller Z_m ?

(QCDSF-UKQCD, SPQcdR, ALPHA)

DWF	NPR	Pert (K_c)
$Z_{V,A}$	0.7574(1)	0.770
$Z_{S,P}$	0.62(5)	0.847

Conclusions

- Performance of Dynamical simulation is updated.
- We need “fair” way to compare different simulations other than fixing a .
- Seemingly promising {new,improved} algorithms.
- Now three $N_F = 2 + 1$ dynamical simulations.
- New dynamical Wilson fermion simulations with NPR.
- New strange quark mass results (4 Wilson, 1 DWF).
- Systematic error (chiral extrapolation, perturbative Z)