Progress in Kaon Phenomenology from Lattice QCD

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[Lattice 2005]

Introduction

Plan:

1. $K \to l3$
    - Three new results
2. Twisted Boundary Conditions
3. B_K
    - Summary of quenched results
    - Dynamical results
    - Theoretical work.
4. $K \to \pi\pi$
    - Twisted mass
    - Two pion final states
    - ϵ-regime.

Kl3

K_{l3} Decay

$$K^0 \to \pi^- L^+ \nu_l \ ; \ K^+ \to \pi^0 L^+ \nu_l$$

where $l \in \{e, \mu\}$.

- $\Gamma_{K_{l3}} \propto |V_{us}|^2 \, |f_+(0)|^2$

$$\langle \pi(p_f) \, |\bar{s}\gamma_\mu u| \, K(p_i) \rangle = (p_i + p_f)_\mu f_+(q) + q_\mu f_-(q) \ ; \ q = p_i - p_f$$

- The most precise determination of $|V_{us}|$.

K_{l3} Decay

Calculating $f_+(0)$

- Ademollo-Gatto theorem: $f_+(0) = 1 - O\left((m_s - m_u)^2\right)$

Expand the form factors in Chiral Perturbation Theory:

$$f_+(q^2) = 1 + f_2 + f_4 + \cdots$$

with f_i of $O(M^i/f^i)$ in ChiPT

- f_2 :
  - f_2 depends on no new low energy constants. Can be worked out from M_K , M_π and f_π .
    -0.023 using values from experiment.

- f_4 :
  - Calculated in Chiral Perturbation Theory by Bijnens and Talavera. In *principle*, can be constrained by the experimentally measured slope of $f_0(q^2)$, but needs better experimental resolution.
  - $-0.016(8)$ from quark model [Leutwyler and Roos, 1984]

Calculating $f_+(0)$...

Need the error on $f_+(0)$ to be $< 1\%$ to be interesting.

The calculation of f_2 from ChiPT/Experiment means that theoretical approaches usually concentrate on calculating

$$\Delta f = f - 1 - f_2$$

$f_+(0)$ from the Lattice

First calculation: [Becirevic et al, hep-lat/0403217].

- This was a quenched calculation at a single volume and lattice spacing
- result:
  - $f_+ = 0.960 \pm 0.005 \pm 0.007$
- to be compared with the currently used [Leutwyler and Roos] number of
  - $f_+ = 0.961(8)$
- I'll cover three new dynamical numbers; all applying the same approach.

$f_+(0)$ from the Lattice...

The double ratio method

Rather than work with the three-point function of interest directly, the double ratio is used. (c.f. [Hashimito *et al*, 2000]).

$$\frac{\langle\pi\,|\bar{s}\gamma_0 u|\,K\rangle\langle K\,|\bar{u}\gamma_0 s|\,\pi\rangle}{\langle\pi\,|\bar{u}\gamma_0 u|\,\pi\rangle\langle K\,|\bar{s}\gamma_0 s|\,K\rangle} = \left[f_0\left(q^2_{max}\right)\right]^2 \frac{(M_K+M_\pi)^2}{4M_K M_\pi}\;;\;\; q^2_{max} = (M_K - M_\pi)^2$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

Scalar form factor:

$$f_0(0, M_\pi, M_K) = f_+(0, M_\pi, M_K)$$

This approach has several advantages:

- Small statistical error ($< 0.1\%$)
- Exactly unity, and exactly $f_+(0)$, on the lattice in $SU(3)$ limit.
  - 1% error not as hard as it sounds

The double ratio method..

This just gives $f_0(q^2, M_\pi, M_K)$. Need to

1. Extrapolate, in q^2 , to $f_0(0) = f_+(0)$ at a fixed (non-physical) mass.

2. Extrapolate to physical masses.

For 1) lattice data with explicit insertion of momenta is needed.

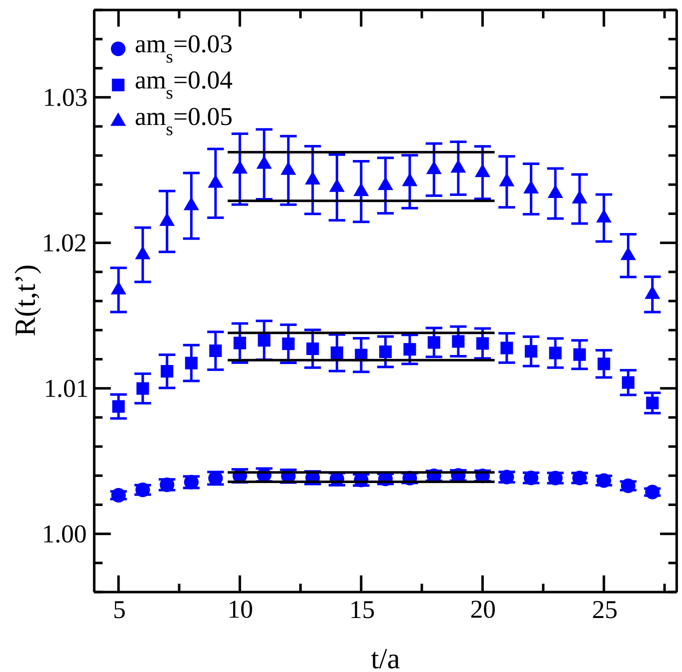
- [Becirevic et al, hep-lat/0403217] show how to use various ratios to allow an extraction with a small enough error-bar to be useful.
  - requires some anzatz to fit to:
    * $f_0(q^2) = f_0(0)/(1 - \lambda^{(pol)} q^2)$
    * $f_0(q^2) = f_0(0)(1 + \lambda^{(1)} q^2)$
    * $f_0(q^2) = f_0(0)(1 + \lambda^{(2)} q^2 + c q^4)$

For 2), it's also usual to use an anzatz for the extrapolation (more later)

RBC

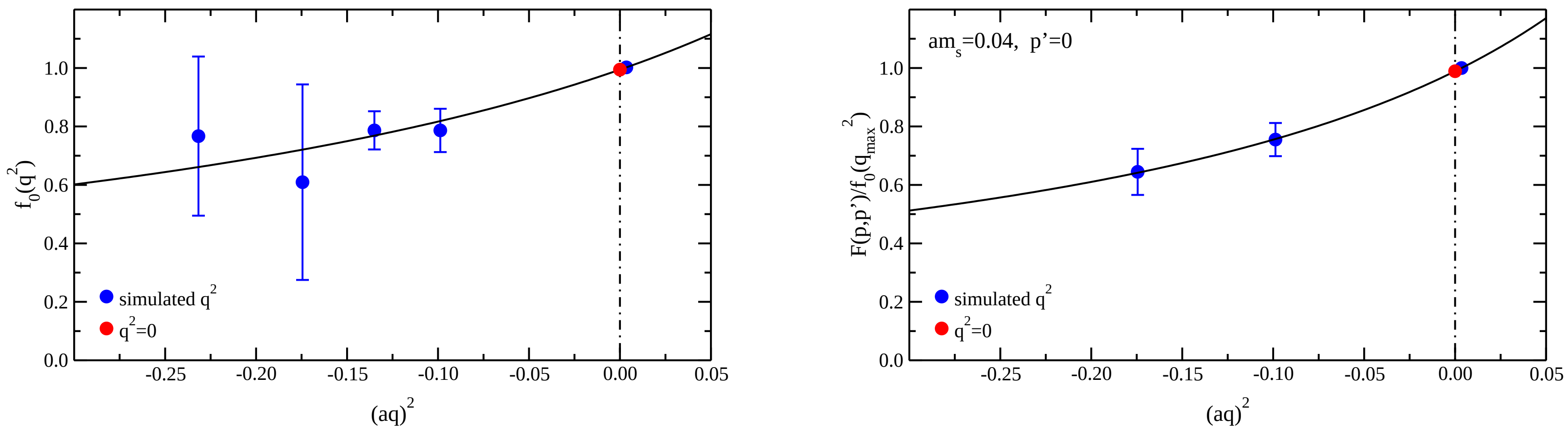
See poster by Takashi Keneko

- Uses RBC $N_F = 2$ domain wall fermions lattices
  - single lattice spacing: $a^{-1} \sim$ 1.7GeV
  - small volume : $16^3 \times 32$.
  - sea quark mass = 0.02 ($\sim m_s/2$).
  - 94 configurations.



- This is the double ratio, with no momentum insertion, versus valence quark mass.

RBC...

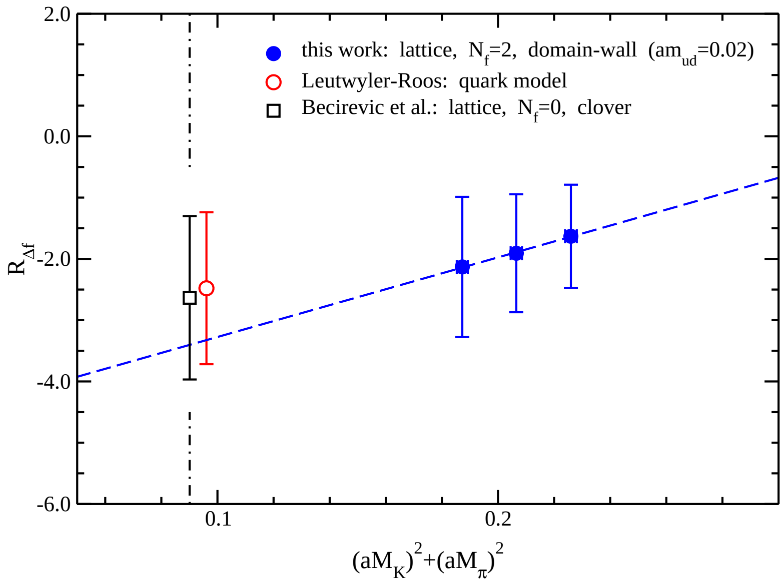


- Comparison of the two different approaches to fitting the q^2 .
  - left: standard ; right CP-PACS (see talk by Naoto Tsutsui)

- Note: a large error in the slope can be tolerated because of the point with no momentum injection, which is very close to $q^2 = 0$.

- But not the physical mass point!

RBC...



- The ratio

$$R = \frac{\Delta f}{M_K^2 - M_\pi^2}$$

  is assumed to have some smooth dependence on the meson masses
- Note: valence extrapolation

- $\Delta f = (A + B(M_K^2 + M_\pi^2))(M_K^2 - M_\pi^2)^2$: $f_+(0) = 0.955(12)$
- $\Delta f = (A + Bm_s)(M_K^2 - M_\pi^2)^2$: $f_+(0) = 0.966(6)$

CP-PACS

See talk of Naoto Tsutsui.

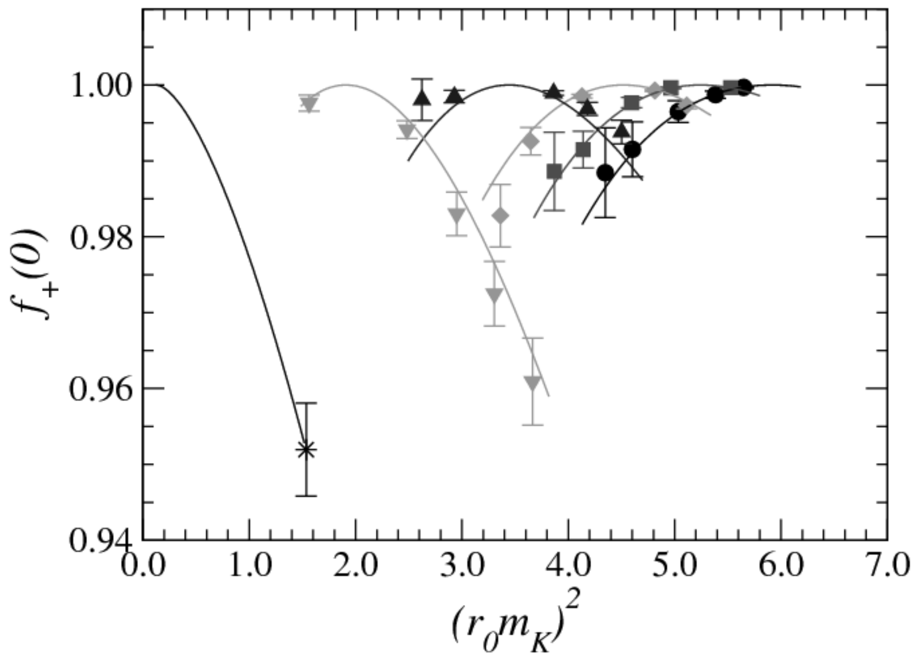
- JLQCD, $N_f = 2$ ensemble
- Non-perturbatively $O(a)$ improved Wilson quarks.
- $20^3 \times 48$, $\beta = 5.2$
- 5 quark masses: pion 500 MeV $\rightarrow$ 1000 MeV
- 1,200 configs (10 trajectory seperation)
- As mentioned before: alternative way of fitting ratios for q^2 dependence.
- Need to $O(a)$ improve vector density when tranfering momenta.

CP-PACS

- Two different forms of the mass extrapolation tried:

  1. Including the f_2 log term + polynomial in masses

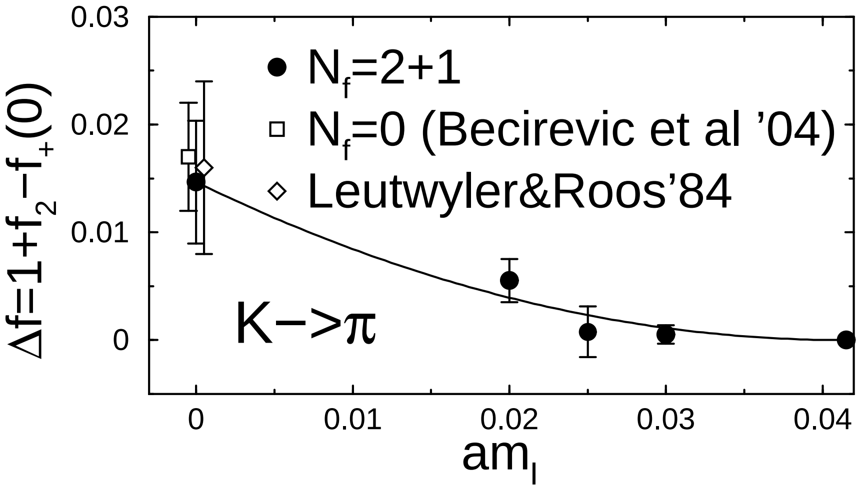
  2. No f_2 term + polynomial in masses "Quadratic"



- Quadratic: 0.967(6)
- ChPT : 0.952(6)

Fermi-lab/HPQCD

See Okomoto, hep-lat/0412044

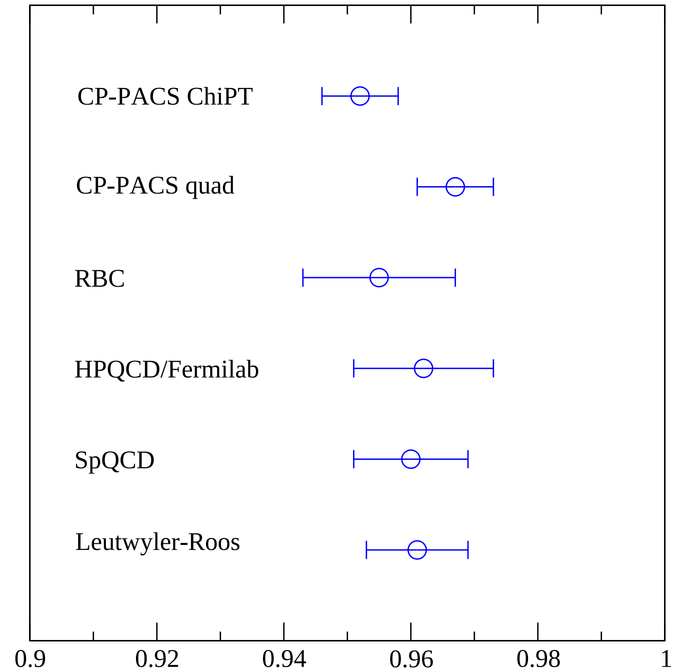


- Gauge ensemble: 2+1 flavours of Staggered quarks
- s-quark using clover improved wilson fermions, naive staggered u and d quarks.

- No data for finite momentum
  - extrapolate to $q^2 = 0$ using pole form and experimental result for coefficient
  - $\lambda_+ = 0.0278(7)$ (PDG) ; 0.026(2) (Becirevic) ; 0.021(2) (CP-PACS)
- Extrapolation to physical masses with $\Delta f = (A + Bm_l)(m_s - m_l)^2$
- final answer: $f_+(0) = 0.962(6)(9)$

Summary of Results

- All these new lattice numbers should be considered as preliminary for various reasons.
- Quenched, $N_f = 2$, and $N_f = 2 + 1$ all agree.
  - also with the Leutwyler-Roos
- Chiral extrapolation for all measurements is over a large range.
- Lattice spacing, Volume effects small ?



- Double edged sword: as you get closer to the physical masses, the momentum extrapolation gets larger.

Twisted Boundary Conditions

See talks of C. Sachrajda, A. Juttner.

- Simple to "twist" the boundary conditions for a quark

$$\psi(x_i + L) = e^{i\theta}\psi(x_i)$$

  and introduce a minimum momenta for the quark ($\theta = \pi$; anti-periodic boundary conditions; minimum momenta π/L)

- Giving a minimum momentum to a pion slightly more complicated:
  - Break flavour symmetry at the boundary: different twist for different flavours (or $\overline{u}\gamma_5 d$ wouldn't change)
  - This is just a change in the boundary conditions. Finite volume effect? (More precisely: is it exponentially supressed with volume).

- Sachrajda and Villaro, hep-lat/0411033 studied twisted boundary conditions in chiral perturbation theory and found
  1. For physical quantities without final state interactions the flavour symmetry breaking effects are exponentially supressed with volume
  2. With final state interactions ($K \to \pi\pi$) it is not generally possible.

! works partially twisted (Numerical Study Flynn, *et al*, hep-lat/0506016)

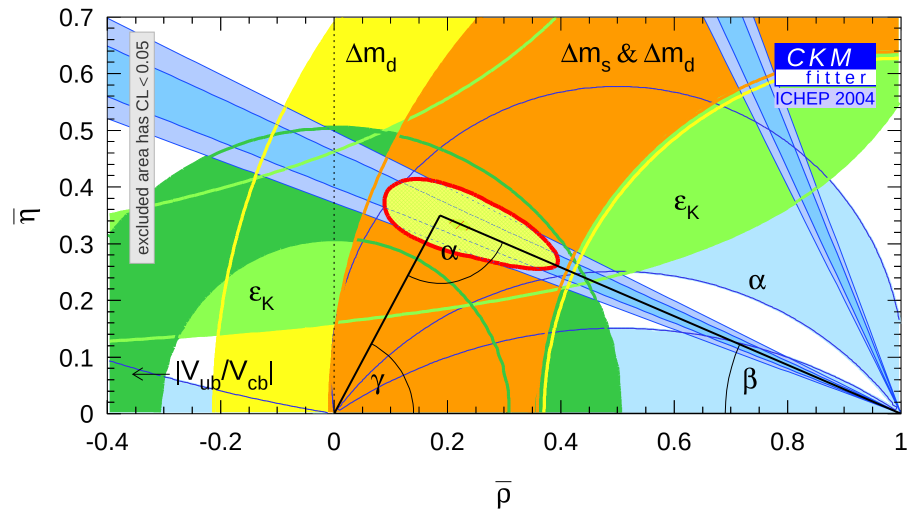
B_K : 2004 numbers

B_K is the low energy matrix element relevant to indirect CP-violation in the $K^0 - \overline{K^0}$ system

$$|\epsilon_K| = C_\epsilon A^2 \lambda^6 \overline{\eta} \left[-\eta_1 S(x_c) + \eta_2 S(x_t)(A^2\lambda^4(1-\overline{\rho}) + \eta_3 S(x_c, x_t) \right] \hat{B}_K$$

- This is the CKMfitter group's plot from ICHEP 2004. The input used was

$$\hat{B}_K = 0.86 \pm 0.06 \pm 0.14$$



- PDG : $\hat{B}_K = 0.68 - 1.06$

- For both these results, the value quoted is the result of lattice calculations; the main error is from the use of the quenched approximation.

B_K definition

In the continuum there is one operator that contributes to B_K It if of the form:

$$O_\Gamma = \overline{s}\Gamma_i d \; \overline{s}\Gamma_i d$$

with the gamma structure:

$$VV + AA \equiv \gamma_\mu \otimes \gamma_\mu + \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5 \;;$$

which is simply the parity conserving part of

$$(V - A) \otimes (V - A)$$

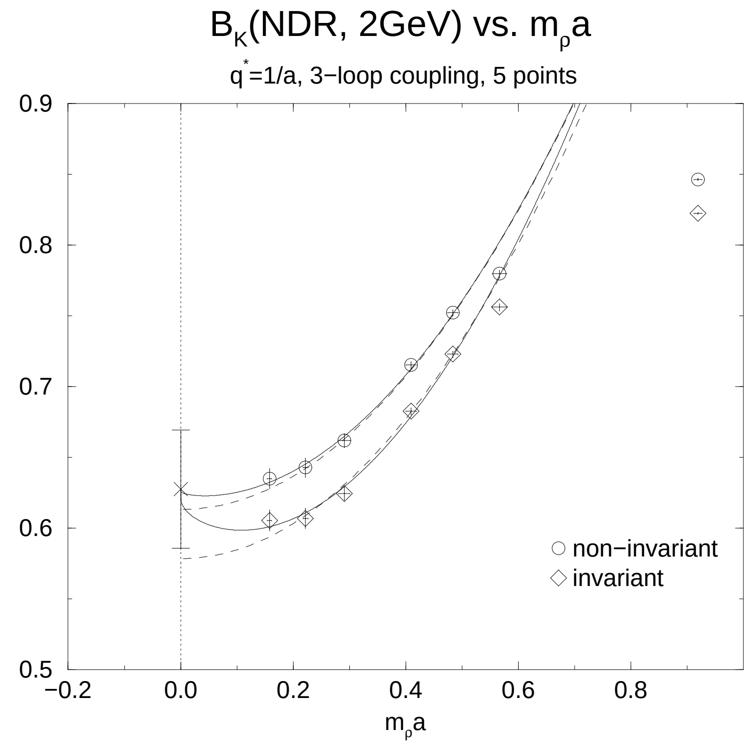
B_K itself is defined as

$$\begin{aligned} B_K &= \frac{\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2} \\ &= \frac{\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle}{\frac{8}{3} \langle \overline{K}^0 | A_\mu | 0 \rangle \langle 0 | A_\mu | K^0 \rangle} \end{aligned}$$

and is most often quoted either renormalised in the NDR,$\overline{MS}$ scheme at 2 GeV, or as, the renormalisation group invariant, $\hat{B}_K$. ($\hat{B}_K \sim 1.4 B_K^{NDR,\overline{MS}}$)

Benchmark Calculation

- JLQCD (Aoki et al, 1997)
  - staggered fermions
  - 7 lattice spacings
    $a \sim 0.24 - 0.04$fm
  - $La \sim 2.3 - 2.5$fm generally
    volume dependance studies at two lattice spacings; $La \sim 1.8 - 3.1$fm



- Two lines values are from differen't operator definitions; should be same to order in perturbation theory and lattice spacing they worked...
  - $O(a^2)$ (lattice spacing errors) and $O(\alpha^2)$ (mixing errors) terms included in the fit
    - $*$ Discretisation errors large; $O(\alpha^2)$ also large and badly constrained.

Operator Mixing and B_K: Staggered Fermions

- Mixing between different tastes causes allows many other operators to mix with the naive continuum operator.

- Standard (one-loop) mixing calculation works with four different operators which contribute the

  – VV, one colour-trace, two colour-trace

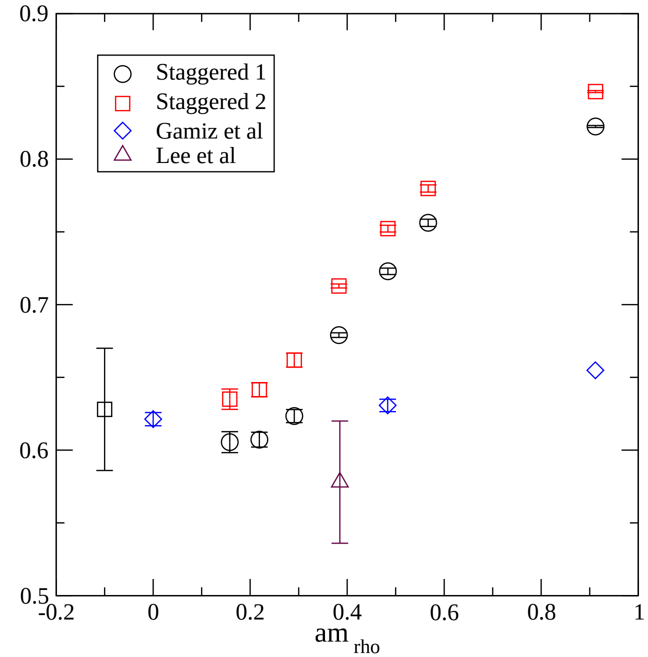
  – AA, one colour-trace, two colour-trace

  constractions to the matrix element, with mixing resolved to $O(\alpha)$

- This list ignores taste breaking operators, a point I'll come back to – briefly – later.

Improved Staggered Results

- Two groups:
  - Gamiz *et al*, hep-lat/0409049
    - Hyp smeared fermion action
    - "Thin-link" operators
    - statistical error only
  - W. Lee *et al*, hep-lat/0409047:
    - Hyp smeared fermion action
    - One-loop perturbation theory
    - Error dominated by estimate of neglected $O(\alpha^2)$ terms.



- Evidence of good scaling
- Large error-bar on renormalised value, estimate of error due to neglected $O(\alpha^2)$ terms in mixing calculation.

Improved Staggered Results...

- The final result from W. Lee *et al* is

$$B_K(\overline{MS},\ 2\ \text{GeV}) = 0.578 \pm 0.018 \pm 0.042$$

  - First error: statistics + chiral extrapolation
  - Second error: using one-loop renormalisation factors.

- Worth understanding the (large) size of this last error:
  - The perturbative calculation used

    $$\alpha_s(q^* = 1/a) = 0.192$$

    Simply estimating the truncation error to be of order $\pm 1 \times (\alpha_s(q^*))^2$ gives a $\sim 4\%$ error.
  - The actual error quoted is around a factor of two larger, due to taking the largest deviation due to seperately varying the coeffcient of the four different operators by terms of order $\pm 1 \times (\alpha_s(q^*))^2$.

Operator Mixing and B_K : Broken Chiral Symmetry

- If chiral symmetry is broken, four other operators can mix (the four other possible gamma matrix structures)

$$\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle_{\text{latt}} \propto \langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle_{\text{ren}} + \sum_{i \geq 2} c_i \langle \overline{K}^0 | O_{MIX,i} | K^0 \rangle_{\text{ren}}$$

These operators, of course, have a different chiral structure.

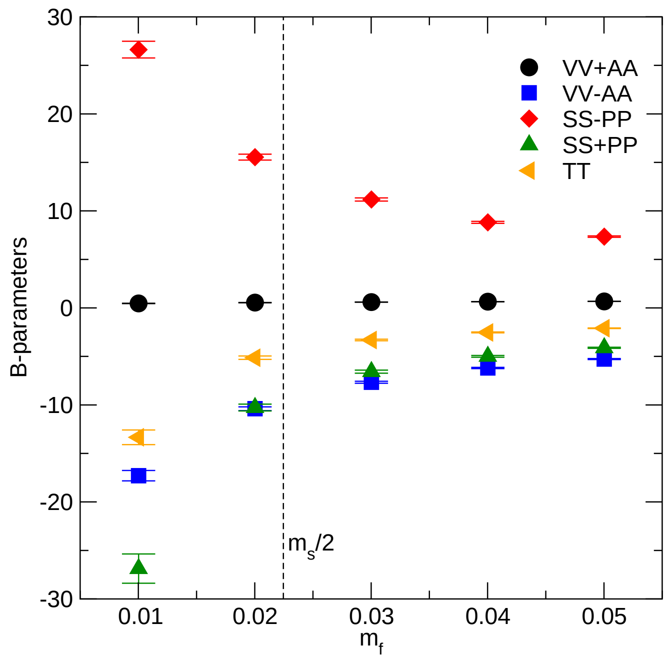
Mixing is hard to control using perturbation theory; First order chiral perturbation theory predicts that

$$\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle \propto M_k^2$$

and,

$$\langle \overline{K}^0 | O_{\text{THE REST}} | K^0 \rangle \propto \text{constant}$$

small enough mass, wrong chirality operators will dominate.



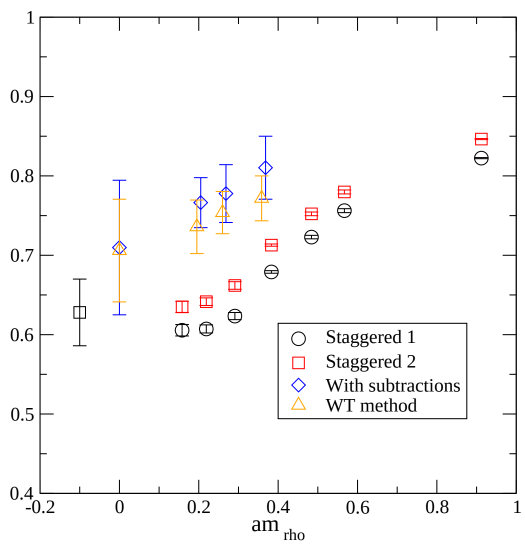
Wilson Fermions

1. Subtract the other operators using (non-perturbatively extracted) mixing coefficients

2. Use Chiral Ward-Takahashi identities to relate the matrix element of interest to that it's parity partner

$$2\langle K O_{VV+AA} K\rangle = 2m\int d^4x \langle P_5(x) K O_{VA+AV} K\rangle + O(a) + \cdots$$

- O_{VA+AV} renormalises multiplicatively (discrete symmetries).



← Becirevic *et al*, hep-lat/0005013, hep-lat/0407004

- Both methods have large error-bars
- variant of the latter : twisted mass...

Twisted Mass : Maximal Twist

Frezzotti *et al*, hep-lat/0101001

- Twisted u and d quarks, standard (Wilson) s :

$$\mathcal{L}_f = \overline{\psi}\left(\not{D} + m + i\mu_q\gamma_5\tau^3\right)\psi + \overline{s}\left(\not{D} + m_s\right)s$$

  $\psi = (u, d)$

- In terms of twisted fields, $\Psi' = e^{i\alpha\gamma_5\tau^3/2}\psi$

$$O'_{VV+AA} = cos(\alpha)O_{VV+AA} - isin(\alpha)O_{VA+AV}$$

  – For $\alpha = \frac{\pi}{2}$ (maximal twist),

$$O'_{VV+AA} = -iO_{VA+AV}$$

    * multiplicatively renormalisable
    * u,d lattice spacing errors cancel ; errors still start at $O(a)$ (strange).
    * non-degenerate easy, but can't take strange quark mass too low

Twisted Mass : $\alpha = \pi/4$

Twisted s and d quarks:

$$O'_{VV+AA} = cos(2\alpha)O_{VV+AA} - isin(2\alpha)O_{VA+AV}$$

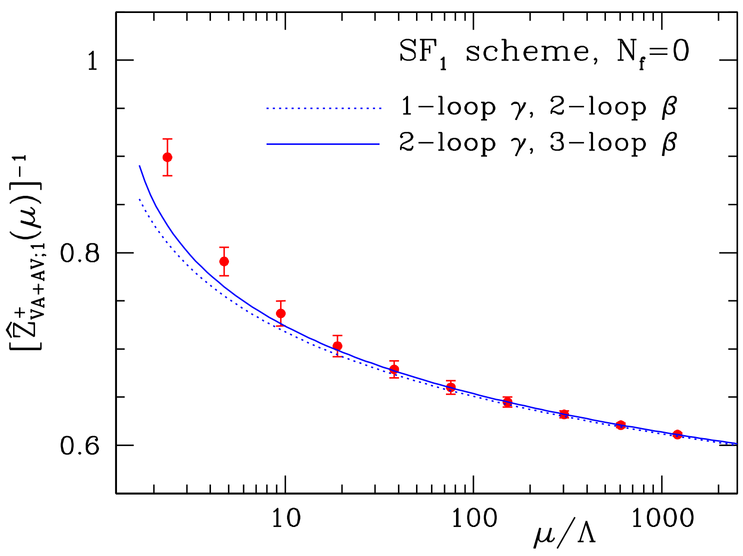
- Take $\alpha = \pi/4$ for multiplicative renormalisation
  - not at maximal twist : no "magic" cancellation of $O(a)$ errors
  - no exeptional configurations
  - s , d degenerate (not a problem in quenched).

Frezzotti and Rossi, hep-lat/0407002 and talk of Frezzotti

Although I wont show any results for it, it is possible to calculate B_K with full $O(a)$ improvement using twisted mass

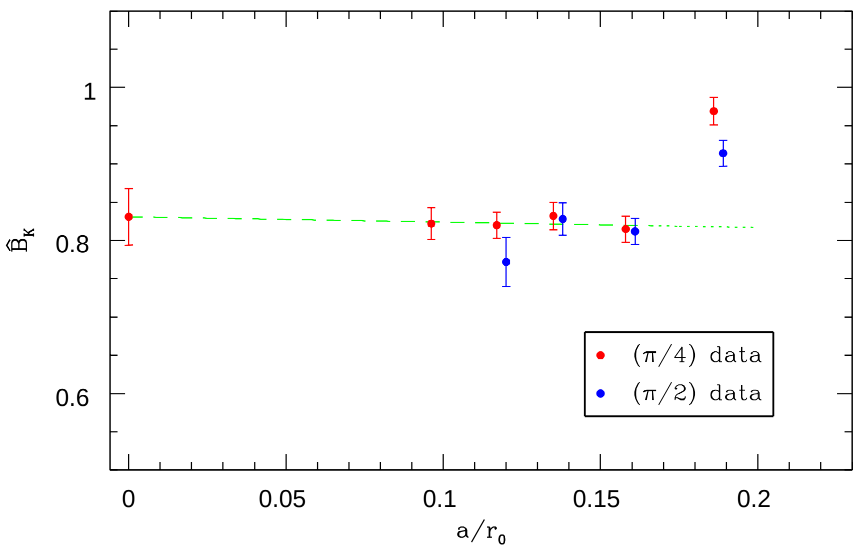
Twisted Mass Renormalisation (Alpha Collaboration)

Non-perturbative renormalisation calculation using the schrodinger functional technique. Guagnelli *et al*, hep-lat/0505002, hep-lat/0505003 and talk by Stefan Sint.



- Gauge invariant
  - don't have to worry about gribov copies
- uses finite volume scheme; $\mu = 1/L$
- Matched – in perturbation theory – to the continuum at large μ.
- work in massesless limit.
  - dirichlet boundary conditions in time direction.

Twisted Mass Results (Alpha Collaboration)

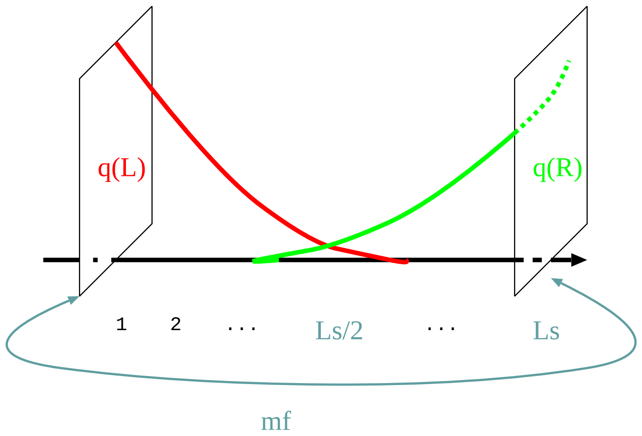


- Wilson gauge: $\beta = 6 - 6.45$
- Zero/Small extrapolation to physical mass
- $16^3 \times 32$ - $32^3 \times 72$ Volume effects checked at $16^3 \times 32$.
- continuum extrapolation linear in a

- Strange jump for $\beta = 6.0$
- Preliminary number: $B_K(\overline{MS}, 2\text{ GeV}) = 0.604(27)$

Domain Wall

- Domain Wall Fermions "almost" preserve chiral symmetry.
  - "almost" no mixing with the wrong chirality operators



- wrong chirality matrix elements $O(10)$ times larger than signal at masses of interest.

- How much chiral symmetry is enough?
  Simple model:
  - One trip through the bulk : supression factor of $O(am_{\text{res}})$
  - Operator of interest is $(V-A)^2$: four left-handed fields.
    wrong chirality operators : two left-handed, two right-handed

$$\rightarrow O((am_{\text{res}})^2) \sim 10^{-6}$$

Domain Wall...

Not so simple... (Golterman and Shamir, hep-lat/0411007)

- Chiral symmetry breaking best understood in terms of the transfer matrix in the fifth dimension

$$T = \frac{1-H}{1+H}$$

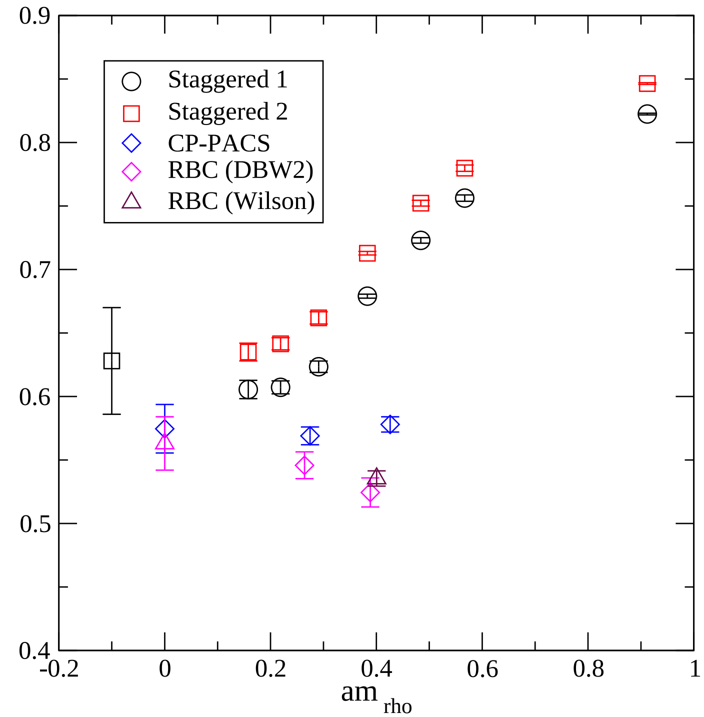
- Two types of contribution to chiral symmetry breaking:
  1. Extended modes: fall-off $\propto \exp(-\lambda_C L_s)$ (λ_C - mobility edge)
  2. Localized modes: includes zero modes of H : unsupressed propagation in fifth dimension.
     - strongly supressed in continuum limit, and by improved actions

- Can estimate the relative size of these two contributions by looking a m_{res} versus L_s , or eigenspectrum of H
  - See talk of Peter Boyle

- Can still estimate size of wrong chirality mixing : rough estimate is the m_{res} gained with $2L_s$. (See talk of Norman Christ).

Domain Wall

- Two groups:
  - CP-PACS collaboration (Ali Khan *et al*)
    - ∗ Perturbative renormalisation factors
    - ∗ Finite volume study
    - ∗ Iwasaki Gauge action
    - ∗ Finite L_s study
  - RBC collaboration (Blum *et al*)
    - ∗ Non-perturbative renormalisation
    - ∗ DBW2 gauge action (One Wilson point)



Both have small time-extents: may have systematic error due to this.

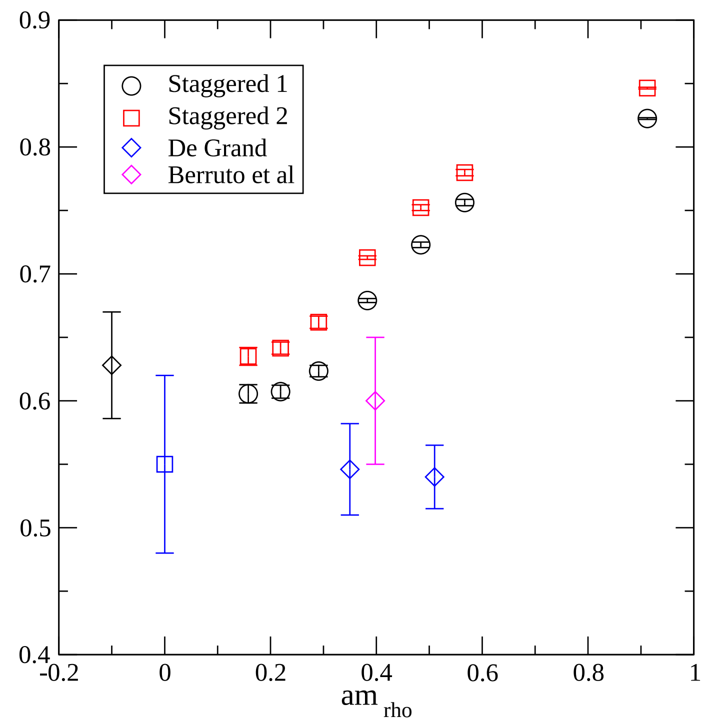
Overlap

- Don't have to worry about any mixing, but expensive.

- Large error-bars simply due to low statistics

Two Groups:

1. DeGrand *et al*, hep-lat/0309026
   - Perturbative Renormalisation
2. Berruto *et al*, hep-lat/0409131
   - NPR



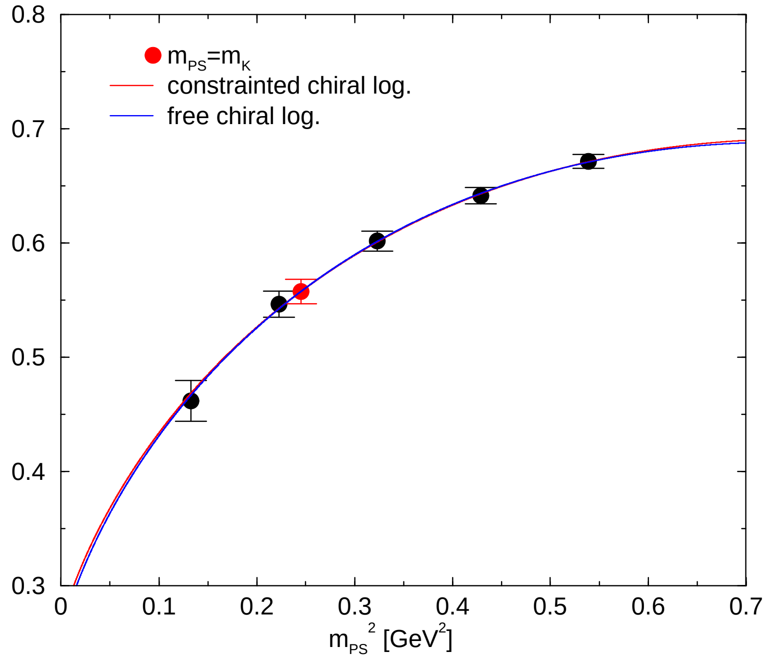
Chiral PT fits

Predicted NLO ChiPT:

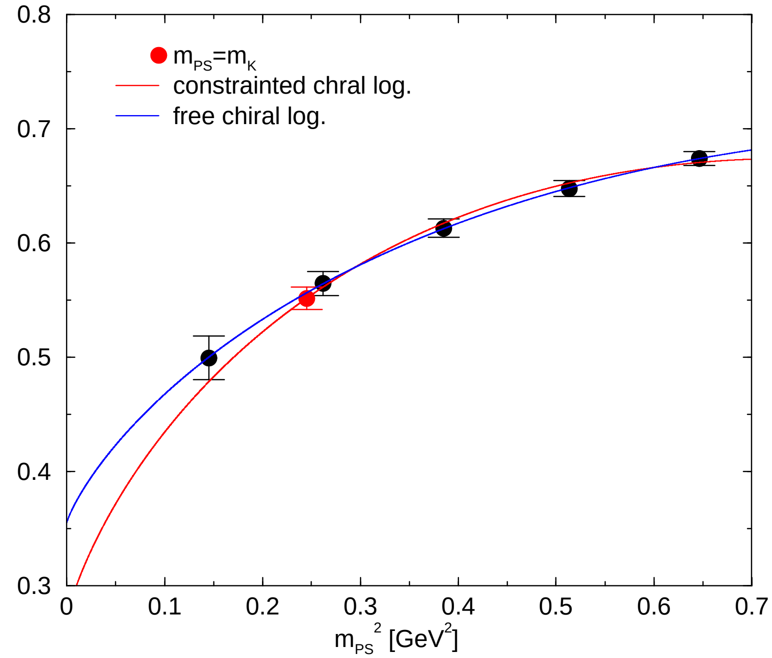
$$B_K = b_0\left(1 - \frac{6}{(4\pi f)^2}M_K^2 \ln\left[\frac{M_K^2}{(4\pi f)^2}\right]\right) + b_1 M_K^2$$

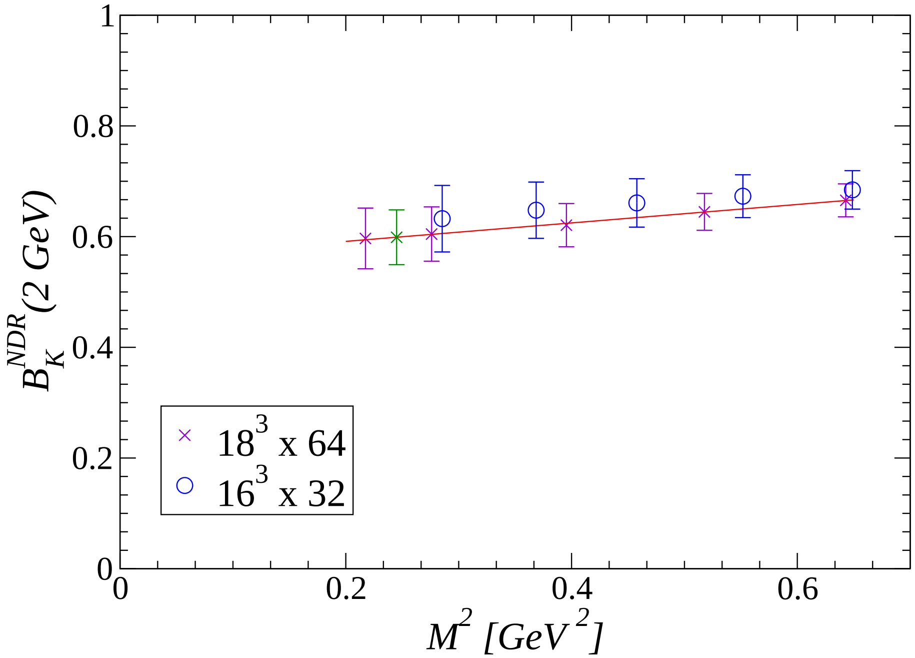
Some groups able to fit to this:

DWF(RBC) $a^{-1} = 2\text{GeV}$



DWF(RBC) $a^{-1} = 3\text{GeV}$



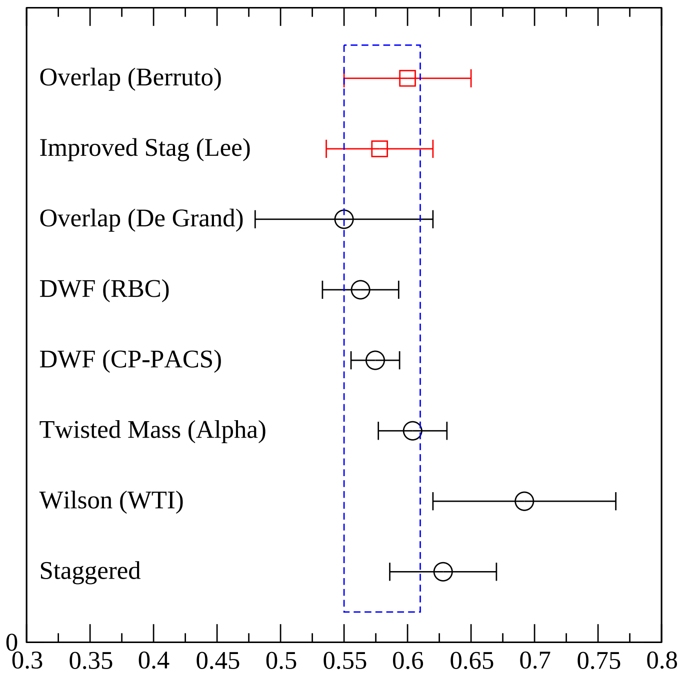


- Plot stolen from hep-lat/0409131
  - Berruto *et al*, Overlap Fermions

- Discretisation Error?

- Since – in the quenched approximation – we are able to either interpolate to the physical point, this difference is not a large effect on the final number.

Quenched Summary



Treat all the errors as statistical: all (continuum extrapolated) combined give

$$B_K^{NDR}(2\text{GeV}) = 0.587(13)$$

All a^2 extrapolated, published gives

$$B_K^{NDR}(2\text{GeV}) = 0.582(17)$$

- both with good χ^2/dof

Errors not all statistical

$$B_K^{NDR}(2\text{GeV}) = 0.58(3)$$

c.f $B_K^{NDR}(2\text{GeV}) = 0.58(4)$ [Shoji Hashimoto (ICHEP 2004)]

Dynamical Work

- As mentioned before: dominant error on the accepted number a guess of the size of the quenching ambiguity

- Recent work using dynamical fermions:

  1. Wilson fermions, UKQCD $N_f = 2$ (very heavy masses)
  2. Domain Wall Fermions, RBC $N_f = 2$ (non-degenerate masses)
  3. Domain Wall Fermions, RBC $N_f = 2 + 1$
  4. Improved Staggered $N_f = 2 + 1$ (underway by two groups)

- Number quoted by RBC is $B_K^{NDR}(2\text{GeV}) = 0.495(18)$, where the last number is a *statistical* error. Various systematics:

  - single lattice spacing
  - fairly heavy dynamical masses
    - $0.5 m_s \to m_s$
  - single (small) volume
  - "less quenched" approximation.

Degenerate fit

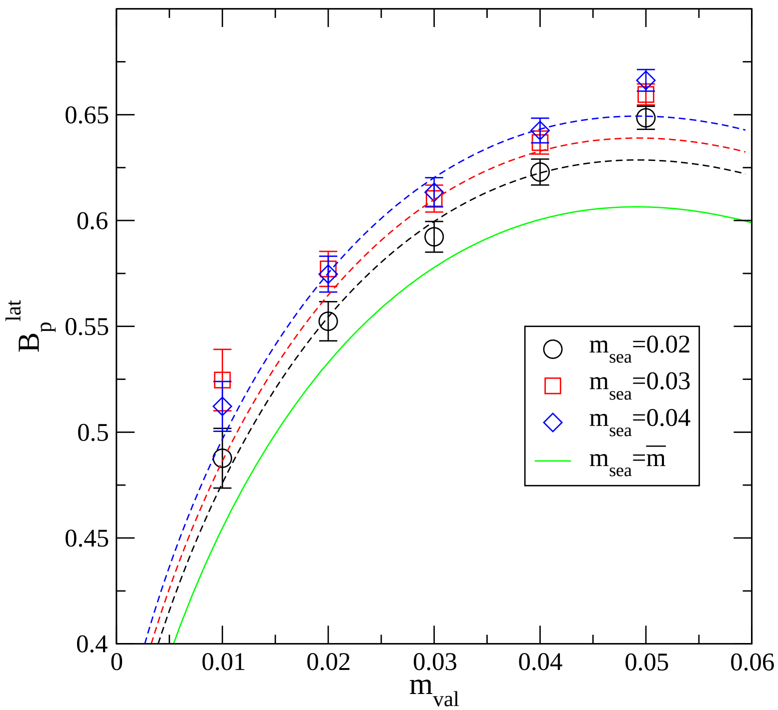
In quenched work ChiPT was used to interpolate; here we use it to extrapolate.

- relying much more on the convergence of ChiPT (known to be bad for other quantities at these masses)

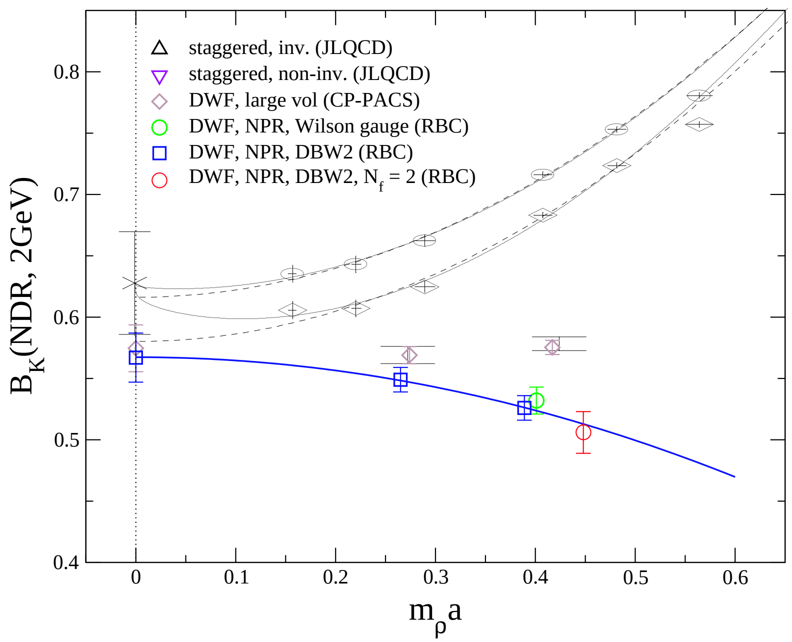
m_{sea} dependence not well resolved between $m_{sea} = 0.03,\ 0.04$.

$m_{sea} = 0.02$ is clearly lower: relevant ChiPT coefficient $\sim 2\sigma$

lighest/heaviest valence points aren't fit well



Lattice Spacing



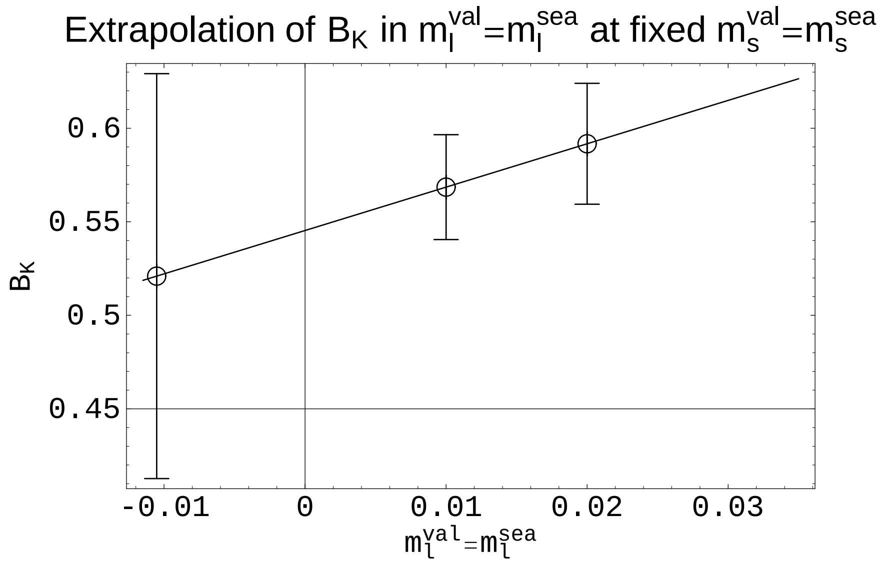
- "Suggestive" graph with the quenched and dynamical DWF results on, with the a^2 extrapolation on it.
  - Not a very sensible thing to plot

- Our dynamical result is only 3% lower than the quenched results closest in lattice spacing.
  - *does* (probably) reduce as we reduce the dynamical mass.

$2+1f$ **Calculations**

Saul Cohen will talk about preliminary results with 2+1f of dynamical domain wall fermions

- Still early days (low statistics)
- $16^3 \times 32 \times 8$
- Coarse lattices, large m_{res}
  - will study larger valence L_s
- Non-degenerate dynamical extrapolation

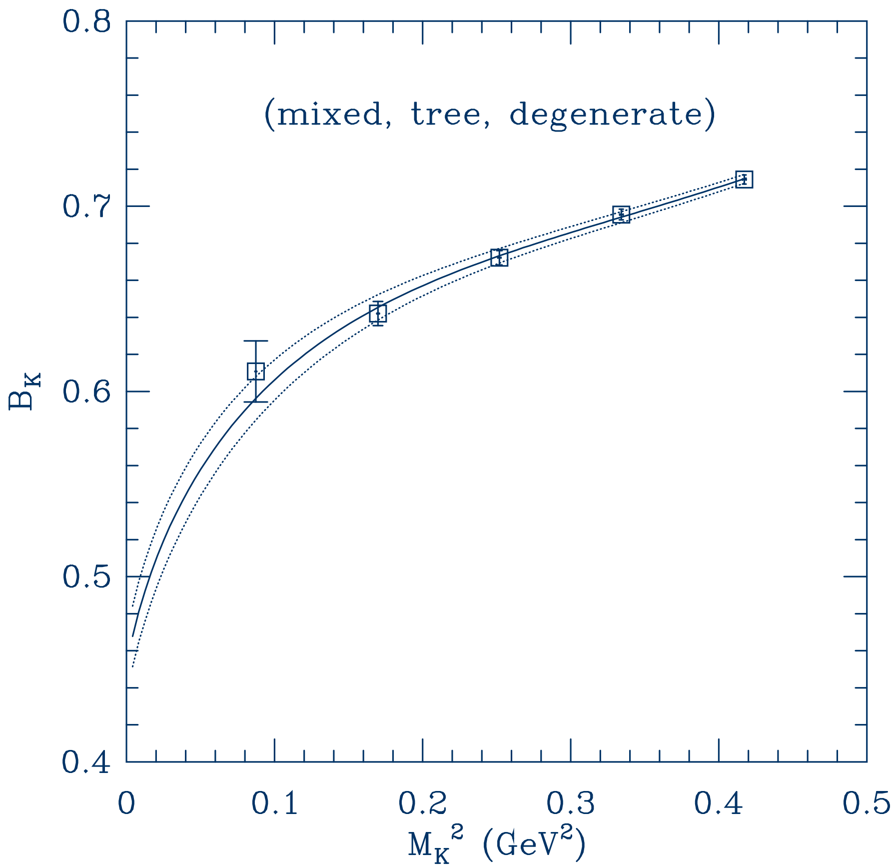


- currently with estimate for the overall renormalisation factor of $\frac{1}{Z_A^2}$ (far from the leading systematic).

- Warm up for
  - $24^3 \times 64 \times 16$, Iwasaki ensembles currently being generated by RBC/UK

$2 + 1f$ **Calculations**

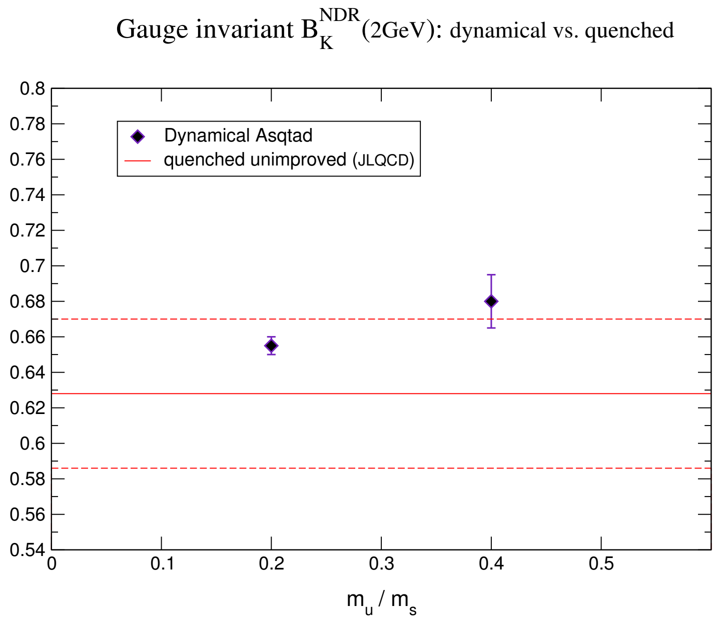
Taegil Bae , Jongjeong Kim, and Weonjong Lee will present posters on the calculation of B_K using the MILC 2+1f ensembles, using a mixing action approach: (HYP-smeared operators; a^2 -tad background.



- Calculation underway
  - Only tree level matching so far
- $20^3 \times 32$, $1/a = 1.588(19)$GeV
  - Two dynamical masses 0.01 ,0.02 .
  - Five valence quark masses 0.01 – 0.05 .
- See deviation from predicted – continuum – chiral log (I'll mention this again later)

$2+1f$ Calculations

See talk of Elvira Gamiz



- Coarse set of MILC configurations : $a \sim 0.125$fm
- Two different dynamical masses: $m_s/2$, $m_s/4$
- One-loop matching (a^2 -tad)
- Degenerate valance quark masses.

$$B_K(2\text{ GeV}) = 0.630(18)(15)(30)(130)$$

- Errors: statistics, chiral fit, discretisation errors (quenched), matching..
  - course lattice/dynamical : $\alpha_V(1/a)$ simply larger
    - $*$ doesn't take into account operator mixing

- Need two-loop / non-perturbative matching (finer lattices)

New Chiral Perturbation Theory Results

See talk of Ruth Van de Water and Van de Water and Sharpe, hep-lat/0507012.

- NLO calculation of B_K in staggered chiral perturbation theory
- Need a power counting scheme to decide which operators to include

$$p^2 \sim m \sim a^2 \sim a_\alpha^2 \sim \alpha/4\pi \sim \alpha^2$$

  - $a_\alpha^2 = a^2 \alpha_V (\pi/a)^2$ is estimate of the size of taste breaking discretisation effect (seen large numerically) [Standard power counting]
  - existing one-loop calculations are $1 + O(1 \times \frac{\alpha}{4\pi})$; estimating next order as $O(\alpha^2)$.
  - the $O(\alpha/4\pi)$ terms are those left out of the matching.

- Upshot:
  - 13 chiral operators (4 in continuum), some with multiple independent coefficients (different orders in a , α).
    - $*$ when fitting different lattice spacings: 37 free parameters.

Approach

37 parameters is... daunting.

- Obvious things:
  - add in $O(\alpha/4\pi)$ and $O(\alpha^2)$ terms to matching.

- Approach suggested by Van de Water and Sharpe
  1. Initially work at a single lattice spacing $\longrightarrow$ 16
  2. for degenerate quarks $\longrightarrow$ 9
  3. perform auxillary calculations with different states on external legs to isolate some of the coefficients $\longrightarrow$ 4
  4. fit degenerate (maybe add back in non-degenerate); remove any coefficient that is "purely an error"
  5. final – 4 parameter – fit to the lattice spacing dependence

- Also noted in paper that the as the log are averaged over the different taste pions, the curvature is much reduced : Weonjong Lee's results.

B_K Conclusions

- Good agreement in the quenched case between various groups/actions
- Preliminary dynamical work underway
- Systematic error due to quenching? (black art)
  - Follow Steve Sharpe (1996): Compare change between quenched and dynamical at same lattice spacing, give error based on size of discretisation effects. Discretisation error (a little) smaller: 15% $\rightarrow$ 10%
  - Previous estimate of $SU(3)$ breaking $\sim$ 5% (we see 3%)
  - Add in quadrature: 16% $\rightarrow$ 11%
  - Could be more sophisticated and fold in the direction of change we have seen (+5% -11% ?) - just wait until dynamical results are here.

0.58(3)(6)

$K \to \pi\pi$

The calculation of $K \to \pi\pi$ decays has long been a goal of lattice QCD.

Interested in:

$$\begin{aligned}
\mathcal{A}\left(K^+ \to \pi^+\pi^0\right) &= \sqrt{3/2}A_2 e^{i\delta_2} \\
\mathcal{A}\left(K^0 \to \pi^+\pi^-\right) &= \sqrt{2/3}A_0 e^{i\delta_2} + \sqrt{1/3}A_2 e^{i\delta_2} \\
\mathcal{A}\left(K^0 \to \pi^0\pi^0\right) &= \sqrt{2/3}A_0 e^{i\delta_0} + \sqrt{4/3}A_2 e^{i\delta_2}
\end{aligned}$$

Where the final state has been decomposed into contributions of isospin 0 and 2

- $\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 1/\omega \sim 22$: the $\Delta I = 1/2$ rule.
- ϵ'/ϵ

$K \to \pi\pi$

Very difficult problem:

- Two particles in the final state.
  - use ChiPT to work out from $K \to \pi$ and $K \to 0$.
  - face Maini-Testa theorem : Lellouch-Luscher

- Use OPE to give effective hamiltonian in terms of four-quark operators. Operator mixing problem:
  - 7 dimension 6 operators to consider (below charm).
  - some operators ($\Delta I = 1/2$) mix with lower dimension operator
    - can be power divergent in a : need non-perturbative subtraction

Operator Mixing/Twisted Mass

A major problem with renormalising the $\Delta S = 1$ hamiltonian is the power-divergent mixing with the operator

$$(m_s + m_s)\,\bar{s}d + (m_s - m_d)\bar{s}\gamma_5 d$$

- Above form requires chiral symmetry. Without chiral symmetry, factor of $m_s + m_d$ lost, $m_s - m_d$ enforced by discrete symmetries.

- When charm is included, the GIM mechanism gives a extra factor of

$$(m_c^2 - m_u^2)$$

  contribution of operator $\rightarrow$ finite: $m_c - m_u$ without chiral symmetry.

Frezzotti and Rossi, hep-lat/0407002 and talk of Frezzotti

Use four flavours of quark, can construct scheme that uses maximal twisting (no $O(a)$ terms) to kill all the power divergent mixings for either parity: have discrete symmetries that enforce a factor of

$$(m_c^2 - m_u^2)(m_s - m_d)$$

- Have four twisted dynamical quarks

- Can even make them non-degenerate (bound!)

Older Calculations

Without these discrete symmetries, chiral symmetry is clearly vital:

- Both the RBC and CP-PACS have used Domain WallFermions
  - need to be careful to avoid power-divergent systematics, but possible.
  - biggest systematics of the RBC/CP-PACS calculations:
    - Quenched Approximation (Extra operators!)
    - Using Chiral PT to move between $K \to 0$, $K \to \pi$ to physical $K \to \pi\pi$ Only works at LO. For NLO need some information from $K \to \pi\pi$.

Ways to improve...

Dynamical Quarks:

- Jun Noaki will talk about results for the electroweak penguin operators using $N_f = 2$ dynamical DWF .

Sorry Jun...

$K \to \pi\pi$ direct:

- Need to deal with two pions in the final state...

ϵ-regime:

- Fit chiral coefficients in the ϵ-regime...

Two Pions in final state: Finite Volume Methods

Miani-Testa no-go theorem: two pion operator has ground state of two pions at rest.

- not the physical decay for $K \to \pi\pi$
  - want $\pi(-p)\ \pi(p)$, $\pi(0)\ \pi(0)$ has lower energy

Luscher, 1986, 1991 $\times$ 2

- Can extract the scattering length by looking at the volume dependence.

Lellouch and Luscher, hep-lat/0003023

- On the lattice have discretised energy levels for the two pion states

$$E_{\pi\pi} = 2\sqrt{m_\pi^2 + 2\pi n/L}\ ;\ n = 0, 1, \cdots$$

  doesn't matter that the physical decay isn't the lowest state: do an exited state fit.

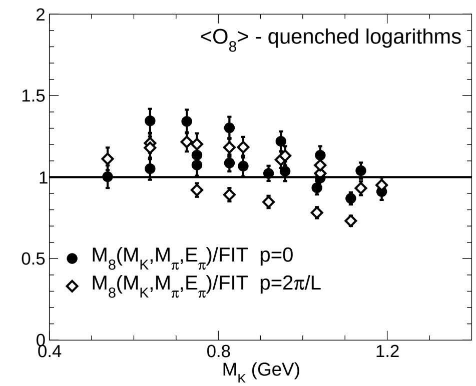
- Formula relating the decay in the centre of mass frame ($P = 0$) in finite volume to the infinite volume result.

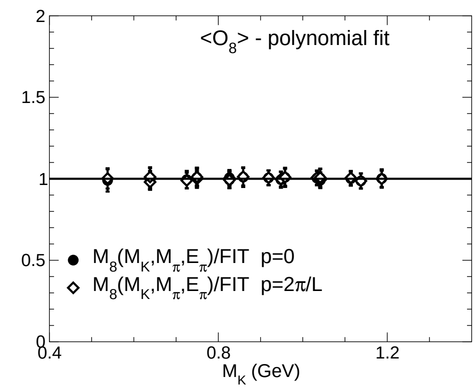
$$|\langle \pi(p)\pi(-p)|H_W|K\rangle_V| \to |\langle \pi(p)\pi(-p)|H_W|K\rangle|$$

$\Delta I = 3/2$

Simpler than $\Delta I = 1/2$, good place to start:
Bouchard *et al*, hep-lat/0412029

- Calculate the $K \to \pi\pi$ directly, but with unphysical kinematics
  - Kaon and one pion a rest, other pion at finite spatial momenta
  - No longer the center of mass frame.
  - is lowest energy state
- Can – in principle – extract all the free parameters at NLO in ChiPT.
- Quenched, Wilson 2 Gev, $24^3 \times 48$, NPR





Plots show the data/fit for the NLO ChiPT and a simple polynomial fit.

- bad convergence (lightest $M_\pi \sim 500$MeV)

Can still extract values by using hybrid ChiPT, polynomial fit; Lellouch-Luscher formula (wrong frame)

$< \pi\pi|O_8^{3/2}|K^0 >= 0.68\pm0.09$ GeV^3 ; $< \pi\pi|O_7^{3/2}|K^0 >= (0.12\pm0.02)$ GeV^3

Need smaller masses, lab-frame formula...

Generalising the Lellouch-Luscher Formula

Would be good to extend the Lellouch-Luscher formalism out of the center of mass frame (total $P = 0$) to lab frame:

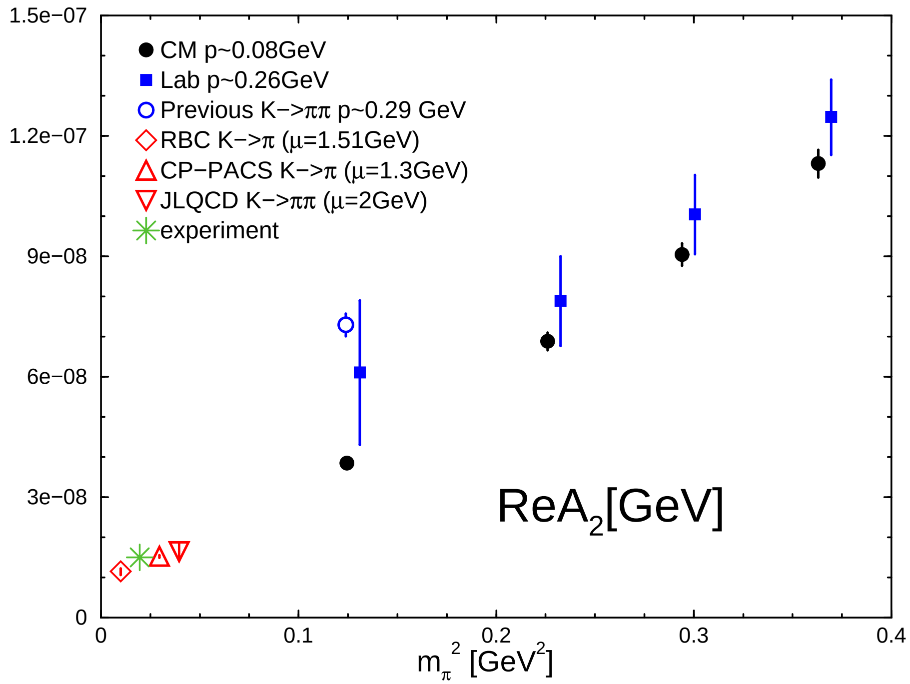
- Two groups: Christ *et al*, hep-lat/0507009 and Kim *et al*, hep-lat/0507006 (See talks of Changhoan Kim and Takeshi Yamazaki) recently did this.

- Luscher:
  - Field theory (Center of mass frame) $\longrightarrow$
  - two-particle quantum mechanics $\longrightarrow$
  - finite/inifinte volume normalisation problem

- Last step already done: Rummukainen and Gottlieb, hep-lat/9503028

Results

Takeshi Keneko will present a poster covering a numerical simulation using this result



- Quenched using DBW2 gauge action
- NPR
- $\beta = 0.87$; $a^1 = 1.3$ GeV
- Lab frame result currently has large error-bar (momentum insertion).

- Can use polynomial anzatz to extrapolate in p^2 and m_π^2

$$1.6(1.3) \times 10^{-8} \text{ GeV}$$

- Good first step, but need to beat down the errors.

ϵ-regime

See talks of P. Hernandez, C. Pena, and Hartmut Wittig.

- Idea of project: study the charm effects on $\Delta I = 1/2$ rule

- Start with $SU(4)$ symmetric theory:

$$\frac{A_0}{A_2} = \frac{1}{\sqrt{2}} \left(\frac{1}{2} + \frac{3}{2} \frac{g_1^-}{g_1^+} \right)$$

- Chiral perturbation theory in ϵ -regime

$$m \Sigma V \sim O(1), F_\pi L \gg 1$$

  (no order ϵ^2 counter-terms).

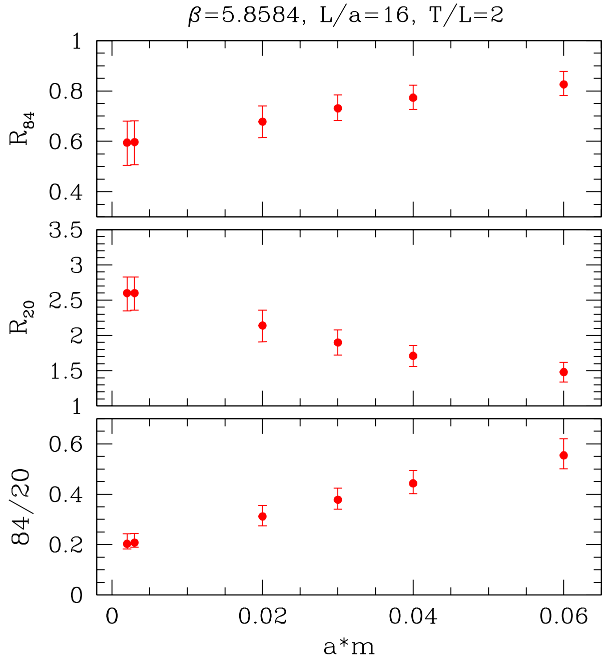
  - relate LEC's to transition amplitudes via ChPT at LO

- mean-field improved perturbation theory

- non-trivial to calculate in ϵ-regime (need to handle zero-modes carefully).

ϵ-regime...

A_0 : factor ~ 2 too small
A_2 : factor ~ 2 too large
A_0/A_2 : factor ~ 4 too small



- Discrepancy? : Breakdown of LO ChiPT, Quenching, Charm quark, finite volume corrections.

Summary

1. $K \to l3$
   - Three new results (Twisted Boundary Conditions could be useful to improve)

2. B_K
   - Good agreement of quenched results
   - Preliminary dynamical results : need to beat down errors

3. $K \to \pi\pi$
   - Twisted mass formulated at maximal twist for $\Delta S = 1$.
   - Lellouch-Luscher formula extended to lab frame
   - First results shown from ϵ-regime study of $\Delta = 1/2$ rule.