AN INTRODUCTION TO THE CHEEGER PROBLEM

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Abstract. Given a bounded domain $\Omega \subset \mathbb{R}^n$ with Lipschitz boundary, the Cheeger problem consists of finding a subset $E$ of $\Omega$ such that its ratio perimeter/volume is minimal among all subsets of $\Omega$. This article is a collection of some known results about the Cheeger problem which are spread in many classical and new papers.

1 Introduction

In 1970, Jeff Cheeger established in his work [9] the following inequality:

$$\lambda_1(\Omega) \geq \left( \frac{h_1(\Omega)}{2} \right)^2,$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain, $\lambda_1(\Omega)$ is the first eigenvalue of the Laplacian under Dirichlet boundary conditions, and $h_1(\Omega)$ is defined as

$$h_1(\Omega) := \inf_{E \subset \Omega} \frac{P(E; \mathbb{R}^n)}{V(E)}.$$

Here $P(E; \mathbb{R}^n)$ is the perimeter of $E$ in distributional sense (see [14]) measured with respect to $\mathbb{R}^n$, while $|E|$ is the $n$-dimensional Lebesgue measure of $E$. $h_1(\Omega)$ is called Cheeger constant of $\Omega$, and a set $C \subset \Omega$ such that

$$\frac{P(C; \mathbb{R}^n)}{|C|} = h_1(\Omega)$$

is a Cheeger set. The task of determining the Cheeger constant of a given domain and of finding a Cheeger set has been considered by many authors. Since the related results are spread in many classical and new papers, it makes sense to collect them in this introductory survey.

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The paper is structured as follows: after introducing the functions of bounded variation in Section 1, we study existence and regularity properties of Cheeger sets (Sections 3 and 4). In Section 5 uniqueness and nonuniqueness issues are discussed, while in Section 6 we treat a quantitative isoperimetric estimate. Finally, we discuss some applications of the Cheeger problem.

2 Functions of bounded variation

Let $\Omega \subset \mathbb{R}^n$ be an open set. The total variation in $\Omega$ of a function $u \in L^1(\Omega)$ is defined as

$$|Du|(\Omega) := \sup \left\{ \int_{\Omega} u \text{div} \varphi \, \vline \, \varphi \in C^1_c(\Omega; \mathbb{R}^n), \|\varphi\|_\infty \leq 1 \right\}.$$ 

A function $u$ such that $|Du|(\Omega) < +\infty$ is said to be of bounded variation. The space of the functions of bounded variation will be denoted by $BV(\Omega)$. It turns out that $BV(\Omega)$ endowed with the norm

$$\|u\|_{BV} := \|u\|_1 + |Du|(\Omega)$$

is a Banach space. A set $E \subset \mathbb{R}^n$ has finite perimeter in $\Omega$ if its characteristic function $\chi_E$ belongs to $BV(\Omega)$, so that

$$P(E; \Omega) := |D\chi_E|(\Omega) < +\infty.$$ 

If $\Omega$ has Lipschitz boundary, then a set $E$ of finite perimeter in $\Omega$ has also finite perimeter in $\mathbb{R}^n$,

$$P(E; \mathbb{R}^n) = P(E; \Omega) + \mathcal{H}^{n-1}(\partial \Omega \cap \partial E),$$

where $\mathcal{H}^{n-1}$ stands for the $(n-1)$-dimensional Hausdorff measure in $\mathbb{R}^n$. In particular,

$$P(\Omega; \mathbb{R}^n) = \mathcal{H}^{n-1}(\partial \Omega).$$

Similarly, if $u \in BV(\Omega)$, then $u \in BV(\mathbb{R}^n)$ (extending it to zero outside $\Omega$), and

$$|Du|(\mathbb{R}^n) = |Du|(\Omega) + \int_{\partial \Omega} |u| \, d\mathcal{H}^{n-1}.$$ 

We will make use of the following results.

**Proposition 2.1.** [14, Theorem 1.9] Let $\{u_k\}$ be a sequence of functions in $BV(\Omega)$ converging in $L^1_{\text{loc}}(\Omega)$ to a function $u$. Then

$$|Du|(\Omega) \leq \liminf_{k \to \infty} |Du_k|(\Omega).$$
Proposition 2.2. [14, Theorem 1.19] Let $\Omega \subset \mathbb{R}^n$ be a domain with Lipschitz boundary, and let $\{u_k\}$ be a sequence of functions in $BV(\Omega)$ such that
\[ \|u_k\|_{BV} \leq M \]
for some $M > 0$. Then there exists a subsequence $\{u_{k_j}\}$ and a function $u \in BV(\Omega)$ such that $u_{k_j} \to u$ in $L^1(\Omega)$.

Proposition 2.3. [14, Theorem 1.23] Let $u \in BV(\Omega)$, and define
\[ E_t := \{ x \in \Omega \mid u(x) > t \}. \]
Then,
\[ |Du|(\Omega) = \int_{-\infty}^{+\infty} P(E_t; \Omega) \, dt. \]

3 Existence of a Cheeger set

In the following, $\Omega \subset \mathbb{R}^n$ will be a bounded domain with Lipschitz boundary. The perimeter of a set will be always measured with respect to $\mathbb{R}^n$, so that we will write
\[ P(E) := P(E; \mathbb{R}^n). \]
We recall that the Cheeger constant is defined as
\[ h_1(\Omega) := \inf_{E \subset \Omega} \frac{P(E)}{|E|}, \]
with the convention that
\[ \frac{P(E)}{|E|} = +\infty \]
whenever $|E| = 0$.

Proposition 3.1. For every bounded domain $\Omega \subset \mathbb{R}^n$ with Lipschitz boundary, there exists at least one Cheeger set.

Proof. Let us define
\[ \tilde{h}_1(\Omega) := \inf_{v \in BV(\Omega) \setminus \{0\}} \frac{|Dv|(\mathbb{R}^n)}{\|v\|_1}. \]
By definition, $\tilde{h}_1(\Omega) \leq h_1(\Omega)$. Moreover, applying the direct method of the Calculus of Variations, the existence of a function $u \in BV(\Omega)$, $u \neq 0$, such that
\[ \frac{|Du|(\mathbb{R}^n)}{\|u\|_1} = \tilde{h}_1(\Omega) \]
follows readily from Propositions 2.1 and 2.2. Since $|D|u||\mathbb{R}^n| \leq |Du|\mathbb{R}^n|$ (see [2, Exercise 3.12]), we can consider without loss of generality $u \geq 0$. Define $E_t := \{x \in \Omega | u(x) > t\}$.

From Proposition 2.3 and Cavalieri’s principle, we have

$$0 = |Du|\mathbb{R}^n - \tilde{h}_1(\Omega)\|u\|_1 = \int_0^{+\infty} [P(E_t) - \tilde{h}_1(\Omega)|E_t|] dt \geq \int_0^{+\infty} [P(E_t) - h_1(\Omega)|E_t|] dt \geq 0.$$ 

It follows that for almost every $t \in \mathbb{R}$ (in the sense of the Lebesgue measure on $\mathbb{R}$),

$$P(E_t) - \tilde{h}_1(\Omega)|E_t| = 0. \quad (3.2)$$

Since $u \not\equiv 0$, there must exist $s \in \mathbb{R}$ such that $|E_s| > 0$ and for which (3.2) holds. This yields at once

$$\tilde{h}_1(\Omega) = h_1(\Omega)$$

as well as the existence of a Cheeger set for $\Omega$. □

**Remark 3.2.** From the proof of Proposition 3.1, it follows that if $u$ is a minimizer for $\tilde{h}_1(\Omega)$, then almost every level set of $u$ with positive Lebesgue measure is a Cheeger set for $\Omega$. In fact, by [6, Theorem 2] this is actually true for all its level sets of positive Lebesgue measure.

**Proposition 3.3.** Let $\Omega \subset \mathbb{R}^n$ have a boundary of class Lipschitz. Then

$$h_1(\Omega) = \inf_{E \subset \subset \Omega} \frac{P(E)}{|E|}.$$ 

This is a straightforward consequence of the following proposition.

**Proposition 3.4 ([23], Theorem 2).** Let $\Omega \subset \mathbb{R}^n$ have a boundary of class Lipschitz, and let $E \subset \Omega$ be a set of finite perimeter. Then there exists a sequence of sets of finite perimeter $\{E_k\}$ such that:

(i) $E_k \subset \subset \Omega$ for every $k$;

(ii) $\chi_{E_k} \rightarrow \chi_E$ in $L^1_{\text{loc}}(\mathbb{R}^n)$ as $k \rightarrow \infty$;

(iii) $P(E_k) \rightarrow P(E)$ as $k \rightarrow \infty$.

**Proof (of Proposition 3.3).** Let $C$ be a Cheeger set for $\Omega$. Then there exists a sequence $\{E_k\}$ of sets of finite perimeter satisfying (i), (ii) and (iii) in Proposition 3.4. By classical results, each $E_k$ can be in its turn be approximated in a similar way by a sequence of sets compactly contained in $\Omega$, but not necessarily in $E_k$, and with smooth boundary (see [14, Theorem 1.24]). Hence the claim follows. □
However, a Cheeger set can not be compactly contained in $\Omega$, as the following proposition states.

**Proposition 3.5.** Let $C$ be a Cheeger set for $\Omega$. Then, $\partial C \cap \partial \Omega \neq \emptyset$.

**Proof.** Suppose, by contradiction, that $C \subset \subset \Omega$. Then it would be possible to find a $t > 1$ such that the set 

$$tC := \{x \in \mathbb{R}^n \mid t^{-1}x \in C\}$$

is still contained in $\Omega$. But then 

$$\frac{P(tC)}{|tC|} = \frac{t^{n-1}P(C)}{t^n|C|} = \frac{1}{t} \frac{P(C)}{|C|} < \frac{P(C)}{|C|},$$

a contradiction to the definition of Cheeger set. Hence, the boundary of $C$ must intersect the boundary of $\Omega$. \hfill $\Box$

## 4 Regularity of Cheeger sets

Let $C$ be a Cheeger set for $\Omega$, and set $V_0 := |C|$. Then, $C$ will be in particular a set which minimizes the perimeter among all the subsets of $\Omega$ with volume $V_0$. Hence, some classical regularity results find application.

**Proposition 4.1.** Let $C$ be a Cheeger set for $\Omega$. Then $\partial C \cap \Omega$ is analytic, possibly except for a closed singular set whose Hausdorff dimension does not exceed $n - 8$.

**Proof.** If $V_0 = |\Omega|$, then $C = \Omega$ and $\partial C \cap \Omega = \emptyset$, so that there is nothing to prove. If $V_0 < |\Omega|$, the result is stated in [15, Theorem 1] (one has to set $\Gamma = \emptyset$ in the notation used there). The idea of the proof is the following: let $E$ be a set of finite perimeter in $\Omega$, $x \in \partial E$, $r > 0$ such that $B_r(x) \subset \Omega$. We define

$$\psi(x, r) := |D\chi_{E}|(B_r(x)) - \inf\{|D\chi_{F}|(B_r(x)) \mid F \Delta E \subset \subset B_r(x)\}$$

The quantity $\psi$ gives a measure of how far the set $E$ is from being a perimeter-minimizing set (without volume constraints). A result of Tamanini ([27, Lemma 3]) states that, if $E$ is a set of finite perimeter with $\psi(x, r) \leq Cr^{n-1+2\alpha}$ for some $x \in \partial E$ and all $0 < r < R$ with given constants $C, R$ and $0 < \alpha < 1$, then the tangent cone to $\partial E$ in $x$, as defined in [14, Theorem 9.3], is area-minimizing. This is what actually happens in this case, since it can be proved (see [16]) that for a set minimizing perimeter under a volume constraint we have

$$\psi(x, r) \leq Cr^{n}$$

for a constant $C > 0$, for each $x \in \partial E$ and for all sufficiently small $r > 0$. The properties of area minimizing tangent cones, which can be found in [14, Chapter

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9], allow us to reason in a way similar to [22] and finally state the claim. The dimension $n - 8$ appearing in the theorem is linked to the following fact: $x \in \partial E$ is a regular point if and only if the tangent cone in $x$ is a half-space. In $\mathbb{R}^n$, $n \leq 7$, the only possible area minimizing tangent cones are half-spaces, while in $\mathbb{R}^8$ there exist nontrivial area minimizing cones such as the so-called Simon’s cone (see [4]).

Another important property of Cheeger sets is the constancy of the mean curvature of $\partial C \cap \Omega$; the result is stated for instance in [13, Theorem 1.22].

**Proposition 4.2.** The mean curvature of $\partial C \cap \Omega$ is constant at every regular point, and equal to $\frac{1}{n-1} \cdot h_1(\Omega)$.

**Proof.** The fact that the mean curvature is constant at every regular point of $\partial C \cap \Omega$ follows from [15, Theorem 2]. To show that it is exactly equal to $h_1(\Omega)$, take a regular point $x_0 \in \partial C \cap \Omega$. Then there exist a ball $B$, an open interval $I$ and a function $f \in C^\infty(B;I)$ such that, if we set $F = B \times I$, then $x_0 \in B$ and $E \cap F$ is the epigraph of $-f$. Take now $g \in C^2_c(B;I)$, and set

$$E_t = (E \setminus F) \cup epi(-(f + tg))$$

where $t \in (-\varepsilon, \varepsilon)$, with $\varepsilon$ so small that $E_t$ is still contained in $\Omega$. As $E$ is a Cheeger set, it follows that the functional

$$I(t) = P(E_t) - h_1(\Omega)|E_t|$$

satisfies $I(0) = 0$, and $I(t) \geq 0$ for $t \in (-\varepsilon, \varepsilon)$. So we have

$$0 \leq I(t) - I(0) = \int_B \sqrt{1 + |D(f + tg)|^2} - h_1(\Omega) \int_B (f + tg)$$

$$- \int_B \sqrt{1 + |Df|^2 + h_1(\Omega)} \int_B f = J(t) - J(0)$$

for every $t \in (-\varepsilon, \varepsilon)$, where

$$J(t) := \int_B \sqrt{1 + |D(f + tg)|^2} - h_1(\Omega) \int_B (f + tg)$$

It follows $J'(0) = 0$, which means, after integrating by parts,

$$- \int_B \text{div} \left( \frac{Df}{\sqrt{1 + |Df|^2}} \right) g = h_1(\Omega) \int_B g$$

and since this relation is valid for every $g \in C^2_c(B;I)$, the theorem is finally proved. $\square$
A Cheeger set enjoys also boundary regularity. More precisely, the following result holds.

**Proposition 4.3.** [15, Theorem 3] Let \( C \) be a Cheeger set for \( \Omega \), and let \( x \in \partial \Omega \) be such that \( \partial \Omega \cap B_r(x) \) is of class \( C^1 \) for some \( r > 0 \). Then there exists a \( \rho \in (0, r) \) such that \( \partial C \cap B_\rho(x) \) is also of class \( C^1 \).

In particular, this implies that \( \partial C \) and \( \partial \Omega \) must meet tangentially at regular points of \( \partial \Omega \).

## 5 Uniqueness and nonuniqueness

A relevant question is whether there can exist more than one Cheeger set for a given domain \( \Omega \). This is not the case if \( \Omega \) is convex. A first result in this direction concerns planar convex domains. Given two sets \( A, B \subset \mathbb{R}^n \), we define

\[
A \oplus B := \{ x \in \mathbb{R}^n \mid x = a + b, \ a \in A, \ b \in B \}.
\]

**Proposition 5.1.** Let \( \Omega \subset \mathbb{R}^2 \) be a convex domain. Then there exists a unique Cheeger set \( C \) for \( \Omega \). Moreover, \( C \) is convex, has boundary of class \( C^{1,1} \), and

\[
C = C_R \oplus B_R,
\]

where

\[
C_R = \{ x \in \Omega \mid \text{dist}(x; \partial \Omega) \leq R \},
\]

\( B_R \) is the disc of radius \( R \), and \( R \) is such that \( |C_R| = \pi R^2 \).

**Proof.** Let \( H_\Omega \) be the union of all discs with largest radius contained in \( \Omega \). If \( C \) is a Cheeger set for \( \Omega \), it follows from [12, Theorem 33] that \( |C| \geq |H_\Omega| \). It is then possible to apply [26, Theorem 3.32] to state the uniqueness and the regularity result. The characterization of \( C \) as union of balls of suitable radius has been established in [19, Theorem 1].

The result was generalized to higher dimensional domains some years later.

**Proposition 5.2.** [1, Theorem 1] Let \( \Omega \subset \mathbb{R}^n \) be a convex domain. Then there exists a unique Cheeger set \( C \) for \( \Omega \). Moreover, \( C \) is convex and has boundary of class \( C^{1,1} \).

In general, if \( n \geq 3 \) it does not hold true that the Cheeger set of a convex domain is the union of balls of suitable radius (see [18, Remark 13]).

If \( \Omega \) is not convex, one can not expect in general uniqueness of the Cheeger set, as shown by simple examples such as the "barbell domain" (see [19]). We observe that the star-shapedness of \( \Omega \) is not a sufficient condition for uniqueness of the Cheeger
set; indeed, there exist L-shaped domains which admit infinitely many Cheeger sets (see [24]). However, an interesting result states that if $\Omega$ is a domain admitting more than one Cheeger set, then it is possible to find a set $\tilde{\Omega}$ arbitrarily close to $\Omega$ and admitting only one Cheeger set. Here is the precise statement.

**Proposition 5.3.** [7, Theorem 1] Let $\Omega \subset \mathbb{R}^n$ be an open set with finite volume. Then, for any compact set $K \subset \Omega$ there exists a bounded open set $\tilde{\Omega}$ such that $K \subset \tilde{\Omega} \subset \Omega$ and $\tilde{\Omega}$ has a unique Cheeger set.

Another property of the class of Cheeger sets is the fact that it is stable under countable union: if $\{C_n\}$ is a sequence of Cheeger sets for $\Omega$, then also $C := \bigcup_n C_n$ is a Cheeger set ([6, Theorem 3]). This allows to define the notion of maximal Cheeger set ([5, Proposition 1.1]), which is a Cheeger set $C$ such that, if $\tilde{C}$ is another Cheeger set, then $\tilde{C} \subset C$. The maximal Cheeger set is always unique. Similarly one can define the notion of minimal Cheeger set ([7, Lemma 2.5]); in this case, there may be more than one minimal Cheeger set, but they are always finitely many.

### 6 Quantitative isoperimetric estimates

A celebrated result of De Giorgi ([10]) states that, if $E$ is a set of finite perimeter in $\mathbb{R}^n$, and $E^*$ is a ball such that $|E^*| = |E|$, then $P(E^*) \leq P(E)$, with equality holding if and only if $E$ is itself a ball. This implies that

$$h_1(\Omega) \geq h_1(\Omega^*).$$
In fact, if $C$ is a Cheeger set for $\Omega$, then $\Omega^*$ contains a ball $C^*$ with the same volume as $C$. Hence,

$$h_1(\Omega) = \frac{P(C)}{|C|} \geq \frac{P(C^*)}{|C^*|} \geq h_1(\Omega^*).$$

The equality sign holds if and only if $\Omega$ is a ball. However, by means of a so-called quantitative isoperimetric inequality, it is possible to say that if the difference $h_1(\Omega) - h_1(\Omega^*)$ is small, then $\Omega$ must be somehow ”near” to be a ball. More precisely, one defines the Fraenkel asymmetry of a set $\Omega$ as

$$A(\Omega) := \inf \left\{ \frac{|\Omega \Delta B|}{|\Omega|} \bigg| B \text{ is a ball with } |B| = |\Omega| \right\}.$$

Observe that $A(\Omega) = 0$ if and only if $\Omega$ is a ball. Then the following result holds.

**Proposition 6.1.** [11] Let $A(\Omega)$ be defined as above. Then,

$$h_1(\Omega) \geq h_1(\Omega^*) \left[ 1 + \frac{A(\Omega)^2}{C} \right],$$

where $C = C(n) > 0$ depends only on the dimension $n$.

### 7 Applications of the Cheeger problem

Besides the well-known Cheeger’s inequality mentioned in the introduction, the Cheeger problem appears in several mathematical contexts. One example is the study of plate failure under stress (see [20]). If $\Omega$ represents the shape of a planar plate subject to a constant uniform pressure $p$, we want to determine the minimal value of $p$ for which the plate breaks down; here we do not consider bending or buckling effects. Let $E \subset \Omega$; the vertical force acting on $E$ will be equal to $p|E|$, while the opposing force exerted on $E$ by the portion of the plate surrounding it can be supposed to have the form $\sigma P(E)$, where $\sigma > 0$ is a constant. Hence, failure will not occur if for every subdomain $E \subset \Omega$ one has

$$p \frac{|E|}{\sigma} \leq \inf_{E \subset \Omega} \frac{P(E)}{|E|} = h_1(\Omega) \Leftrightarrow p \leq \sigma h_1(\Omega).$$

This is equivalent to ask that

$$p \leq \frac{\inf_{E \subset \Omega} P(E)}{|E|} = h_1(\Omega) \Leftrightarrow p \leq \sigma h_1(\Omega).$$

Thus, failure will occur for $p = \sigma h_1(\Omega)$ along a Cheeger set for $\Omega$.

Another application concerns the asymptotic behaviour of the first eigenvalue of the $p$-Laplacian for $p \to 1$, as shown in [18]. Define for $p > 1$

$$\lambda_1(p; \Omega) := \inf_{v \in W_{\alpha,p}(\Omega) \setminus \{0\}} \frac{\int_{\Omega} |\nabla v|^p}{\int_{\Omega} |v|^p}.$$
One can easily show that the infimum is actually attained, and that a minimizer is a weak solution of the equation

$$\begin{cases}
-\Delta_p u &= \lambda |u|^{p-2} u \quad \text{in } \Omega, \\
u &= 0 \quad \text{on } \partial \Omega,
\end{cases}$$

where $\lambda = \lambda_1(p; \Omega)$ and $\Delta_p u = \text{div}(|\nabla u|^{p-2} \nabla u)$ is the $p$-Laplacian. On one hand, it is possible to generalize Cheeger’s inequality to the $p$-Laplacian as follows (see [21, Appendix]):

$$\lambda_1(p; \Omega) \geq \left( \frac{h_1(\Omega)}{p} \right)^p.$$ 

On the other hand, one can show ([18, Corollary 6]) that

$$\limsup_{p \to 1} \lambda_1(p; \Omega) \leq h_1(\Omega),$$

which finally yields

$$\lim_{p \to 1} \lambda_1(p; \Omega) = h_1(\Omega).$$

Moreover, the first eigenfunctions converge in $L^1(\Omega)$ to a minimizer of (3.1), and hence to a function whose level sets are Cheeger sets for $\Omega$. Consequently, if $\Omega$ admits only one Cheeger set $C$, then the first eigenfunctions converge to a suitably scaled characteristic function of $C$.

We also mention the interpretation given by Gilbert Strang in [25] in the context of maximal flow-minimal cut problems. Given a bounded, planar domain $\Omega$, and given two functions $F, c : \Omega \to \mathbb{R}$, we want to find the maximal value of $\lambda \in \mathbb{R}$ such that there exists a vector field $v : \Omega \to \mathbb{R}^2$ satisfying

$$\begin{cases}
\text{div } v &= \lambda F \\
|v| &\leq c.
\end{cases}$$

The problem can be interpreted as follows: given a source or sink term $F$, we want to find the maximal flow in $\Omega$ under the capacity constraint given by $c$. It turns out that if $F \equiv 1$ and $c \equiv 1$, then the maximal value of $\lambda$ is equal to the Cheeger constant of $\Omega$, while the boundary of a Cheeger set is the associated minimal cut. This kind of results have found an interesting application in medical image processing (see [3]).

The Cheeger problem can be extended by considering its weighted version. More precisely, given a function $g \in C^1(\overline{\Omega})$ with $g \geq g_0$ for a constant $g_0 > 0$, one defines the weighted total variation of a function $u \in L^1(\Omega)$:

$$|Du|_g(\Omega) := \sup \left\{ \int_{\Omega} u \text{div}(g\varphi) \left| \varphi \in C^1_c(\Omega; \mathbb{R}^n), \|\varphi\|_{\infty} \leq 1 \right. \right\}.$$ 

Then one tries to find

$$h_{g}^{f}(\Omega) := \inf_{u \in BV_g(\Omega)} \frac{|Du|_g(\mathbb{R}^n)}{\int_{\Omega} fu},$$

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where $f \in L^\infty(\Omega)$ with $f \geq f_0$ for a constant $f_0 > 0$, and $BV_g(\Omega)$ is the space of functions with finite weighted total variation. This problem was introduced in [17] in connection to landslide modelling. Extentions of the Cheeger problem involving anisotropic norms and anisotropic total variation turned out to be useful in image processing (see [8]).

References


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