POSITIVE DEFINITE SOLUTION OF TWO KINDS OF NONLINEAR MATRIX EQUATIONS

Xuefeng Duan, Zhenyun Peng and Fujian Duan

Abstract. Based on the elegant properties of the Thompson metric, we prove that the following two kinds of nonlinear matrix equations $X = m \sum_{i=1}^{m} A_i^* X^\delta_i A_i$ and $X = m \sum_{i=1}^{m} (A_i^* X A_i)^\delta_i$, $(0 < |\delta_i| < 1)$ always have a unique positive definite solution. Iterative methods are proposed to compute the unique positive definite solution. We show that the iterative methods are more effective as $\delta = \max\{|\delta_i|, \ i = 1, 2, \cdots, m\}$ decreases. Perturbation bounds for the unique positive definite solution are derived in the end.

Full text

References


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