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Localization with change of the base space in uniform bundles and sheaves

Localización con cambio del espacio base en campos uniformes y haces

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To Professor Jairo A. Charris in memoriam

Abstract. In this paper a localization (germination) process with change of the base space is presented. The data consist of two topological spaces T and S, a continuous function $\varphi : T \longrightarrow S$, a surjective function $p : E \longrightarrow T$, a directed family $(d_i)_{i\in I}$ of bounded pseudometrics for p generating a Hausdorff uniformity and a family Σ of global selections for p. In terms of these data, a uniform bundle is constructed over the base space S, whose fibers are colimits in a category of uniform spaces. Similar results follow for the case of sheaves of sets. This localization process leads to a universal arrow in a context described in terms of a category of uniform bundles.

Key words and phrases. Uniform bundle, sheaf of sets, localization, colimit.

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Resumen. En este artículo se presenta un proceso de localización (germinación) con cambio del espacio base. El conjunto de datos consta de dos espacios topológicos T y S, una función continua $\varphi : T \longrightarrow S$, una función sobreyectiva $p : E \longrightarrow T$, una familia dirigida $(d_i)_{i\in I}$ de seudométricas acotadas para p que genera una uniformidad de Hausdorff y una familia Σ de selecciones globales para p. En términos de estos datos, se construye un campo uniforme sobre el espacio base S, cuyas fibras son colímites en una categoría de espacios uniformes. Como aplicación inmediata, se obtienen resultados similares para el caso particular de los haces de conjuntos. Este proceso de localización da lugar a una flecha universal en un contexto apropiado que es descrito en términos de una categoría de campos uniformes.

Palabras y frases clave. Campo uniforme, haz de conjuntos, localización, colímite.

1. Introduction

In the theories of sheaves of sets and sheaves of groups one resorts to the classical germination process to construct fibers of a sheaf from data provided by a given presheaf. In [7] K. H. Hofmann presented a localization process to obtain a uniform bundle over a topological space C from a uniform bundle (E, π, B), where B is a subset of C, and from a full set of global selections for π. This method was just slightly different from the process presented in [5], where J. Dauns and K. H. Hofmann considered a surjective function $\varphi : B \longrightarrow C$ instead of the inclusion map. In [2] S. Bautista et al, gave a localization process in the context of bundles of Hausdorff uniform spaces over a fixed topological space T, their procedure led to the construction of a bundle of uniform spaces over T, in terms of a given presheaf of local selections defined on open subsets of T. The fibers of these bundles are colimits in the category of Hausdorff uniform spaces. In this paper, the function $\varphi : T \longrightarrow S$, introduced as a new ingredient in the above localization process, allows to have at our disposal an alternative and eventually better behaved and more interesting base space S, in place of the given space T, that could enjoy some desirable topological properties like compactness. In the present article, the data for the generalized localization process consists of a pair of topological spaces T and S, a continuous function φ from T to S (non necessarily surjective), a surjective function p from a set E onto T, a directed family of bounded pseudometrics for p and a family of global selections for p. In terms of these data a uniform bundle over the base space S is contructed, its fibers are colimits in a category consisting of sets equipped with a family of pseudometrics generating a Hausdorff uniformity and appropriate morphisms between them. In particular, one can start with a sheaf over T and construct a sheaf over S via the continuous function φ, as described in Example 1 below. This localization process leads to a universal arrow in a context described in terms of the category of uniform bundles.

2. Preliminaries

Definitions and results that are required in what follows are given in this sections, all of them are treated in detail in the references [7], [10], [11] and [12].

2.1. **Uniform bundles.** Let E and T be topological spaces and $p : E \longrightarrow T$ be a surjective function. For each $t \in T$, the set $E_t = p^{-1}(t) = \{a \in E : p(a) = t\}$ is called the *fiber above* t. Observe that E is the disjoint union of the family $(E_t)_{t \in T}$. A *local selection* for p is a function $\sigma : Q \longrightarrow E$ such that $Q \subset T$ is an open subset and $p \circ \sigma$ is the identity map id_Q of Q. If $Q = T$, σ is a *global selection.* A *local section* for p is a continuous local selection and a *global section* is a continuous global selection. For each open subset Q of T, let $\Gamma_Q(p) := \{\sigma \mid \sigma : Q \longrightarrow E \text{ is continuous and } p \circ \sigma = id_Q\}$. If $Q = T$, one writes $\Gamma(p)$ instead of $\Gamma_T(p)$. A set Σ of local sections is said to be *full*, if for each $x \in E$, there exists $\sigma \in \Sigma$ such that $\sigma(p(x)) = x$.

Let $E \times_T E := \{(u,v) \in E \times E : p(u) = p(v)\}$. The function $d : E \times_T E \longrightarrow \mathbb{R}$ is called a *pseudometric for* p provided that the restriction of d to $E_t \times E_t$ is a pseudometric on E_t, for each $t \in T$. A family of pseudometrics for p, $(d_i)_{i\in I}$, is *directed* if for each pair $i_1,\ i_2 \in I$, there exists $i \in I$ such that $d_{i_1}(u,v) \leqslant d_i(u,v)$ and $d_{i_2}(u,v) \leqslant d_i(u,v)$, for each $(u,v) \in E \times_T E$.

Definition 1. *Let $(d_i)_{i\in I}$ be a directed family of pseudometrics for p, σ be a local selection, $i \in I$ and $\epsilon > 0$, the set $\mathcal{T}^i_\epsilon(\sigma) = \{u \in E : d_i(u, \sigma(p(u))) < \epsilon\}$ is called* the ϵ-tube around σ with respect to d_i.

Definition 2. *Let E and T be topological spaces, $p : E \longrightarrow T$ be a surjective function and $(d_i)_{i\in I}$ a family of pseudometrics for p. The quadruplet $(E, p, T, (d_i)_{i\in I})$ is said to be a* bundle of uniform spaces *or simply a* uniform bundle *provided that:*

(1) *For each $u \in E$, each $\epsilon > 0$ and each $i \in I$, there exists a local section σ such that $u \in \mathcal{T}^i_\epsilon(\sigma)$.*
(2) *The collection of all tubes around local sections for p form a base for the topology of E.*

The space T is called the base space and the space E is called the fiber space *or the* bundle space.

Observe that if $(E, p, T, (d_i)_{i\in I})$ is a uniform bundle, then the function p is continuous and open.

Definition 2 secures the upper semicontinuity of the distance functions $s \longmapsto d_i(\sigma(s), \tau(s)) : Q \longrightarrow \mathbb{R}^+ \cup \{0\}$, where σ and τ are arbitrary local sections for p, Q is any open subset of $Dom\, \sigma \cap Dom\, \tau$ and $i \in I$.

2.2. **Existence Theorem of Uniform Bundles.** The following result, cf. [12], is an indispensable tool in the constructions that follow.

Theorem 1. *Let T be a topological space and $p : E \longrightarrow T$ be a surjective function. Consider a set Σ of local selections for p and a directed family $(d_i)_{i\in I}$ of pseudometrics for p. Assume the following conditions:*

a) *For each $u \in E$, each $i \in I$ and each $\epsilon > 0$, there exists $\alpha \in \Sigma$ such that $u \in \mathcal{T}^i_\epsilon(\alpha)$.*
b) *The function $s \longmapsto d_i(\alpha(s), \beta(s)) : Dom\, \alpha \cap Dom\, \beta \longrightarrow \mathbb{R}$ is upper semicontinuous, for each $i \in I$ and each $(\alpha, \beta) \in \Sigma \times \Sigma$.*

Then there exists a topology $\mathcal{S}$ over E such that:

1) *The topology $\mathcal{S}$ has a base consisting of the sets of the form $\mathcal{T}^i_\epsilon(\alpha_Q)$, where $i \in I$, $\epsilon > 0$ and α_Q is the restriction to an open subset $Q \subset Dom\, \alpha$ of a local selection $\alpha \in \Sigma$.*
2) *Each $\alpha \in \Sigma$ is a local section.*
3) *$(E, p, T, (d_i)_{i\in I})$ is a uniform bundle.*

3. Localization with change of the base space

Let T and S be topological spaces, $\varphi : T \longrightarrow S$ a continuous function (not necessarily surjective), $p : E \longrightarrow T$ a surjective function and $(d_i)_{i\in I}$ a directed family of bounded pseudometrics for p. Suppose that Σ is a full set of global selections for p. A uniform bundle over S is to be constructed in terms of these data by means of the map φ.

For each $s \in S$, define the relation R_s in the set Σ as follows: for each pair $\sigma,\ \tau \in \Sigma$,

$$\sigma R_s \tau, \text{ if and only if, } \inf_{V\in\mathcal{V}(s)} \sup_{r\in\varphi^{-1}(V)} d_i(\sigma(r),\tau(r)) = 0, \text{ for each } i \in I. \tag{i}$$

Note that if $s \in (S \smallsetminus \varphi(T))^\circ$, there exists a neighbourhood V of s such that $\varphi^{-1}(V) = \emptyset$. If this is the case, by convention, one takes $\sup_{r\in\emptyset} d_i(\sigma(r),\tau(r)) = 0$, for each $i \in I$.

For each $s \in S$, R_s is an equivalence relation on Σ. Since d_i is a pseudometric, then R_s is reflexive and symmetric. The transitivity is also straightforward.

Denote by $[\sigma]_s$ the equivalence class of σ module R_s. To be noted that, for $s \in (S \smallsetminus \varphi(T))^\circ$, the set Σ/R_s reduces to a single point.

Let $\widehat{E}$ be the disjoint union of the family $\{\Sigma/R_s : s \in S\}$ and $\widehat{p}$ be the function

$$\begin{aligned} \widehat{p} : \widehat{E} &\longrightarrow S \\ [\sigma]_s &\longmapsto s. \end{aligned} \tag{ii}$$

For each $i \in I$, define

$$\begin{aligned} \widehat{d_i} : \widehat{E} \times_S \widehat{E} &\longrightarrow \mathbb{R} \\ ([\sigma]_s,[\tau]_s) &\longmapsto \inf_{V\in\mathcal{V}(s)} \sup_{r\in\varphi^{-1}(V)} d_i(\sigma(r),\tau(r)). \end{aligned} \tag{iii}$$

All the $\widehat{d_i}$, $i \in I$, are bounded pseudometrics for $\widehat{p}$, indeed:
to show that the function $\widehat{d_i}$ is well defined, let $\sigma_1 R_s \sigma$, $\tau_1 R_s \tau$,

$$m := \inf_{V\in\mathcal{V}(s)} \sup_{r\in\varphi^{-1}(V)} d_i(\sigma(r),\tau(r))$$

and

$$l := \inf_{V\in\mathcal{V}(s)} \sup_{r\in\varphi^{-1}(V)} d_i(\sigma_1(r),\tau_1(r)).$$

For a given $\epsilon > 0$, there exists a neighbourhood V of s such that

$$\sup_{r\in\varphi^{-1}(V)} d_i(\sigma(r),\sigma_1(r)) < \frac{\epsilon}{3},$$

$$\sup_{r\in\varphi^{-1}(V)} d_i(\tau(r),\tau_1(r)) < \frac{\epsilon}{3}$$

and

$$\sup_{r\in\varphi^{-1}(V)} d_i(\sigma(r),\tau(r)) < m + \frac{\epsilon}{3}.$$

Hence, for each $r \in \varphi^{-1}(V)$,

$$\begin{aligned} d_i(\sigma_1(r), \tau_1(r)) &\leqslant d_i(\sigma_1(r), \tau(r)) + d_i(\tau(r), \tau_1(r)) \\ &\leqslant d_i(\sigma_1(r), \sigma(r)) + d_i(\sigma(r), \tau(r)) + d_i(\tau(r), \tau_1(r)) \\ &< \frac{\epsilon}{3} + m + \frac{\epsilon}{3} + \frac{\epsilon}{3} \\ &= m + \epsilon. \end{aligned}$$

It follows that, for each $\epsilon > 0$, there exists a neighbourhood V of s such that

$$\sup_{r \in \varphi^{-1}(V)} d_i(\sigma_1(r), \tau_1(r)) \leqslant m + \epsilon,$$

thus

$$l = \inf_{V \in \mathcal{V}(s)} \sup_{r \in \varphi^{-1}(V)} d_i(\sigma_1(r), \tau_1(r)) \leqslant m.$$

A similar argument shows that conversely

$$m = \inf_{V \in \mathcal{V}(s)} \sup_{r \in \varphi^{-1}(V)} d_i(\sigma(r), \tau(r)) \leqslant l.$$

The family $\left(\widehat{d_i}\right)_{i \in I}$ of pseudometrics for $\widehat{p}$ is directed, in fact, let $i_1,\ i_2 \in I$, since by hypothesis the family $(d_i)_{i \in I}$ is directed, there exists a $j \in I$ such that $d_{i_1}(a,b),\ d_{i_2}(a,b) \leqslant d_j(a,b)$, for each $(a,b) \in E \times_T E$. It is immediate that each pair $([\sigma]_s, [\tau]_s) \in \widehat{E} \times_S \widehat{E}$ satisfies

$$\widehat{d}_{i_1}([\sigma]_s, [\tau]_s),\ \widehat{d}_{i_2}([\sigma]_s, [\tau]_s) \leqslant \widehat{d}_j([\sigma]_s, [\tau]_s).$$

Define now a set of selections for $\widehat{p}$: for each $\sigma \in \Sigma$, let $\widehat{\sigma} : S \longrightarrow \widehat{E}$ be the function defined by $\widehat{\sigma}(s) = [\sigma]_s$. The set $\widehat{\Sigma} = \{\widehat{\sigma} : \sigma \in \Sigma\}$ is a full set of global selections for $\widehat{p}$. It remains to be seen that $\widehat{p}$, the family $\widehat{\Sigma}$ and the family $\left(\widehat{d_i}\right)_{i \in I}$ satisfy the hypotheses of the Existence Theorem of Uniform Bundles.

a) Let $[\sigma]_s \in \widehat{E}$, $i \in I$ and $\epsilon > 0$. It is immediate that $[\sigma]_s = \widehat{\sigma}(s) \in \mathcal{T}^i_\epsilon(\widehat{\sigma})$.

b) Let $i \in I$, $\left(\widehat{\alpha}, \widehat{\beta}\right) \in \widehat{\Sigma} \times \widehat{\Sigma}$ and $\epsilon > 0$. If $\widehat{d_i}\left(\widehat{\alpha}(s_0), \widehat{\beta}(s_0)\right) < \epsilon$ then we have that $\inf_{V \in \mathcal{V}(s_0)} \sup_{t \in \varphi^{-1}(V)} d_i\left(\alpha(t), \beta(t)\right) < \epsilon$, hence there exists an open neighbourhood V of s_0 such that $\sup_{t \in \varphi^{-1}(V)} d_i(\alpha(t), \beta(t)) < \epsilon$, thus one has that for each $s \in V$, $\widehat{d_i}\left(\widehat{\alpha}(s), \widehat{\beta}(s)\right) < \epsilon$.

It follows that the sets of the form $\mathcal{T}^i_\epsilon\left(\widehat{\sigma}_Q\right)$, where $i \in I$, $\epsilon > 0$, $\widehat{\sigma} \in \widehat{\Sigma}$, Q is an open subset of the domain of $\widehat{\sigma}$ and $\widehat{\sigma}_Q$ is the restriction of $\widehat{\sigma}$ to Q, form a base for a topology on $\widehat{E}$ such that $\left(\widehat{E}, \widehat{p}, S, \left(\widehat{d_i}\right)_{i \in I}\right)$ is a uniform bundle and each element of $\widehat{\Sigma}$ is a section.

On the other hand, for each $i \in I$, the functions $D_i : \Sigma \times \Sigma \longrightarrow \mathbb{R}$ and $\widehat{D}_i : \widehat{\Sigma} \times \widehat{\Sigma} \longrightarrow \mathbb{R}$ defined by $D_i(\sigma, \tau) = \sup_{t \in T} d_i(\sigma(t), \tau(t))$ and $\widehat{D}_i\left(\widehat{\sigma}, \widehat{\tau}\right) = \sup_{s \in S} \widehat{d}_i(\widehat{\sigma}(s), \widehat{\tau}(s))$ are pseudometrics on Σ and $\widehat{\Sigma}$ respectively.

The function

$$\psi : \Sigma \longrightarrow \widehat{\Sigma}, \ \sigma \longmapsto \widehat{\sigma} \tag{iv}$$

is an isometry with respect to D_i and $\widehat{D}_i$. In fact, for each $t \in T$ and each neighbourhood V of $\varphi(t)$ in S, one has that

$$d_i(\sigma(t), \tau(t)) \leqslant \sup_{r \in \varphi^{-1}(V)} d_i(\sigma(r), \tau(r)),$$

then

$$\begin{aligned} d_i(\sigma(t), \tau(t)) &\leqslant \inf_{V \in \mathcal{V}(\varphi(t))} \sup_{r \in \varphi^{-1}(V)} d_i(\sigma(r), \tau(r)) \\ &= \widehat{d}_i\left(\widehat{\sigma}(\varphi(t)), \widehat{\tau}(\varphi(t))\right) \\ &\leqslant \sup_{s \in S} \widehat{d}_i(\widehat{\sigma}(s), \widehat{\tau}(s)), \end{aligned}$$

therefore $\sup_{t \in T} d_i(\sigma(t), \tau(t)) \leqslant \sup_{s \in S} \widehat{d}_i(\widehat{\sigma}(s), \widehat{\tau}(s))$, thus $D_i(\sigma, \tau) \leqslant \widehat{D}_i(\widehat{\sigma}, \widehat{\tau})$. On the other hand, since S is a neighbourhood of s for each $s \in S$ and $\varphi^{-1}(S) = T$, it follows that

$$\inf_{V \in \mathcal{V}(s)} \sup_{r \in \varphi^{-1}(V)} d_i(\sigma(r), \tau(r)) \leqslant \sup_{r \in T} d_i(\sigma(r), \tau(r)) = D_i(\sigma, \tau),$$

for each $s \in S$, thus $\sup_{s \in S} \left(\inf_{V \in \mathcal{V}(s)} \sup_{r \in \varphi^{-1}(V)} d_i(\sigma(r), \tau(r))\right) \leqslant D_i(\sigma, \tau)$. Hence $\widehat{D}_i(\widehat{\sigma}, \widehat{\tau}) \leqslant D_i(\sigma, \tau)$. Then the function ψ satisfies the identity

$$\widehat{D}_i(\psi(\sigma), \psi(\tau)) = D_i(\sigma, \tau), \text{ for each } i \in I.$$

If $\psi(\sigma) = \psi(\tau)$ then $\widehat{D}_i(\widehat{\sigma}, \widehat{\tau}) = 0$, for each $i \in I$, hence

$$D_i(\sigma, \tau) = \sup_{t \in T} d_i(\sigma(t), \tau(t)) = 0,$$

for each $i \in I$, then $d_i(\sigma(t), \tau(t)) = 0$, for each $t \in T$. It follows that, if the family of pseudometrics $(d_i)_{i \in I}$ is Hausdorff, that is, $a = b$ if $d_i(a, b) = 0$, for each $i \in I$, then $\sigma = \tau$. If that is the case, the function ψ is one to one. Summing up, if the uniformity for p determined by the family $(d_i)_{i \in I}$ is Hausdorff (equivalently if each fiber is a Hausdorff space), one can identify Σ with $\widehat{\Sigma}$.

In terms of the definitions given by (i), (ii), (iii) and (iv) above, the following proposition makes precise the preceding arguments, cf. [10][p. 44].

Proposition 1. *Let T and S be two topological spaces, $\varphi : T \longrightarrow S$ be a continuous function, $p : E \longrightarrow T$ be a surjective function, $(d_i)_{i \in I}$ be a directed family of bounded pseudometrics for p generating a Hausdorff uniformity and Σ be a full set of global selections for p.*

(1) *For each $s \in S$, the relation R_s is an equivalence relation.*

(2) $\left(\widehat{d}_i\right)_{i \in I}$ *is a directed family of bounded pseudometrics for $\widehat{p}$.*

(3) *The quadruplet* $\left(\widehat{E}, \widehat{p}, S, \left(\widehat{d_i}\right)_{i\in I}\right)$ *is a uniform bundle whose set of global sections contains an isomorphic copy* $\widehat{\Sigma}$ *of* Σ.

For the particular case $\varphi = id_T$, cf. [10][p. 34].

Example 1. *The case of a sheaf of sets, cf.* [10][p. 48].

Consider a sheaf of sets (E, p, T), here $p : E \longrightarrow T$ is a local homeomorphism. The sheaf with the family of pseudometrics reduced to a single element d that when restricted to each fiber is the discrete metric, can be considered as a uniform bundle.
Suppose that $\varphi : T \longrightarrow S$ is a continuous function and that Σ, the set of all global sections of the sheaf, is full and let $\left(\widehat{E}, \widehat{p}, S, \widehat{d}\right)$ be the uniform bundle obtained by localization by means of φ.
To show that $\left(\widehat{E}, \widehat{p}, S\right)$ is a sheaf of sets it suffices to verify that each fiber is endowed with the discrete metric since in this case, if $[\sigma]_s \in \widehat{E}$ then the restriction of $\widehat{p}$ to $\mathcal{T}_{\frac{1}{2}}(\widehat{\sigma}) = \{\widehat{\sigma}(s) : s \in Dom\,\widehat{\sigma}\}$ is a homeomorphism of $\mathcal{T}_{\frac{1}{2}}(\widehat{\sigma})$ onto S.
Let s be any element of S and consider two different elements $[\sigma]_s$ and $[\tau]_s$ of the fiber $\widehat{E}_s$. Since

$$\widehat{d}\,([\sigma]_s, [\tau]_s) = \inf_{V\in\mathcal{V}(s)} \sup_{r\in\varphi^{-1}(V)} d(\sigma(r), \tau(r)),$$

and $[\sigma]_s \neq [\tau]_s$, then $\inf_{V\in\mathcal{V}(s)} \sup_{r\in\varphi^{-1}(V)} d(\sigma(r), \tau(r)) > 0$, hence

$$\sup_{r\in\varphi^{-1}(V)} d(\sigma(r), \tau(r)) > 0,$$

for each neighbourhood V of s, thus if V is a neighbourhood of s, there exists $r \in \varphi^{-1}(V)$ such that $d(\sigma(r), \tau(r)) = 1$. Then $\sup_{r\in\varphi^{-1}(V)} d(\sigma(r), \tau(r)) = 1$ and it follows that

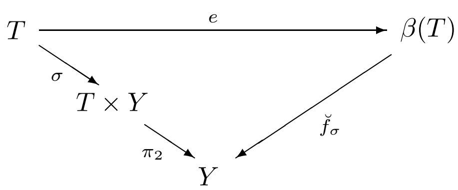
$$\widehat{d}([\sigma]_s, [\tau]_s) = \inf_{V\in\mathcal{V}(s)} \sup_{r\in\varphi^{-1}(V)} d(\sigma(r), \tau(r)) = 1.$$

This shows that the triplet $\left(\widehat{E}, \widehat{p}, S\right)$, obtained by localization by means of φ is also a sheaf of sets.

Example 2. *The case of the Stone-Čech compactification, cf.* [10][p. 47].

Two uniform bundles over the Stone-Čech compactification of a space T that turn out to be essentially the same can be constructed, one resorting to change of the base space and the other one directly.

Let T be a topological space, $\beta(T)$ its Stone-Čech compactification and $e : T \longrightarrow \beta(T)$ its canonical function. Consider a Hausdorff uniform space Y whose uniformity is defined by a directed family of bounded pseudometrics $(d_i)_{i\in I}$, $E = T \times Y$ and let $p : E \longrightarrow T$ be the first projection. The set Λ of all global selections for p can be identified with Y^T, by means of the bijection $f \longmapsto \sigma_f : Y^T \longrightarrow \Lambda$ where $\sigma_f(t) = (t, f(t))$, whose inverse is the function $\sigma \longmapsto f_\sigma : \Lambda \longrightarrow Y^T$, where $f_\sigma(t) = \pi_2(\sigma(t))$. Consider a full set Σ of global selections for p containing only elements of the form σ_f, such that $f : T \longrightarrow Y$ is continuous and $\overline{Im\, f}$ is compact. If $\sigma \in \Sigma$ there exists a unique continuous function $\breve{f}_\sigma$ from $\beta(T)$ to Y such that diagram



commutes.

There are two ways leading to the same uniform bundle with base space $\beta(T)$: first by localization with change of the base space by means of $e : T \longrightarrow \beta(T)$, consider Σ in the set of data and define for each $s \in \beta(T)$ the equivalence relation R_s on Σ by $\sigma_f R_s \sigma_g$, if and only if, $\inf_{V\in\mathcal{V}(s)} \sup_{t\in e^{-1}(V)} d_i(\sigma_f(t), \sigma_g(t)) = 0$, for each $i \in I$. As a second alternative one can obtain the same bundle directly, without resorting to the expedient of change of the base space as follows: define, for each $s \in \beta(T)$, an equivalence relation $\breve{R}_s$ on the set $\breve{\Sigma} = \left\{\breve{f}_\sigma : \sigma \in \Sigma\right\}$ by $\breve{f}_\sigma \breve{R}_s \breve{f}_\tau$, if and only if, for each $i \in I$, we have $\inf_{V\in\mathcal{V}(s)} \sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) = 0$.

For each $s \in \beta(T)$ the corresponding fibers in each one of the resulting bundles are isomorphic, indeed, if $s \in \beta(T)$ the function

$$\psi : \Sigma/R_s \longrightarrow \breve{\Sigma}/\breve{R}_s$$
$$[\sigma_f]_s \longmapsto \left[\breve{f}_\sigma\right]_s$$

is an isometry for the families of pseudometrics defined on those quotients. Indeed, the function ψ is surjective and for each $i \in I$ and each σ_f, τ_f in Σ we have that

$$\inf_{V\in\mathcal{V}(s)} \sup_{t\in e^{-1}(V)} d_i(\sigma_f(t), \tau_f(t)) = \inf_{V\in\mathcal{V}(s)} \sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right).$$

In fact, let

$$m = \inf_{V\in\mathcal{V}(s)} \sup_{t\in e^{-1}(V)} d_i(\sigma_f(t), \tau_f(t)),$$
$$l = \inf_{V\in\mathcal{V}(s)} \sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right)$$

and let $\epsilon > 0$. There exists $V \in \mathcal{V}(s)$ such that $\sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) < l+\epsilon$. If $t \in e^{-1}(V)$ then $e(t) \in V$, thus $d_i\left(\breve{f}_\sigma(e(t)), \breve{f}_\tau(e(t))\right) < l + \epsilon$, therefore

$d_i(\sigma_f(t), \tau_f(t)) < l + \epsilon$, hence $\sup_{t\in e^{-1}(V)} d_i(\sigma_f(t), \tau_f(t)) \leqslant l + \epsilon$. Then

$$\inf_{V\in\mathcal{V}(s)} \sup_{t\in e^{-1}(V)} d_i(\sigma_f(t), \tau_f(t)) \leq l + \epsilon.$$

This implies that $m \leqslant l$.
On the other hand, let again $\epsilon > 0$. There exist $V \in \mathcal{V}(s)$ and $\delta > 0$ such that

$$\sup_{t\in e^{-1}(V)} d_i(\sigma_f(t), \tau_f(t)) < \delta < m + \epsilon.$$

For an indirect argument, suppose that for a $k \in V$, $\gamma = d_i\left(\breve{f}_\sigma(k), \breve{f}_\tau(k)\right) \geq m+\epsilon > \delta$. Let $\alpha = \dfrac{\gamma-\delta}{2}$. Since $\breve{f}_\sigma$ and $\breve{f}_\tau$ are continuous functions, there exists a neighbourhood W of k with $W \subset V$ such that if $q \in W$ then $d_i\left(\breve{f}_\sigma(q), \breve{f}_\sigma(k)\right) < \alpha$ and $d_i\left(\breve{f}_\tau(q), \breve{f}_\tau(k)\right) < \alpha$.
Again by contradiction, if for some $q \in W$, $d_i\left(\breve{f}_\sigma(q), \breve{f}_\tau(q)\right) \leqslant \delta$, we have that

$$d_i\left(\breve{f}_\sigma(q), \breve{f}_\sigma(k)\right) + d_i\left(\breve{f}_\tau(q), \breve{f}_\tau(k)\right) + d_i\left(\breve{f}_\sigma(q), \breve{f}_\tau(q)\right) < \alpha + \alpha + \delta = \gamma,$$

thus $d_i\left(\breve{f}_\sigma(q), \breve{f}_\sigma(k)\right) + d_i\left(\breve{f}_\tau(k), \breve{f}_\sigma(q)\right) < \gamma$, then $d_i\left(\breve{f}_\sigma(k), \breve{f}_\tau(k)\right) < \gamma$ which is a contradiction. Then for each $q \in W$ we have that $d_i\left(\breve{f}_\sigma(q), \breve{f}_\tau(q)\right) > \delta$. Since $\overline{e(T)} = \beta(T)$ there exists $t \in T$ such that $e(t) \in W \subset V$. Then $d_i\left(\breve{f}_\sigma(e(t)), \breve{f}_\tau(e(t))\right) > \delta$, therefore $d_i(\sigma_f(t), \tau_f(t)) > \delta$, which is not possible, then $\sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) \leqslant m+\epsilon$, thus $\inf_{V\in\mathcal{V}(s)} \sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) \leqslant m + \epsilon$ and we have $l \leqslant m$.
Note that $\inf_{V\in\mathcal{V}(s)} \sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) = 0$, for each $i \in I$, implies that for $\epsilon > 0$ there exists a neighbourhood V of s such that $\sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) < \epsilon$. Therefore for each $i \in I$, we have $d_i\left(\breve{f}_\sigma(s), \breve{f}_\tau(s)\right) = 0$ then $\breve{f}_\sigma(s) = \breve{f}_\tau(s)$, since the family $(d_i)_{i\in I}$ is Hausdorff. On the other hand, if $\breve{f}_\sigma(s) = \breve{f}_\tau(s)$, $i \in I$ and $\epsilon > 0$, then, since $\breve{f}_\sigma$ and $\breve{f}_\tau$ are continuous functions, there exists a neighbourhood V of s such that if $r \in V$ then $d_i\left(\breve{f}_\sigma(s), \breve{f}_\sigma(r)\right) < \dfrac{\epsilon}{2}$ and $d_i\left(\breve{f}_\tau(s), \breve{f}_\tau(r)\right) < \dfrac{\epsilon}{2}$. Therefore for each $r \in V$ we have that $d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) < \epsilon$. We conclude that $\sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) \leqslant \epsilon$ and that

$$\inf_{V\in\mathcal{V}(s)} \sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) = 0.$$

It follows that $\inf_{V\in\mathcal{V}(s)} \sup_{r\in V} d_i\left(\breve{f}_\sigma(r), \breve{f}_\tau(r)\right) = 0$ if and only if $\breve{f}_\sigma(s) = \breve{f}_\tau(s)$. This shows that the quotient $\breve{\Sigma}/\breve{R}_s$, and hence also Σ/R_s, coincides with the space Y, for each $s \in \beta(T)$. Therefore these two bundles over the base space $\beta(T)$ are the same.

Next, we want to look at the sections corresponding to functions having a singularity at a point a, and their values in the fiber over that point, in a bundle obtained by localization by change of the base space.

Example 3. *The bundle of continuous functions around a singular point.*

Let S be a topological space, $a \in S$ be any point of S, $T = S \setminus \{a\}$, $E = T \times \mathbb{R}$ and $p : E \longrightarrow T$ the map defined by $p(t,x) = t$. Let d be the pseudometric for p defined by $d((t,x_1),(t,x_2)) = |x_1 - x_2|$.

For each continuous function $f : T \longrightarrow \mathbb{R}$ denote by σ_f the selection for p, defined by $\sigma_f(t) = (t, f(x))$, let $\Sigma = \{\sigma_f \mid f : T \longrightarrow \mathbb{R} \text{ is continuous}\}$ and denote by φ the inclusion map from T into S.

Consider the bundle obtained by localization over S from the given data. For each $z \in S$ define the equivalence relation R_z in Σ by

$$\sigma_f \; R_z \; \sigma_g \quad \text{if and only if} \quad \inf_{V \in \mathcal{V}(z)} \sup_{t \in \varphi^{-1}(V)} d(\sigma_f(t), \sigma_g(t)) = 0;$$

that is, if and only if $\inf_{V \in \mathcal{V}(z)} \sup_{t \in \varphi^{-1}(V)} |f(t) - g(t)| = 0$. Denote by $[\sigma_f]_z$ the equivalent class of σ_f module R_z.

Take $z \in T$, that is $z \neq a$. It is apparent that if $\sigma_f \; R_z \; \sigma_g$, then $f(z) = g(z)$. Conversely, if $f, \; g : T \longrightarrow \mathbb{R}$ are continuous, if $f(z) = g(z)$ and if $\epsilon > 0$, by the continuity of $f - g$ there exists an open neighbourhood V of z in T, such that $|f(t) - g(t)| < \dfrac{\epsilon}{2}$, for each $t \in V$. Taking into account that V is also a neighbourhood of z in S, it follows that $\sup_{t \in \varphi^{-1}(V)} |f(t) - g(t)| < \epsilon$, then $\inf_{V \in \mathcal{V}(z)} \sup_{t \in \varphi^{-1}(V)} |f(t) - g(t)| < \epsilon$ and since ϵ was chosen arbitrarily, one concludes that $\inf_{V \in \mathcal{V}(z)} \sup_{t \in \varphi^{-1}(V)} |f(t) - g(t)| = 0$. That is, $\sigma_f \; R_z \; \sigma_g$. Thus, if $z \neq a$, it turns out that $\sigma_f \; R_z \; \sigma_g$ if and only if $f(z) = g(z)$. Moreover, by using similar arguments, it follows that if $f, \; g : T \longrightarrow \mathbb{R}$ are continuous functions, then $\widehat{d}\,([\sigma_f]_z, [\sigma_g]_z) = |f(z) - g(z)|$. Indeed, if $\epsilon > 0$, there exists an open neighbourhood V of z in T such that $|f(t) - g(t)| < |f(z) - g(z)| + \frac{\epsilon}{2}$ for each $t \in V$. Then $\sup_{t \in \varphi^{-1}(V)} |f(t) - g(t)| < |f(z) - g(z)| + \epsilon$, hence $\inf_{V \in \mathcal{V}(z)} \sup_{t \in \varphi^{-1}(V)} |f(t) - g(t)| < |f(z) - g(z)| + \epsilon$. therefore $\widehat{d}\,([\sigma_f]_z, [\sigma_g]_z) = \inf_{V \in \mathcal{V}(z)} \sup_{t \in \varphi^{-1}(V)} |f(t) - g(t)| = |f(z) - g(z)|$. One can now assert that the map $\psi : \widehat{E}_z = \Sigma / R_z \longrightarrow \mathbb{R}$ defined by $\psi\,([\sigma_f]_z) = f(z)$ is an isometry. In other words, if $z \in T$, then the fiber over z in the bundle $\left(\widehat{E}, \widehat{p}, \mathbb{R}, \widehat{d}\right)$ is $\mathbb{R}$.

Since each continuous function $f : S \setminus \{a\} \longrightarrow \mathbb{R}$ can be identified with the element σ_f of Σ and since σ_f is identified with the section $\widehat{\sigma_f}$ of the bundle $\left(\widehat{E}, \widehat{p}, \mathbb{R}, \widehat{d}\right)$ obtained by localization, it follows that each continuous function defined from $S \setminus \{a\}$ in $\mathbb{R}$ can be extended to a (continuous) section from S to $\widehat{E}$.

In particular if $S = \mathbb{R}$ and $a = 0$, the functions $\sin z$ and $1/z$ can be extended, despite their non removable discontinuity at a, to a (continuous) sections to the domain S that does include the point a. This is an indication of the awesome intricacy of the fiber above the point a.

4. An observation regarding the categorical setting

In this section it is shown that the fibers of the uniform bundle constructed by localization with change of the base space are colimits of direct systems in a certain category of uniform spaces.
The following definitions are required.

Definition 3. *Let Λ be a directed set viewed as a category by means of its preorder. A directed system indexed by Λ in a category $\mathcal{X}$ is a covariant functor F, from Λ to $\mathcal{X}$, that sends each object α of Λ into an object X_α of $\mathcal{X}$, and such that, for $\alpha \leqslant \beta$, there exists a morphism $f_{\alpha\beta} : X_\alpha \longrightarrow X_\beta$ of $\mathcal{X}$ satisfying:*

(1) *For each $\alpha \in \Lambda$, $f_{\alpha\alpha} = 1_{X_\alpha}$.*
(2) *If $\alpha \leqslant \beta \leqslant \gamma$ then $f_{\beta\gamma} \circ f_{\alpha\beta} = f_{\alpha\gamma}$.*

Denote by $((X_\alpha)_{\alpha\in\Lambda}, (f_{\alpha\beta})_{\alpha\leqslant\beta})$ the direct system that corresponds to F.

Definition 4. *Let $((X_\alpha)_{\alpha\in\Lambda}, (f_{\alpha\beta})_{\alpha\leqslant\beta})$ be a direct system in a category $\mathcal{X}$.*

(1) *An inductive cone for this direct system is a pair $(X, (\mathcal{F}_\alpha)_{\alpha\in\Lambda})$ consisting of an object X of $\mathcal{X}$ and a family of morphisms $\mathcal{F}_\alpha : X_\alpha \longrightarrow X$ of $\mathcal{X}$ such that for each $\alpha,\ \beta \in \Lambda$, with $\alpha \leqslant \beta$, $\mathcal{F}_\beta \circ f_{\alpha\beta} = \mathcal{F}_\alpha$.*
(2) *A direct limit or colimit for the direct system is an inductive cone $(X, (\mathcal{F}_\alpha)_{\alpha\in\Lambda})$ satisfying the following universal property: given any other inductive cone $(Y, (\mathcal{G}_\alpha)_{\alpha\in\Lambda})$ in $\mathcal{X}$, for this direct system, there exists a unique morphism $\varphi : X \longrightarrow Y$, such that for every $\alpha \in \Lambda$, $\varphi \circ \mathcal{F}_\alpha = \mathcal{G}_\alpha$.*

Following S. Bautista [1], consider the category $Umet_H$. Its objects are pairs consisting of a set X and a family of bounded pseudometrics $(d_i)_{i\in I}$ defined on X, such that $x_1 = x_2$ if and only if $d_i(x_1, x_2) = 0$, for each $i \in I$, that is, the uniformity associated with the family of pseudometrics is Hausdorff, and a morphism between two given objects $(X, (d_i)_{i\in I})$ and $(Y, (m_j)_{j\in J})$, is a pair of functions (f, l), where $f : X \longrightarrow Y$ and $l : J \longrightarrow I$ satisfy $m_j(f(x_1), f(x_2)) \leqslant d_{l(j)}(x_1, x_2)$, for each $x_1,\ x_2 \in X$ and each $j \in J$. If $(X, (d_i)_{i\in I})$ is an object of $Umet_H$, the identity morphism $id_{(X,(d_i)_{i\in I})}$ is the pair (id_X, id_I), and if $(f, l) : (X, (d_i)_{i\in I}) \longrightarrow (Y, (m_j)_{j\in J})$ and $(f', l') : (Y, (m_j)_{j\in J}) \longrightarrow (Z, (n_k)_{k\in K})$ are morphisms of $Umet_H$, then the composition of (f, l) and (f', l') is given by $(f', l') \circ (f, l) = (f'f, ll')$.
Let T be a topological space, $p : E \longrightarrow T$ be a surjective function, $(d_i)_{i\in I}$ be a directed family of bounded pseudometrics for p, such that the fiber $E_t = \{a \in E : p(a) = t\}$ is a Hausdorff space for each $t \in T$, Σ be a full set of global selection for p and $\varphi : T \longrightarrow S$ be a continuous function.
Take $s \in S$ and for each $V \in \mathcal{V}(s)$ let $\Sigma_V = \left\{\sigma \restriction_{\varphi^{-1}(V)} : \sigma \in \Sigma\right\}$ and $\left(d_i^V\right)_{i\in I}$ be the family of pseudometrics over Σ_V, where for each $i \in I$, $d_i^V : \Sigma_V \times \Sigma_V \longrightarrow \mathbb{R}$

is defined by $d_i^V\left(\sigma\restriction_{\varphi^{-1}(V)},\tau\restriction_{\varphi^{-1}(V)}\right)=\sup_{t\in\varphi^{-1}(V)}d_i(\sigma(t),\tau(t))$, then the pair $\left(\Sigma_V,(d_i^V)_{i\in I}\right)$ is an object of $Umet_H$.
Now consider the order relation $\leqslant$ over the neighbourhood filter of s, given by $V\leqslant W$, if and only if, $W\subset V$. For each $V,\quad W\in\mathcal{V}(s)$ such that $V\leqslant W$, let $f_{VW}:\Sigma_V\longrightarrow\Sigma_W$ and $l_{VW}:I\longrightarrow I$ be the functions defined by $f_{VW}\left(\sigma\restriction_{\varphi^{-1}(V)}\right)=\sigma\restriction_{\varphi^{-1}(W)}$ and $l_{VW}(i)=i$. For each $i\in I$, one has that

$$\begin{aligned}d_i^W\left(\sigma\restriction_{\varphi^{-1}(W)},\tau\restriction_{\varphi^{-1}(W)}\right)&=\sup_{t\in\varphi^{-1}(W)}d_i(\sigma(t),\tau(t))\\&\leqslant\sup_{t\in\varphi^{-1}(V)}d_i(\sigma(t),\tau(t))\\&=d^V_{l_{VW}(i)}\left(\sigma\restriction_{\varphi^{-1}(V)},\tau\restriction_{\varphi^{-1}(V)}\right),\end{aligned}$$

thus the pair (f_{VW},l_{VW}) is a morphism of $Umet_H$.
Then the pair $\left(\left(\Sigma_V,\left(d_i^V\right)_{i\in I}\right)_{V\in\mathcal{V}(s)},(f_{VW},l_{VW})_{W\subset V}\right)$ is a direct system in $Umet_H$.
Let $\left(\widehat{E},\widehat{p},S,\left(\widehat{d}_i\right)_{i\in I}\right)$ be the uniform bundle over the topological space S, constructed by localization from the topological space T, the surjective function $p:E\longrightarrow T$, the directed family of bounded pseudometrics for p, $(d_i)_{i\in I}$ such that $E_t=\{a\in E:p(a)=t\}$ is a Hausdorff space, for each $t\in T$, the set Σ of global selections for p and the continuous function $\varphi:T\longrightarrow S$.
For $s\in S$, the equivalence relation R_s in Σ is defined by, $\sigma R_s\tau$, if and only if, $\inf_{V\in\mathcal{V}(s)}\sup_{t\in\varphi^{-1}(V)}d_i(\sigma(t),\tau(t))=0$, for each $i\in I$, and the fiber $\widehat{E}_s$ is the set Σ/R_s. Consider the family of pseudometrics $\left(\widehat{d}_i\right)_{i\in I}$ on $\widehat{E}_s$, where $\widehat{d}_i\left([\sigma]_s,[\tau]_s\right)=\inf_{V\in\mathcal{V}(s)}\sup_{t\in\varphi^{-1}(V)}d_i(\sigma(t),\tau(t))$, for each $i\in I$. The pair $\left(\widehat{E}_s,\left(\widehat{d}_i\right)_{i\in I}\right)$ is then an object of $Umet_H$.
For each $V\in\mathcal{V}(s)$, let $\hbar_V:\Sigma_V\longrightarrow\widehat{E}_s$ and $\xi_V:I\longrightarrow I$, the functions defined by $\hbar_V(\sigma\restriction_{\varphi^{-1}(V)})=[\sigma]_s$ (class of σ module R_s) and $\xi_V(i)=i$ respectively. It follows that

$$\begin{aligned}\widehat{d}_i\left(\hbar_V\left(\sigma\restriction_{\varphi^{-1}(V)}\right),\hbar_V\left(\tau\restriction_{\varphi^{-1}(V)}\right)\right)&=\widehat{d}_i\left([\sigma]_s,[\tau]_s\right)\\&=\inf_{U\in\mathcal{V}(s)}\sup_{t\in\varphi^{-1}(U)}d_i\left(\sigma(t),\tau(t)\right)\\&\leqslant\sup_{t\in\varphi^{-1}(V)}d_i\left(\sigma(t),\tau(t)\right)\\&=d_i^V\left(\sigma\restriction_{\varphi^{-1}(V)},\tau\restriction_{\varphi^{-1}(V)}\right)\\&=d^V_{\xi_V(i)}\left(\sigma\restriction_{\varphi^{-1}(V)},\tau\restriction_{\varphi^{-1}(V)}\right),\end{aligned}$$

then $(\hbar_V, \xi_V)$ is a morphism of $Umet_H$.
Note that if $V,\ W \in \mathcal{V}(s)$ and $V \leqslant W$, then for each $\sigma \restriction_{\varphi^{-1}(V)} \in \Sigma_V$,

$$\hbar_W \left(f_{VW} \left(\sigma \restriction_{\varphi^{-1}(V)}\right)\right) = \hbar_W \left(\sigma \restriction_{\varphi^{-1}(W)}\right) = [\sigma]_s = \hbar_V \left(\sigma \restriction_{\varphi^{-1}(V)}\right),$$

thus $\hbar_W f_{VW} = \hbar_V$, therefore $\left(\left(\widehat{E}_s, \left(\widehat{d}_i\right)_{i\in I}\right), (\hbar_V, \xi_V)_{V\in\mathcal{V}(s)}\right)$ is an inductive cone for the direct system

$$\left(\left(\Sigma_V, (d_i^V)_{i\in I}\right)_{V\in\mathcal{V}(s)}, (f_{VW}, l_{VW})_{W\subset V}\right).$$

Now suppose that $\left(\left(Y, (m_j)_{j\in J}\right), (\eth_V, \chi_V)_{V\in\mathcal{V}(s)}\right)$ is an other inductive cone for this direct system.
For each $V \in \mathcal{V}(s)$, the functions $\eth_V : \Sigma_V \longrightarrow Y$ and $\chi_V : J \longrightarrow I$ satisfy $m_j \left(\eth_V \left(\sigma \restriction_{\varphi^{-1}(V)}\right), \eth_V \left(\tau \restriction_{\varphi^{-1}(V)}\right)\right) \leqslant d^V_{\chi_V(j)} \left(\sigma \restriction_{\varphi^{-1}(V)}, \tau \restriction_{\varphi^{-1}(V)}\right)$, for each $\sigma \restriction_{\varphi^{-1}(V)},\ \tau \restriction_{\varphi^{-1}(V)} \in \Sigma_V$ and each $j \in J$, and if $V,\ W \in \mathcal{V}(s)$ and $V \leqslant W$, then $\eth_W f_{VW} = \eth_V$ and $l_{VW}\chi_W = \chi_V$. Therefore, for each $V \in \mathcal{V}(s)$ and each $\sigma \in \Sigma$, $\eth_V(f_{SV}(\sigma)) = \eth_S(\sigma)$, that is, $\eth_V \left(\sigma \restriction_{\varphi^{-1}(V)}\right) = \eth_S(\sigma)$. Furthermore, for each $j \in J$, $l_{SV}(\chi_V(j)) = \chi_S(j)$, hence $\chi_V(j) = \chi_S(j)$.
Define $\psi : \widehat{E}_s \longrightarrow Y$ by $\psi([\sigma]_s) = \eth_S(\sigma)$, suppose that $[\sigma]_s = [\tau]_s$ and let $j \in J$ and $\epsilon > 0$. There exists a $V \in \mathcal{V}(s)$ such that $\sup_{t\in\varphi^{-1}(V)} d_{\chi_S(j)}(\sigma(t), \tau(t)) < \epsilon$. Then

$$\begin{aligned} m_j\left(\psi\left([\sigma]_s\right), \psi\left([\tau]_s\right)\right) &= m_j\left(\eth_S(\sigma), \eth_S(\tau)\right) \\ &= m_j\left(\eth_V\left(\sigma \restriction_{\varphi^{-1}(V)}\right), \eth_V\left(\tau \restriction_{\varphi^{-1}(V)}\right)\right) \\ &\leqslant d^V_{\chi(j)}\left(\sigma \restriction_{\varphi^{-1}(V)}, \tau \restriction_{\varphi^{-1}(V)}\right) \\ &< \epsilon, \end{aligned}$$

This means that $m_j(\psi([\sigma]_s), \psi([\tau]_s)) = 0$, for each $j \in J$, thus $\eth_S(\sigma) = \eth_S(\tau)$ since Y is a Hausdorff space. It follows that ψ is well defined. Define $\zeta : J \longrightarrow I$ by $\zeta(j) = \chi_S(j)$. It is immediate that the pair (ψ, ζ) is a morphism of $Umet_H$ and is the only morphism such that for each $V \in \mathcal{V}(s)$, $(\psi, \zeta) \circ (\hbar_V, \xi_V) = (\eth_V, \chi_V)$.

It follows that $\left(\left(\widehat{E}_s, \left(\widehat{d}_i\right)_{i\in I}\right), (\hbar_V, \xi_V)_{V\in\mathcal{V}(s)}\right)$ is the colimit of the direct system $\left(\left(\Sigma_V, (d_i^V)_{i\in I}\right)_{V\in\mathcal{V}(s)}, (f_{VW}, l_{VW})_{W\subset V}\right)$ in $Umet_H$.

In terms of the definitions given above the following proposition summarizes the preceding considerations.

Proposition 2. *Let T be a topological space, $p : E \longrightarrow T$ a surjective function, $(d_i)_{i\in I}$ a directed family of bounded pseudometrics for p, such that the fiber $E_t = \{a \in E : p(a) = t\}$ is a Hausdorff space for each $t \in T$, Σ a full set of global selection for p and $\varphi : T \longrightarrow S$ a continuous function. Consider the* *uniform bundle* $\left(\widehat{E}, \widehat{p}, S, \left(\widehat{d}_i\right)_{i\in I}\right)$ *constructed by localization from the data above and let* $s \in S$*. Then:*

(1) *The pair* $\left(\Sigma_V, \left(d_i^V\right)_{i\in I}\right)$ *is an object of* $Umet_H$*.*

(2) *The pair* (f_{VW}, l_{VW}) *is a morphism of* $Umet_H$ *and the pair*

$$\left(\left(\Sigma_V, \left(d_i^V\right)_{i\in I}\right)_{V\in\mathcal{V}(s)}, (f_{VW}, l_{VW})_{W\subset V}\right)$$

is a direct system in $Umet_H$*.*

(3) *The pair* $(\hbar_V, \xi_V)$ *is a morphism of* $Umet_H$ *and the inductive cone* $\left(\left(\widehat{E}_s, \left(\widehat{d}_i\right)_{i\in I}\right), (\hbar_V, \xi_V)_{V\in\mathcal{V}(s)}\right)$ *is the colimit of the direct system*

$$\left(\left(\Sigma_V, \left(d_i^V\right)_{i\in I}\right)_{V\in\mathcal{V}(s)}, (f_{VW}, l_{VW})_{W\subset V}\right)$$

in $Umet_H$*.*

Remark 1. *From now on, the following set of data is to be considered: a topological space* T*, a surjective function* $p : E \longrightarrow T$*, a directed family* $(d_i)_{i\in I}$ *of bounded pseudometrics for* p *such that the fiber* $E_t = \{a \in E : p(a) = t\}$ *is a Hausdorff space for each* $t \in T$*, a full set* Σ *of global selection for* p *and a continuous function* $\varphi : T \longrightarrow S$*.*

5. A category of presheaves

In order to present a universal arrow associated with the process of localization with change of the base space, the data (provided to construct the uniform bundle) ought to be presented as a presheaf $\mathbb{M}$ over the space S. If $\mathcal{H}$ denotes the functor that to each uniform bundle over S associates the presheaf of its local sections, it will be shown that the uniform bundle $\left(\widehat{E}, \widehat{p}, S, \left(\widehat{d}_i\right)_{i\in I}\right)$ obtained by localization from the data, together with an obvious morphism of presheaves ϕ between $\mathbb{M}$ and $\mathcal{H}\left(\widehat{E}, \widehat{p}, S, \left(\widehat{d}_i\right)_{i\in I}\right)$, establish a universal arrow from $\mathbb{M}$ to the functor $\mathcal{H}$.

This and the following sections outline the categorical context in which we can state without ambiguity the aforementioned universal arrow.

Definition 5. *Let* (X, τ) *be a topological space. A presheaf* $\mathbb{F}$ *of Hausdorff uniform spaces over* X *is a contravariant functor defined on* τ *with values in the category* $Umet_H$ *as follows:*

(1) *To each open set* $U \in \tau$ *it is assigned a set* $F(U)$ *and a family* $(d_i)_{i\in I_U}$ *of bounded pseudometrics generating a Hausdorff uniformity.*

(2) *To each pair of open sets U, $V \in \tau$, with $V \subset U$, it is assigned a pair (f_{UV}, l_{UV}) of functions $f_{UV} : F(U) \longrightarrow F(V)$, $l_{UV} : I_V \longrightarrow I_U$ in such a way that $d_i(f_{UV}(x), f_{UV}(y)) \leq d_{l_{UV}(i)}(x, y)$, for each x, $y \in F(U)$ and each $i \in I_V$.*

Notice that a presheaf is a direct system in $Umet_H$.

Example 4. The data defined in Remark 1 give rise to a presheaf $\mathbb{M}$ of Hausdorff uniform spaces in the following way:

(1) Given an open set U of S, let $M(U) = \Sigma_{\varphi^{-1}(U)}$ and let $\left(d_i^U\right)_{i \in I}$ the family of pseudometrics over $M(U)$, where for each $i \in I$ it holds that

$$d_i^U\left(\sigma_{\restriction \varphi^{-1}(U)}, \tau_{\restriction \varphi^{-1}(U)}\right) = \sup_{t \in \varphi^{-1}(U)} d_i\left(\sigma_{\restriction_{\varphi^{-1}(U)}}(t), \tau_{\restriction_{\varphi^{-1}(U)}}(t)\right).$$

(2) If U and V are open subsets of S and $V \subset U$, define the functions

$$\begin{aligned} m_{UV} : \Sigma_{\varphi^{-1}(U)} &\longrightarrow \Sigma_{\varphi^{-1}(V)} \\ \sigma_{\restriction \varphi^{-1}(U)} &\longmapsto \sigma_{\restriction \varphi^{-1}(V)} \end{aligned}$$

and

$$\begin{aligned} l_{UV}^M : I &\longrightarrow I \\ i &\longmapsto i. \end{aligned}$$

It follows that

$$\begin{aligned} d_i^V(m_{UV}(\sigma_{\restriction \varphi^{-1}(U)}), m_{UV}(\tau_{\restriction \varphi^{-1}(U)})) &= d_i^V\left(\sigma_{\restriction \varphi^{-1}(V)}, \tau_{\restriction \varphi^{-1}(V)}\right) \\ &= \sup_{t \in \varphi^{-1}(V)} d_i\left(\sigma_{\restriction \varphi^{-1}(V)}, \tau_{\restriction \varphi^{-1}(V)}\right) \\ &\leq \sup_{t \in \varphi^{-1}(U)} d_i\left(\sigma_{\restriction \varphi^{-1}(U)}, \tau_{\restriction \varphi^{-1}(U)}\right) \\ &= d_i^U\left(\sigma_{\restriction \varphi^{-1}(U)}, \tau_{\restriction \varphi^{-1}(U)}\right). \end{aligned}$$

Example 5. Let $\left(F, q, S, (m_j)_{j \in J}\right)$ be a uniform bundle where $(m_j)_{j \in J}$ is a directed family of bounded pseudometrics. The presheaf $\mathbb{F}$ of local sections for q is defined as follows:

(1) Given an open set U of S let $F(U) = \Gamma_U$ the set of all local sections for q with domain U and let $\left(d_j^U\right)_{j \in J}$ be the family of pseudometrics over $F(U)$, where for each $j \in J$, $m_j^U(\sigma, \tau) = \sup_{s \in U} m_j(\sigma(s), \tau(s))$.

(2) If U and V are open subsets of S and if $V \subset U$, define

$$\begin{aligned} f_{UV} : \Gamma_U &\longrightarrow \Gamma_V \\ \sigma &\longmapsto \sigma_{\restriction V} \end{aligned}$$

and

$$\begin{aligned} l_{UV}^F : J &\longrightarrow J \\ j &\longmapsto j. \end{aligned}$$

In the particular case of the bundle $\left(\widehat{E}, \widehat{p}, S, \left(\widehat{d_i}\right)_{i\in I}\right)$ obtained by localization, denote by $\widehat{\mathbb{E}}$ the presheaf of local sections for $\widehat{p}$ and by e_{UV} and $l^{\widehat{E}}_{UV}$ the functions f_{UV} and l^{F}_{UV} respectively.

5.1. **The category $\mathcal{P}reh$.** Extending the ideas of S. Bautista [1], consider as objects of the category $\mathcal{P}reh$ all presheaves of Hausdorff uniform spaces over the topological space (S, τ).

Let $\mathbb{F} : \tau \longrightarrow Umet_H$ and $\mathbb{G} : \tau \longrightarrow Umet_H$ be presheaves over S. For $U,\ V \in \tau$, with $V \subset U$, denote by f_{UV} and l_{UV} the functions defining $\mathbb{F}$ and by g_{UV} and h_{UV} those defining $\mathbb{G}$.

A morphism from $\mathbb{F}$ to $\mathbb{G}$ is given by a natural transformation ϕ from $\mathbb{F}$ to $\mathbb{G}$ sending each object $U \in \tau$ into the morphism $(\phi_U, b_U) : F(U) \times J_U \longrightarrow G(U) \times I_U$ in $Umet_H$ in such a way that if $U,\ V \in \tau$ and $V \subset U$, then the diagrams

$$\begin{array}{ccc} F(U) & \xrightarrow{\ \phi_U\ } & G(U) \\ {\scriptstyle f_{UV}}\downarrow & & \downarrow{\scriptstyle g_{UV}} \\ F(V) & \xrightarrow{\ \phi_V\ } & G(V) \end{array}$$

and

$$\begin{array}{ccc} I_V & \xrightarrow{\ b_V\ } & J_V \\ {\scriptstyle l_{UV}}\downarrow & & \downarrow{\scriptstyle h_{UV}} \\ I_U & \xrightarrow{\ b_U\ } & J_U \end{array}$$

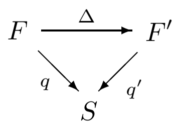
commute.

6. The category of Hausdorff uniform bundles

In this section it is shown that the uniform bundle obtained by localization with change of the base space satisfies a universal property in a category of uniform bundles as it was announced in the first paragraph of Section 5.

The localization process, studied in the previous sections, does not require any hypothesis on the base spaces T and S, but now for the purpose of the present section, consider Hausdorff uniform bundles with Hausdorff base space. Denote by $\mathfrak{U}$ the category of Hausdorff uniform bundles. The objects of $\mathfrak{U}$ are bundles of uniform spaces with base space S, with uniformities determined by directed families of bounded pseudometrics, having a full set of global sections and whose fibers are Hausdorff spaces.

Let $\left(F, q, S, (m_j)_{j\in J}\right)$ and $\left(F', q', S, \left(m'_{j'}\right)_{j'\in J'}\right)$ be two objects of $\mathfrak{U}$. A morphism $(\Delta, \ell) : \left(F, q, S, (m_j)_{j\in J}\right) \longrightarrow \left(F', q', S, \left(m'_{j'}\right)_{j'\in J'}\right)$ in the category $\mathfrak{U}$ is a pair of maps $\Delta : F \longrightarrow F'$, $\ell : J' \longrightarrow J$ such that Δ is continuous, $m'_{j'}(\Delta(a), \Delta(b)) \leq m_{\ell(j')}(a, b)$, for each a, $b \in F$ and each $j' \in J'$ and the diagram



commutes (that is, Δ preserves fibers).

6.1. **The functor $\mathcal{H}$.** Define the functor $\mathcal{H}$ from the category $\mathfrak{U}$ to $\mathcal{P}reh$ as follows:

(1) To each element $\left(F, q, S, (m_j)_{j\in J}\right)$ of $\mathfrak{U}$, $\mathcal{H}$ assigns the presheaf $\mathbb{F}$ of local sections for q.

(2) To each morphism $(\Delta, \ell) : \left(F, q, S, (m_j)_{j\in J}\right) \longrightarrow \left(F', q', S, \left(m'_{j'}\right)_{j'\in J'}\right)$ in $\mathfrak{U}$ assigns the morphism of presheaves given by the natural transformation $\phi^{(\Delta,\ell)}$ from $\mathbb{F}$ to $\mathbb{F}'$ that sends each open set U of S into the morphism of $Umet_H$

$$\phi_U^{(\Delta,\ell)} : \left(\Gamma_U, \left(m_j^U\right)_{j\in J}\right) \longrightarrow \left(\Gamma'_U, \left({m'}_{j'}^{U}\right)_{j'\in J'}\right)$$

defined by the functions

$$\begin{aligned} f_U^{(\Delta,\ell)} : \Gamma_U &\longrightarrow \Gamma'_U \\ \sigma &\longmapsto \sigma', \end{aligned}$$

where $\sigma'(s) = \Delta(\sigma(s))$, for each $s \in S$ and

$$\begin{aligned} l_U^{(\Delta,\ell)} : J' &\longrightarrow J \\ j' &\longmapsto \ell(j'). \end{aligned}$$

Notice that

$$\begin{aligned} {m'}_{j'}^{U}\left(f_U^{(\Delta,\ell)}(\sigma), f_U^{(\Delta,\ell)}(\tau)\right) &= {m'}_{j'}^{U}(\sigma', \tau') \\ &= \sup_{s\in U} m'_{j'}(\sigma'(s), \tau'(s)) \\ &= \sup_{s\in U} m'_{j'}(\Delta(\sigma(s)), \Delta(\tau(s))) \\ &\leq \sup_{s\in U} m_{\ell(j')}(\sigma(s), \tau(s)) \\ &= m^U_{l_U^{(\Delta,\ell)}}(\sigma, \tau). \end{aligned}$$

Furthermore, if U and V are open sets of S and $V \subset U$, then the diagram

$$\begin{array}{ccc} \Gamma_U & \xrightarrow{f_U^{(\Delta,\ell)}} & \Gamma'_U \\ {\scriptstyle f_{UV}}\downarrow & & \downarrow{\scriptstyle f'_{UV}} \\ \Gamma_V & \xrightarrow{f_V^{(\Delta,\ell)}} & \Gamma'_V \end{array}$$

commutes. In fact, if $\sigma \in \Gamma_U$, then

$$\begin{aligned} f'_{UV}\left(f_U^{(\Delta,\ell)}(\sigma)\right) &= f'_{UV}(\sigma') \\ &= \sigma'_{\restriction V} \\ &= \Delta\sigma_{\restriction V} \\ &= f_V^{(\Delta,\ell)}(\sigma_{\restriction V}) \\ &= f_V^{(\Delta,\ell)}(f_{UV}(\sigma)). \end{aligned}$$

6.2. **A universal arrow.** Consider the morphism of presheaves $\phi : \mathbb{M} \longrightarrow \widehat{\mathbb{E}}$ assigning to each open set U in S the morphism of $Umet_H$,

$$\phi_U : \left(\Sigma_{\varphi^{-1}(U)}, \left(d_i^U\right)_{i\in I}\right) \longrightarrow \left(\widehat{\Gamma}_U, \left(\widehat{d}_i^U\right)_{i\in I}\right)$$

defined by the functions

$$\begin{aligned} f_U : \Sigma_{\varphi^{-1}(U)} &\longrightarrow \widehat{\Gamma}_U \\ \sigma_{\restriction\varphi^{-1}(U)} &\longmapsto \widehat{\sigma}_{\restriction U} \end{aligned}$$

and

$$\begin{aligned} l_U : I &\longrightarrow I \\ i &\longmapsto i. \end{aligned}$$

Notice that for each $\sigma,\ \tau \in \Sigma$,

$$\begin{aligned} \widehat{d}_i^U\left(f_U\left(\sigma_{\restriction\varphi^{-1}(U)}\right), f_U\left(\tau_{\restriction\varphi^{-1}(U)}\right)\right) &= \widehat{d}_i^U\left(\widehat{\sigma}_{\restriction U}, \widehat{\tau}_{\restriction U}\right) \\ &= \sup_{s\in U} \widehat{d}_i\left(\widehat{\sigma}_{\restriction U}(s), \widehat{\tau}_{\restriction U}(s)\right) \\ &= \sup_{s\in U}\left(\inf_{V\in\mathcal{V}(s)} \sup_{t\in\varphi^{-1}(V)} d_i(\sigma(t), \tau(t))\right) \\ &\leq \sup_{s\in U}\left(\sup_{t\in\varphi^{-1}(U)} d_i(\sigma(t), \tau(t))\right) \\ &= \sup_{t\in\varphi^{-1}(U)} d_i(\sigma(t), \tau(t)) \\ &= d_i^U\left(\sigma_{\restriction\varphi^{-1}(U)}, \tau_{\restriction\varphi^{-1}(U)}\right), \end{aligned}$$

it follows that ϕ_U is a morphism in $Umet_H$.

It remains to be seen that $<(\widehat{E},\widehat{p},S,(\widehat{d}_i)_{i\in I}),\phi>$ is a universal arrow from $\mathbb{M}$ to the functor $\mathcal{H}$.

Suppose that $\left(F,q,S,(m_j)_{j\in J}\right)$ is an object of $\mathfrak{U}$ and for each open set U of S denote by Γ_U the set of all local sections for q with domain U.

Let $\gamma:\mathbb{M}\longrightarrow\mathbb{F}$ be a morphism of presheaves assigning to each open set U of S the morphism $\gamma_U:\left(\Sigma_{\varphi^{-1}(U)},\left(d_i^U\right)_{i\in I}\right)\longrightarrow\left(\Gamma_U,\left(m_j^U\right)_{j\in J}\right)$ in $Umet_H$ determined by the functions $g_U:\Sigma_{\varphi^{-1}(U)}\longrightarrow\Gamma_U$ and $h_U:J\longrightarrow I$.

Notice that $m_j^U\left(g_U\left(\sigma_{\restriction\varphi^{-1}(U)}\right),g_U\left(\tau_{\restriction\varphi^{-1}(U)}\right)\right)\le d^U_{h_U(j)}\left(\sigma_{\restriction\varphi^{-1}(U)},\tau_{\restriction\varphi^{-1}(U)}\right)$ for each σ, $\tau\in\Sigma$ and each $j\in J$. Furthermore, since γ is a morphism of presheaves, if U and V are open subsets of S and $V\subset U$, then $h_V(j)=h_U(j)$, for each $j\in J$ and $f_{UV}\left(g_U\left(\sigma_{\restriction\varphi^{-1}(U)}\right)\right)=g_V\left(m_{UV}\left(\sigma_{\restriction\varphi^{-1}(U)}\right)\right)$, for each $\sigma\in\Sigma$. That is, $g_U\left(\sigma_{\restriction\varphi^{-1}(U)}\right)_{\restriction V}=g_V\left(\sigma_{\restriction\varphi^{-1}(V)}\right)$, for each $\sigma\in\Sigma$. In particular, $g_S(\sigma)_{\restriction U}=g_U\left(\sigma_{\restriction\varphi^{-1}(U)}\right)$, for each $\sigma\in\Sigma$ and each open subset U of S.

Consider the morphism

$$(\Delta,\ell):\left(\widehat{E},\widehat{p},S,\left(\widehat{d}_i\right)_{i\in I}\right)\longrightarrow\left(F,q,S,(m_j)_{j\in J}\right)$$

defined by $\Delta([\sigma]_s)=g_S(\sigma)(s)$ and $\ell(j)=h_S(j)$.

The function Δ is well defined, in fact: if $[\sigma]_s=[\tau]_s$ then

$$\inf_{V\in\mathcal{V}(s)}\sup_{t\in\varphi^{-1}(V)}d_i(\sigma(t),\tau(t))=0,$$

for each $i\in I$. Let $j\in J$, for each open neighbourhood U of s one can assert that:

$$m_j^U\left(g_U\left(\sigma_{\restriction\varphi^{-1}(U)}\right),g_U\left(\tau_{\restriction\varphi^{-1}(U)}\right)\right)\le d^U_{h_U(j)}\left(\sigma_{\restriction\varphi^{-1}(U)},\tau_{\restriction\varphi^{-1}(U)}\right),$$

then

$$\sup_{r\in U}m_j\left(g_U\left(\sigma_{\restriction\varphi^{-1}(U)}\right)(r),g_U\left(\tau_{\restriction\varphi^{-1}(U)}\right)(r)\right)\le\sup_{t\in\varphi^{-1}(U)}d_{h_U(j)}\left(\sigma(t),\tau(t)\right).$$

In particular,

$$m_j\left(g_U\left(\sigma_{\restriction\varphi^{-1}(U)}\right)(s),g_U\left(\tau_{\restriction\varphi^{-1}(U)}\right)(s)\right)\le\sup_{t\in\varphi^{-1}(U)}d_{h_U(j)}\left(\sigma(t),\tau(t)\right)$$

and since $U\subset S$,

$$m_j\left(g_S(\sigma(s)),g_S(\tau(s))\right)\le\sup_{t\in\varphi^{-1}(U)}d_{h_U(j)}\left(\sigma(t),\tau(t)\right),$$

that is,

$$m_j\left(g_S(\sigma(s)),g_S(\tau(s))\right)\le\sup_{t\in\varphi^{-1}(U)}d_{h_S(j)}\left(\sigma(t),\tau(t)\right),$$

then

$$m_j\left(g_S(\sigma(s)),g_S(\tau(s))\right)\le\inf_{U\in\mathcal{V}(s)}\sup_{t\in\varphi^{-1}(U)}d_{h_S(j)}\left(\sigma(t),\tau(t)\right)=0.$$

This shows that $g_S(\sigma(s))=g_S(\tau(s))$, hence Δ is well defined.

The commutativity of the diagram

$$\begin{array}{ccc} \mathbb{M} & \xrightarrow{\phi} & \widehat{\mathbb{E}} \\ & {\scriptstyle\gamma}\searrow \quad \swarrow{\scriptstyle\phi^{(\Delta,\ell)}} & \\ & \mathbb{F} & \end{array}$$

is secured. Indeed, if U is an open subset of S, $\sigma \in \Sigma$ and $s \in U$, then

$$\begin{aligned} \Delta(\widehat{\sigma}_{\restriction U}(s)) &= \Delta([\sigma]_s) \\ &= g_S(\sigma)_{\restriction u}(s) \\ &= g_U(\sigma_{\restriction \varphi^{-1}(U)})(s). \end{aligned}$$

This means that the diagram

$$\begin{array}{ccc} \Sigma_{\varphi^{-1}}(U) & \xrightarrow{f_U} & \widehat{\Gamma}_U \\ & {\scriptstyle g_U}\searrow \quad \swarrow{\scriptstyle f_U^{(\Delta,\ell)}} & \\ & \Gamma_U & \end{array}$$

commutes.

The commutativity of the diagram

$$\begin{array}{ccc} J & \xrightarrow{h_U} & I \\ & {\scriptstyle l_U^{(\Delta\ell)}}\searrow \quad \swarrow{\scriptstyle l_U} & \\ & I & \end{array}$$

and the uniqueness of γ are immediate.

It follows that $\left\langle \left(\widehat{E}, \widehat{p}, S, \left(\widehat{d}_i\right)_{i\in I} \right), \phi \right\rangle$ is a universal arrow from $\mathbb{M}$ to the functor $\mathcal{H}$.

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