Abstract. We prove a version of Koszul duality and the induced derived equivalence for Adams connected $A_\infty$-algebras that generalizes the classical Beilinson–Ginzburg–Soergel Koszul duality. As an immediate consequence, we give a version of the Bernstein–Gel’fand–Gel’fand correspondence for Adams connected $A_\infty$-algebras.

We give various applications. For example, a connected graded algebra $A$ is Artin–Schelter regular if and only if its Ext-algebra $\text{Ext}_A^*(k, k)$ is Frobenius. This generalizes a result of Smith in the Koszul case. If $A$ is Koszul and if both $A$ and its Koszul dual $A'$ are noetherian satisfying a polynomial identity, then $A$ is Gorenstein if and only if $A'$ is. The last statement implies that a certain Calabi–Yau property is preserved under Koszul duality.