Abstract. This series explores a new notion of T-homotopy equivalence of flows. The new definition involves embeddings of finite bounded posets preserving the bottom and the top elements and the associated cofibrations of flows. In this third part, it is proved that the generalized T-homotopy equivalences preserve the branching and merging homology theories of a flow. These homology theories are of interest in computer science since they detect the nondeterministic branching and merging areas of execution paths in the time flow of a higher-dimensional automaton. The proof is based on Reedy model category techniques.